Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.1-Sine/76-4.1.3.1-a+b-sin-^m-c+d-sin-^n-A+B-sin-

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.44 (356)	0.56 (2)
Mathematica	93.02 (333)	6.98 (25)
Maple	82.68 (296)	17.32 (62)
Giac	80.45 (288)	19.55 (70)
Fricas	76.82 (275)	23.18 (83)
Mupad	49.72 (178)	50.28 (180)
Maxima	37.15 (133)	62.85 (225)
Sympy	28.49 (102)	71.51 (256)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

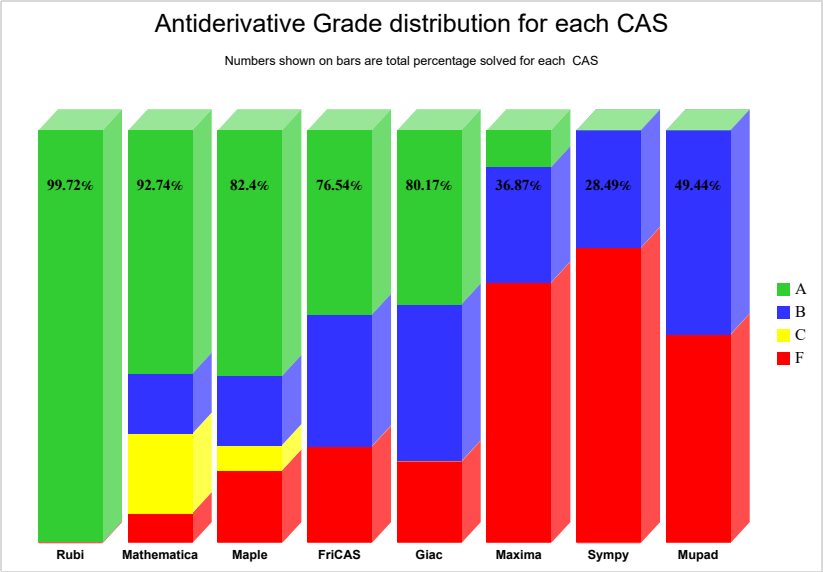
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

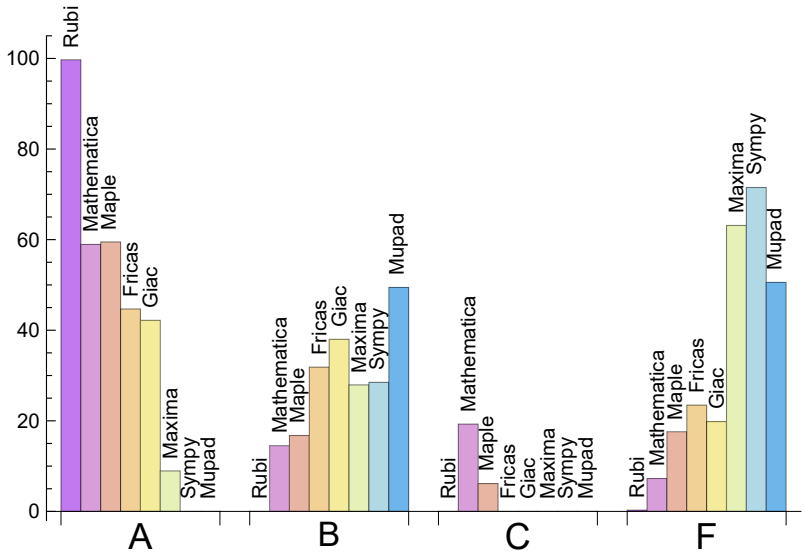
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.162	0.000	0.000	0.838
Maple	59.497	16.760	6.145	17.598
Mathematica	58.939	14.525	19.274	7.263
Fricas	44.693	31.844	0.000	23.464
Giac	42.179	37.989	0.000	19.832
Maxima	8.939	27.933	0.000	63.128
Mupad	0.000	49.441	0.000	50.559
Sympy	0.000	28.492	0.000	71.508

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	25	96.00	4.00	0.00
Maple	62	100.00	0.00	0.00
Giac	70	77.14	12.86	10.00
Fricas	83	98.80	1.20	0.00
Mupad	180	0.00	100.00	0.00
Maxima	225	86.67	4.44	8.89
Sympy	256	41.80	57.81	0.39

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.50
Fricas	0.51
Rubi	0.87
Giac	1.84
Mathematica	6.30
Maple	7.88
Sympy	15.22
Mupad	15.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	177.22	0.99	150.00	1.00
Mathematica	321.07	1.72	204.00	1.33
Fricas	527.80	2.59	234.00	1.77
Giac	578.75	8.93	276.50	1.72
Maxima	782.45	5.48	506.00	4.15
Mupad	833.52	4.61	298.50	2.09
Sympy	2842.08	19.47	1616.50	14.42
Maple	10659.20	15.38	166.50	1.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

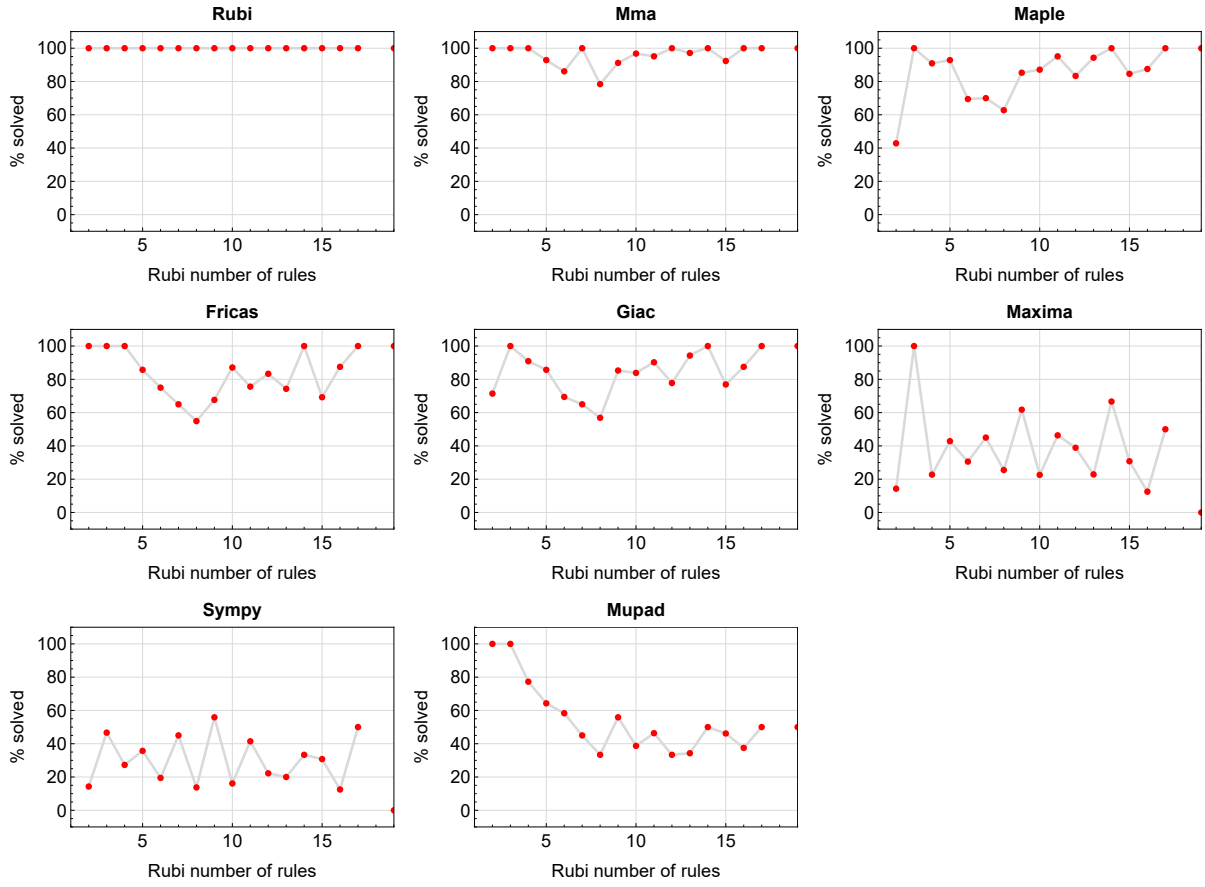


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

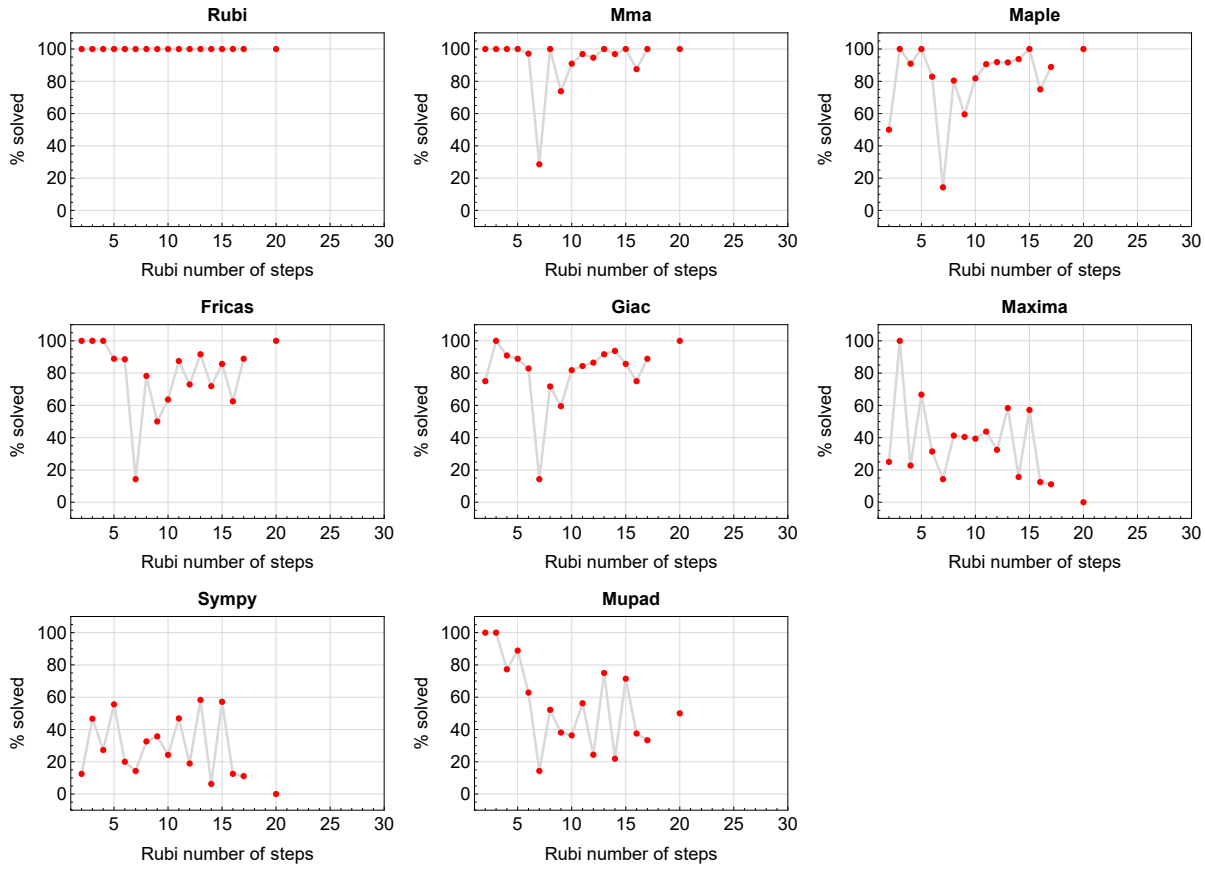


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

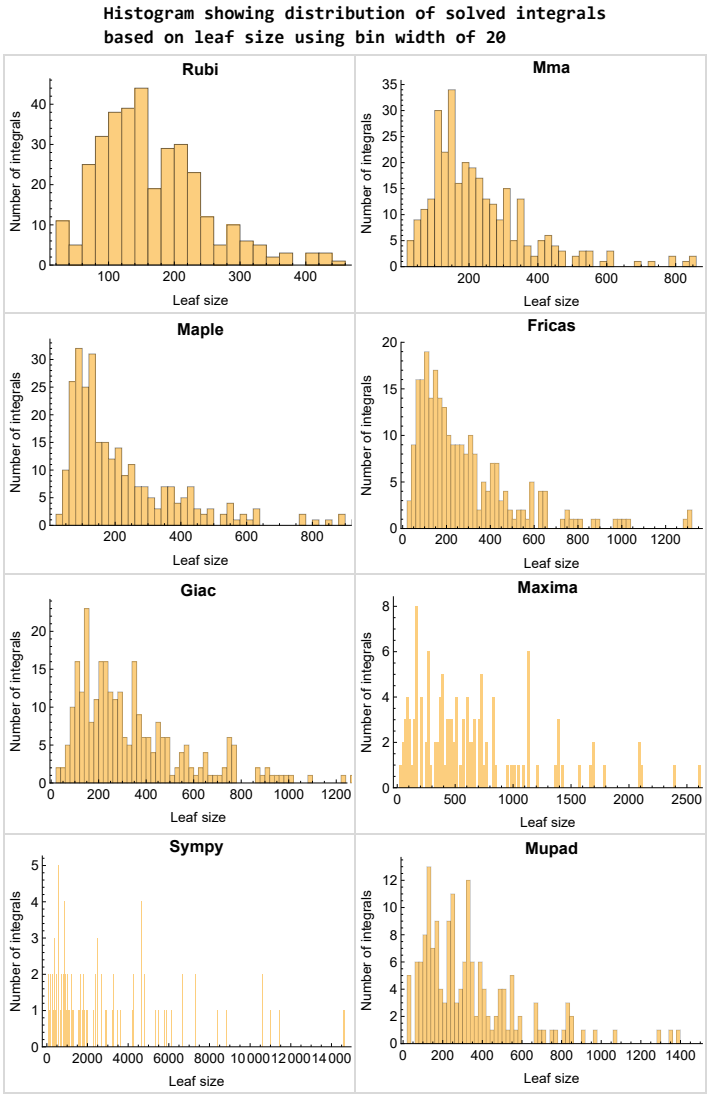


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

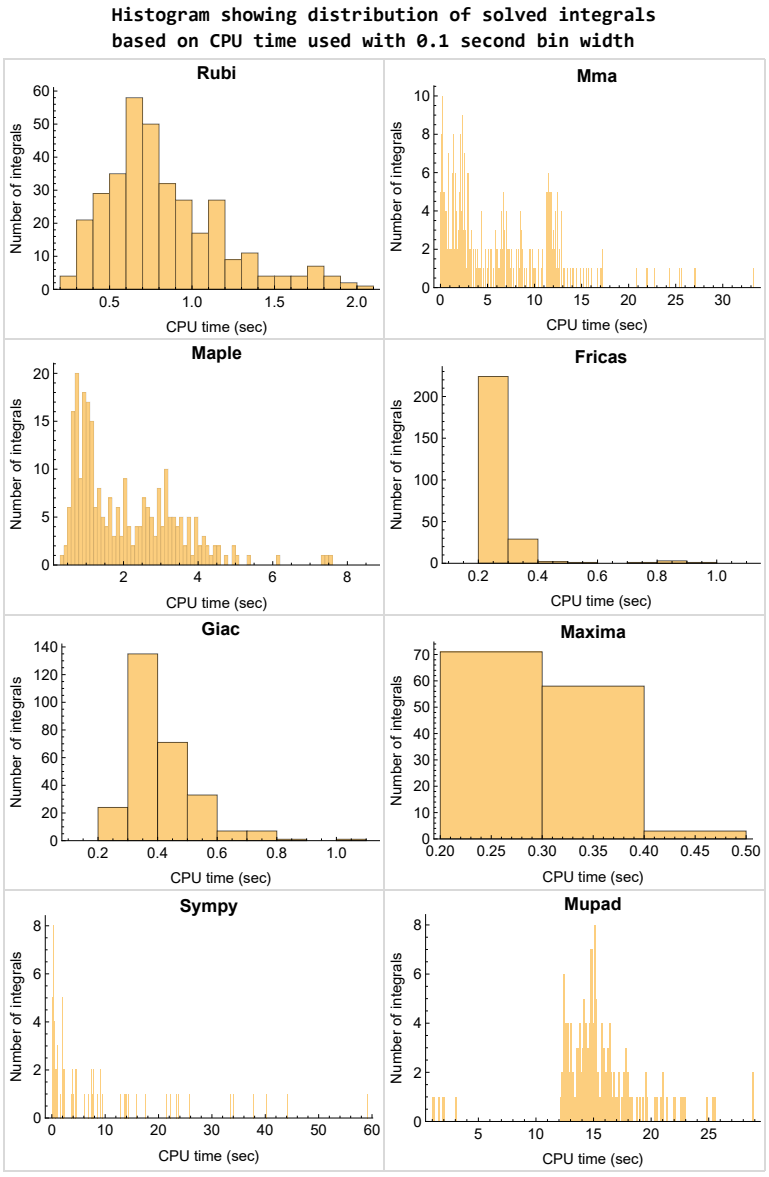


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

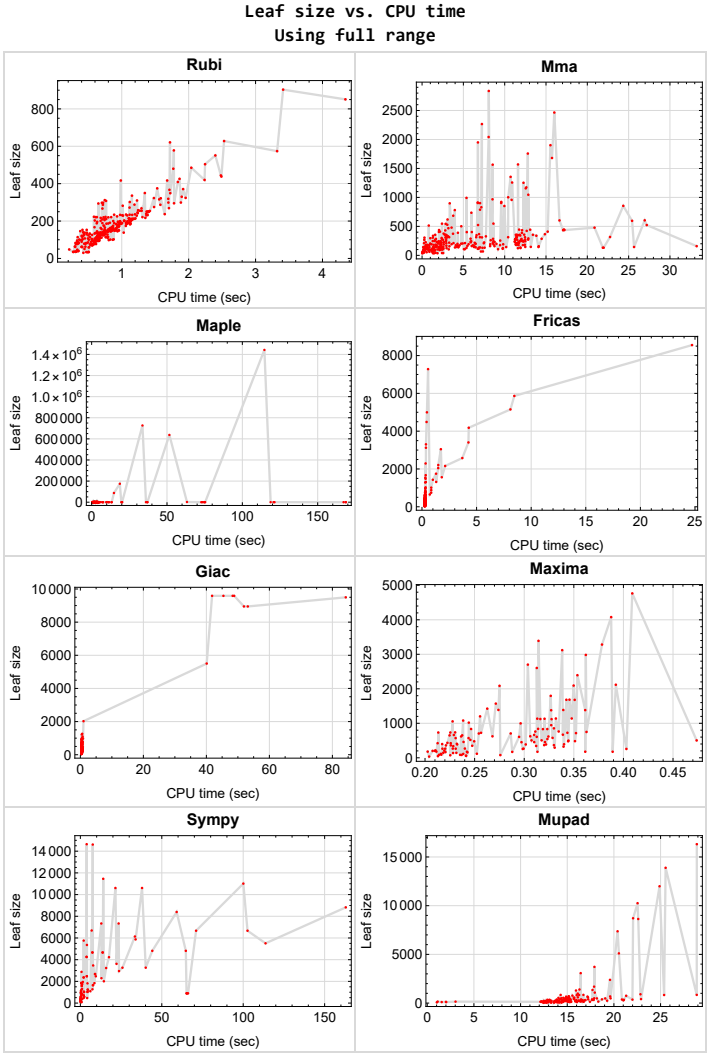


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{358}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{7, 8, 11, 196, 197, 306, 320, 326, 327, 334, 335, 337, 353, 354, 355, 356, 357\}$

Maple $\{353, 354, 355, 356, 357\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

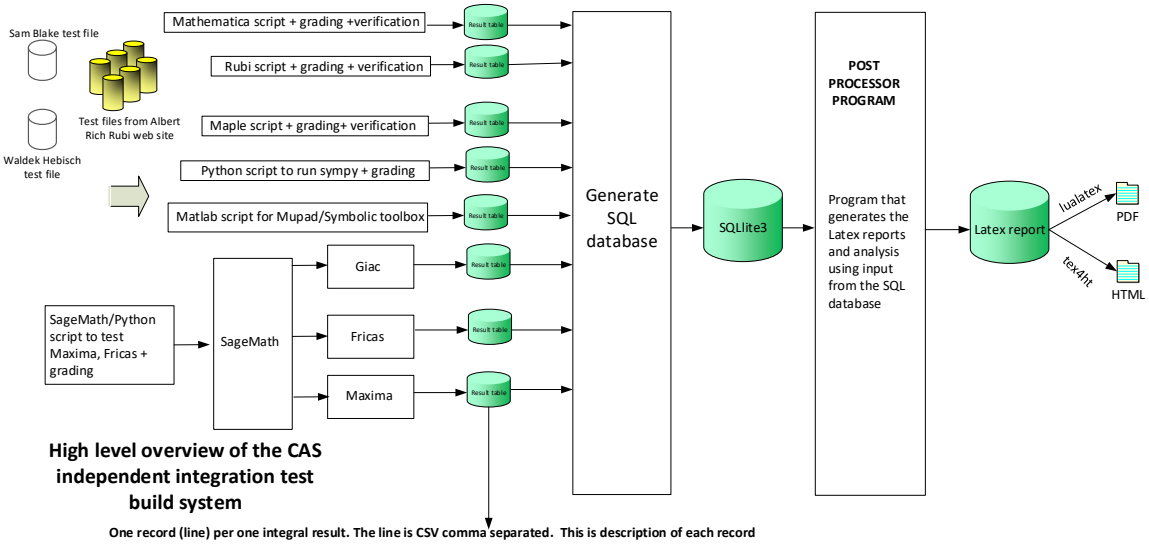
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 21
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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { 307, 313 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 85, 86, 87, 88, 91, 92, 99, 101, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 236, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 301, 302, 303, 332, 334, 350, 351, 352 }

B grade { 8, 11, 14, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 84, 89, 90, 98, 100, 170, 171, 172, 232, 233, 234, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 335, 353, 354, 355, 356, 357 }

C grade { 9, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 196, 197, 198, 199, 214, 215, 235, 237, 248, 249, 250, 290, 291, 292, 297, 298, 299, 300, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 336, 337, 338 }

F normal fail { 12, 13, 195, 200, 201, 202, 216, 217, 328, 329, 330, 331, 333, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { 93 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 16, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 60, 61, 62, 63, 70, 71, 72, 73, 74, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 169, 171, 172, 173, 177, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 193, 194, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 352 }

B grade { 48, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 135, 143, 153, 165, 168, 170, 174, 175, 176, 178, 179, 184, 192, 250, 257, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312,

313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 353, 354, 355, 356, 357 }

C grade { 22, 25, 37, 51, 57, 58, 59, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 79, 80, 237, 239, 282 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351 }

B grade { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 357 }

F(-1) timedout fail { 353 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261 }

B grade { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 128, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 321, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 121, 122, 129, 130, 173, 313, 319, 320, 326, 327 }

F(-2) exception fail { 16, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 352 }

2.1.6 Giac

A grade { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 77, 81, 82, 83, 84, 89, 112, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 147, 148, 152, 153, 155, 156, 159, 163, 164, 166, 167, 168, 169, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 192, 193, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 287, 288, 289, 290, 291, 294, 295, 296, 301, 303, 309 }

B grade { 14, 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 76, 78, 79, 80, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 141, 146, 149, 150, 151, 157, 158, 160, 161, 162, 165, 170, 171, 178, 184, 218, 219, 220, 221, 222, 223, 229, 232,

234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 286, 292, 293, 297, 298, 299, 300, 302, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 10, 11, 203, 204, 208, 334, 335, 350, 351 }

F(-2) exception fail { 154, 177, 185, 194, 209, 210, 316 }

2.1.7 Mupad

A grade { }

B grade { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 218, 219, 220, 221, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 251, 252, 253, 254, 258, 259, 260, 261, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 100, 101, 102, 110, 111, 112, 113, 119, 120, 133, 134, 135, 136, 137, 142, 143, 144, 145, 175, 176, 177, 178, 179, 182, 183, 184, 185, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 203, 204, 207, 208, 209, 211, 212, 213, 214, 216, 217, 222, 223, 228, 229, 230, 231, 240, 241, 242, 243, 286, 287, 288, 289, 293, 294, 295, 296, 302, 303, 307, 308, 309, 310, 315, 316, 317, 324, 333, 334, 335, 336, 337, 338, 343, 344, 345, 353, 354, 355, 356 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 16, 81, 87, 88, 89, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 180, 181, 186, 187, 188, 189, 193, 194, 202, 205, 206, 210, 215, 232, 233, 234, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 300, 301, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 339, 340, 341, 342, 346, 347, 348, 349, 350, 351, 352, 357 }

F(-2) exception fail { 358 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	367	248	0	0	0	0	0	0
N.S.	1	0.98	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.713	1.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	272	204	0	0	0	0	0	0
N.S.	1	0.98	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.109	0.978	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	188	145	0	0	0	0	0	0
N.S.	1	0.98	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	203	159	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.610	0.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	282	220	0	0	0	0	0	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.015	2.578	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	375	288	0	0	0	0	0	0
N.S.	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.550	5.871	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	323	596	0	0	0	0	0	0
N.S.	1	0.96	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.470	25.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	229	225	478	0	0	0	0	0	0
N.S.	1	0.98	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	20.872	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	131	409	0	0	0	0	0	0
N.S.	1	0.96	2.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.512	2.167	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.945	12.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	226	231	523	0	0	0	0	0	0
N.S.	1	1.02	2.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.383	27.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.852	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	9496	61
N.S.	1	1.00	2.89	0.00	0.00	1.11	0.00	256.65	1.65
time (sec)	N/A	0.306	5.069	0.000	0.000	0.288	0.000	84.241	12.925

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	38
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	1.09
time (sec)	N/A	0.280	1.446	0.000	0.000	0.267	0.000	40.100	14.390

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	189	147	212	0	804	0	356	3718
N.S.	1	1.24	0.96	1.39	0.00	5.25	0.00	2.33	24.30
time (sec)	N/A	0.894	1.969	0.903	0.000	0.332	0.000	0.661	17.913

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	159	141	119	336	123	853	178	454
N.S.	1	0.87	0.77	0.65	1.85	0.68	4.69	0.98	2.49
time (sec)	N/A	0.826	1.354	2.016	0.228	0.262	0.442	0.313	15.295

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	124	123	98	200	102	486	140	389
N.S.	1	0.87	0.87	0.69	1.41	0.72	3.42	0.99	2.74
time (sec)	N/A	0.657	1.279	1.497	0.210	0.264	0.299	0.319	14.194

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	86	105	78	179	82	396	110	345
N.S.	1	0.89	1.08	0.80	1.85	0.85	4.08	1.13	3.56
time (sec)	N/A	0.494	0.804	1.261	0.211	0.263	0.204	0.318	13.867

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	47	48	50	73	43	138	55	122
N.S.	1	0.96	0.98	1.02	1.49	0.88	2.82	1.12	2.49
time (sec)	N/A	0.325	0.248	0.742	0.225	0.255	0.125	0.295	14.573

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	107	67	265	116	828	117	111
N.S.	1	1.09	1.91	1.20	4.73	2.07	14.79	2.09	1.98
time (sec)	N/A	0.396	5.167	0.592	0.299	0.266	1.027	0.306	12.744

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	160	80	456	162	700	87	132
N.S.	1	1.07	2.22	1.11	6.33	2.25	9.72	1.21	1.83
time (sec)	N/A	0.572	6.011	0.642	0.321	0.260	2.034	0.321	12.125

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	107	147	94	737	183	1035	131	172
N.S.	1	1.03	1.41	0.90	7.09	1.76	9.95	1.26	1.65
time (sec)	N/A	0.599	6.277	0.750	0.230	0.271	4.342	0.327	12.455

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	143	174	130	1080	251	1831	176	228
N.S.	1	1.01	1.23	0.92	7.61	1.77	12.89	1.24	1.61
time (sec)	N/A	0.736	6.165	0.914	0.238	0.250	8.618	0.359	12.495

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	175	200	166	1425	305	3232	251	310
N.S.	1	0.99	1.14	0.94	8.10	1.73	18.36	1.43	1.76
time (sec)	N/A	0.860	6.468	1.039	0.263	0.256	15.825	0.375	13.045

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	189	219	159	571	158	1586	270	661
N.S.	1	0.83	0.96	0.69	2.49	0.69	6.93	1.18	2.89
time (sec)	N/A	0.988	3.280	3.059	0.231	0.275	0.872	0.381	14.881

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	154	163	137	460	135	1210	237	553
N.S.	1	0.81	0.86	0.72	2.43	0.71	6.40	1.25	2.93
time (sec)	N/A	0.803	5.482	2.321	0.239	0.284	0.616	0.381	15.146

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	116	137	119	360	114	910	202	542
N.S.	1	0.79	0.93	0.81	2.45	0.78	6.19	1.37	3.69
time (sec)	N/A	0.626	7.029	2.126	0.226	0.279	0.441	0.357	14.575

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	77	54	78	164	75	372	113	238
N.S.	1	0.87	0.61	0.88	1.84	0.84	4.18	1.27	2.67
time (sec)	N/A	0.447	0.113	1.165	0.221	0.275	0.278	0.333	14.882

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	83	115	82	179	77	396	107	339
N.S.	1	0.85	1.17	0.84	1.83	0.79	4.04	1.09	3.46
time (sec)	N/A	0.479	0.696	1.186	0.203	0.257	0.201	0.319	13.880

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	108	191	92	624	177	2365	156	244
N.S.	1	0.92	1.63	0.79	5.33	1.51	20.21	1.33	2.09
time (sec)	N/A	0.646	9.819	0.602	0.306	0.274	1.977	0.334	15.250

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	106	238	107	839	237	2474	129	246
N.S.	1	0.97	2.18	0.98	7.70	2.17	22.70	1.18	2.26
time (sec)	N/A	0.663	11.230	0.769	0.314	0.269	3.927	0.320	15.255

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	117	278	126	1139	277	1647	151	233
N.S.	1	1.04	2.48	1.12	10.17	2.47	14.71	1.35	2.08
time (sec)	N/A	0.663	11.275	0.769	0.332	0.260	7.776	0.322	15.853

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	191	133	1571	263	2008	217	269
N.S.	1	1.00	2.55	1.77	20.95	3.51	26.77	2.89	3.59
time (sec)	N/A	0.521	8.251	1.024	0.271	0.260	14.429	0.343	14.132

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	113	261	171	2087	335	3262	285	331
N.S.	1	0.98	2.27	1.49	18.15	2.91	28.37	2.48	2.88
time (sec)	N/A	0.657	8.308	0.987	0.275	0.260	25.838	0.354	15.177

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	149	285	219	2604	407	4816	353	423
N.S.	1	0.96	1.83	1.40	16.69	2.61	30.87	2.26	2.71
time (sec)	N/A	0.792	8.592	1.491	0.313	0.254	44.161	0.356	15.724

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	185	313	248	3120	475	6669	421	500
N.S.	1	0.94	1.59	1.26	15.84	2.41	33.85	2.14	2.54
time (sec)	N/A	0.971	10.419	1.562	0.338	0.273	71.059	0.403	14.894

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	215	255	189	661	181	1948	337	812
N.S.	1	0.81	0.96	0.71	2.49	0.68	7.35	1.27	3.06
time (sec)	N/A	1.132	9.459	4.209	0.229	0.292	1.514	0.376	16.014

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	180	232	164	617	158	1753	292	705
N.S.	1	0.81	1.05	0.74	2.78	0.71	7.90	1.32	3.18
time (sec)	N/A	0.928	8.776	2.642	0.324	0.301	1.143	0.370	15.153

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	142	209	149	571	137	1579	265	661
N.S.	1	0.78	1.15	0.82	3.15	0.76	8.72	1.46	3.65
time (sec)	N/A	0.722	7.467	2.940	0.244	0.278	0.874	0.356	15.200

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	103	64	102	264	92	682	155	325
N.S.	1	0.88	0.55	0.87	2.26	0.79	5.83	1.32	2.78
time (sec)	N/A	0.529	0.166	1.955	0.218	0.266	0.540	0.320	14.899

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	113	133	115	360	106	910	198	536
N.S.	1	0.82	0.96	0.83	2.61	0.77	6.59	1.43	3.88
time (sec)	N/A	0.606	6.886	2.080	0.221	0.273	0.452	0.321	14.056

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	122	141	97	200	100	486	140	390
N.S.	1	0.87	1.01	0.69	1.43	0.71	3.47	1.00	2.79
time (sec)	N/A	0.636	0.894	1.395	0.208	0.263	0.299	0.437	14.337

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	131	223	110	1139	218	4255	223	323
N.S.	1	0.84	1.43	0.71	7.30	1.40	27.28	1.43	2.07
time (sec)	N/A	0.759	11.644	0.655	0.348	0.260	3.789	0.384	14.853

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	148	280	166	1386	286	4665	222	341
N.S.	1	0.91	1.72	1.02	8.50	1.75	28.62	1.36	2.09
time (sec)	N/A	0.800	11.291	0.788	0.339	0.261	7.398	0.435	14.918

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	145	316	157	1685	337	4665	215	336
N.S.	1	0.95	2.07	1.03	11.01	2.20	30.49	1.41	2.20
time (sec)	N/A	0.798	11.412	0.918	0.346	0.262	13.602	0.440	14.757

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	154	356	177	2118	363	2951	202	316
N.S.	1	1.02	2.36	1.17	14.03	2.40	19.54	1.34	2.09
time (sec)	N/A	0.794	11.472	1.033	0.392	0.266	23.735	0.499	16.544

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	283	173	2701	331	3262	285	346
N.S.	1	0.97	3.68	2.25	35.08	4.30	42.36	3.70	4.49
time (sec)	N/A	0.513	12.042	1.164	0.304	0.263	40.133	0.502	14.760

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	113	313	211	3390	405	4816	353	408
N.S.	1	0.96	2.65	1.79	28.73	3.43	40.81	2.99	3.46
time (sec)	N/A	0.654	12.462	1.305	0.314	0.273	64.758	0.758	14.776

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	149	339	250	4078	475	6669	421	500
N.S.	1	0.96	2.17	1.60	26.14	3.04	42.75	2.70	3.21
time (sec)	N/A	0.828	13.834	1.690	0.388	0.266	102.643	0.432	14.357

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	185	365	298	4765	541	8821	489	577
N.S.	1	0.94	1.85	1.51	24.19	2.75	44.78	2.48	2.93
time (sec)	N/A	0.956	14.866	2.005	0.409	0.286	162.970	0.453	14.899

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	171	274	209	1796	261	6690	326	397
N.S.	1	0.90	1.44	1.10	9.45	1.37	35.21	1.72	2.09
time (sec)	N/A	0.942	12.479	0.808	0.327	0.275	6.941	0.323	15.293

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	132	220	110	1120	218	4255	224	319
N.S.	1	0.84	1.40	0.70	7.13	1.39	27.10	1.43	2.03
time (sec)	N/A	0.740	11.649	0.753	0.328	0.277	3.833	0.323	14.872

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	108	188	94	608	179	2365	157	241
N.S.	1	0.92	1.59	0.80	5.15	1.52	20.04	1.33	2.04
time (sec)	N/A	0.652	8.656	0.638	0.313	0.270	2.031	0.309	15.332

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	63	127	67	256	117	828	115	110
N.S.	1	1.11	2.23	1.18	4.49	2.05	14.53	2.02	1.93
time (sec)	N/A	0.395	5.372	0.602	0.403	0.261	1.063	0.463	13.592

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	30	35	42	35	28	83	39	39
N.S.	1	0.86	1.00	1.20	1.00	0.80	2.37	1.11	1.11
time (sec)	N/A	0.418	0.022	0.529	0.204	0.239	0.717	0.333	12.487

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	60	108	86	266	73	578	97	118
N.S.	1	0.95	1.71	1.37	4.22	1.16	9.17	1.54	1.87
time (sec)	N/A	0.513	1.279	0.713	0.228	0.256	2.234	0.351	12.206

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	94	157	111	423	107	1236	167	178
N.S.	1	0.92	1.54	1.09	4.15	1.05	12.12	1.64	1.75
time (sec)	N/A	0.646	1.640	0.997	0.226	0.249	4.554	0.319	12.237

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	128	240	136	619	141	2468	223	239
N.S.	1	0.90	1.69	0.96	4.36	0.99	17.38	1.57	1.68
time (sec)	N/A	0.802	2.022	0.964	0.236	0.253	9.496	0.324	13.141

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	209	354	252	2982	370	10608	391	500
N.S.	1	0.87	1.48	1.05	12.42	1.54	44.20	1.63	2.08
time (sec)	N/A	1.129	11.800	1.368	0.362	0.276	21.523	0.367	15.165

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	166	311	193	2094	322	7337	349	414
N.S.	1	0.92	1.73	1.07	11.63	1.79	40.76	1.94	2.30
time (sec)	N/A	0.936	11.466	1.124	0.350	0.290	12.857	0.331	15.039

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	146	274	162	1378	291	4665	222	336
N.S.	1	0.90	1.69	1.00	8.51	1.80	28.80	1.37	2.07
time (sec)	N/A	0.795	11.303	0.940	0.327	0.268	7.416	0.315	14.789

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	105	234	107	833	242	2474	130	242
N.S.	1	0.97	2.17	0.99	7.71	2.24	22.91	1.20	2.24
time (sec)	N/A	0.619	11.225	0.739	0.323	0.260	3.979	0.329	14.634

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	156	81	452	166	702	87	133
N.S.	1	1.08	2.17	1.12	6.28	2.31	9.75	1.21	1.85
time (sec)	N/A	0.563	6.303	0.687	0.297	0.263	2.094	0.340	12.677

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	59	110	86	265	69	578	97	117
N.S.	1	0.95	1.77	1.39	4.27	1.11	9.32	1.56	1.89
time (sec)	N/A	0.500	1.266	0.664	0.220	0.249	2.225	0.333	12.714

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	51	53	70	47	41	469	82	82
N.S.	1	0.82	0.85	1.13	0.76	0.66	7.56	1.32	1.32
time (sec)	N/A	0.434	0.087	0.720	0.213	0.251	2.043	0.325	12.461

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	77	237	136	651	116	2674	221	183
N.S.	1	0.83	2.55	1.46	7.00	1.25	28.75	2.38	1.97
time (sec)	N/A	0.536	2.003	1.235	0.238	0.253	9.027	0.341	12.537

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	113	285	161	835	151	4228	277	197
N.S.	1	0.84	2.11	1.19	6.19	1.12	31.32	2.05	1.46
time (sec)	N/A	0.679	2.395	1.780	0.246	0.255	17.577	0.462	12.666

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	149	329	184	998	187	5868	333	337
N.S.	1	0.85	1.88	1.05	5.70	1.07	33.53	1.90	1.93
time (sec)	N/A	0.833	3.147	2.380	0.296	0.253	33.910	0.529	13.105

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	206	388	240	3282	429	10608	357	501
N.S.	1	0.85	1.60	0.99	13.51	1.77	43.65	1.47	2.06
time (sec)	N/A	1.103	12.081	1.412	0.378	0.273	37.877	0.507	15.073

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	184	348	201	2394	392	7337	290	419
N.S.	1	0.92	1.73	1.00	11.91	1.95	36.50	1.44	2.08
time (sec)	N/A	0.965	11.630	1.102	0.354	0.269	23.489	0.509	15.723

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	143	308	157	1679	338	4665	215	333
N.S.	1	0.93	2.01	1.03	10.97	2.21	30.49	1.41	2.18
time (sec)	N/A	0.778	11.419	1.007	0.351	0.269	13.774	0.449	15.367

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	272	127	1134	279	1647	151	230
N.S.	1	1.05	2.47	1.15	10.31	2.54	14.97	1.37	2.09
time (sec)	N/A	0.637	11.252	0.783	0.313	0.280	7.716	0.437	16.202

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	106	139	92	733	191	1035	130	172
N.S.	1	1.03	1.35	0.89	7.12	1.85	10.05	1.26	1.67
time (sec)	N/A	0.591	6.473	0.762	0.213	0.261	4.419	0.655	13.632

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	94	156	111	423	106	1236	165	178
N.S.	1	0.92	1.53	1.09	4.15	1.04	12.12	1.62	1.75
time (sec)	N/A	0.647	1.684	0.933	0.213	0.259	4.536	0.328	12.985

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	76	237	136	650	108	2674	221	183
N.S.	1	0.84	2.63	1.51	7.22	1.20	29.71	2.46	2.03
time (sec)	N/A	0.527	1.991	1.174	0.225	0.268	9.003	0.466	13.083

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	63	65	83	60	56	1098	126	126
N.S.	1	0.75	0.77	0.99	0.71	0.67	13.07	1.50	1.50
time (sec)	N/A	0.442	0.138	1.166	0.241	0.266	6.104	0.353	15.425

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	89	325	186	1019	150	6135	333	217
N.S.	1	0.74	2.69	1.54	8.42	1.24	50.70	2.75	1.79
time (sec)	N/A	0.539	2.893	2.396	0.244	0.261	33.492	0.378	13.098

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	125	373	211	1201	185	8396	389	231
N.S.	1	0.77	2.30	1.30	7.41	1.14	51.83	2.40	1.43
time (sec)	N/A	0.683	3.746	3.192	0.255	0.261	59.132	0.366	13.689

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	161	401	236	1387	221	11011	445	474
N.S.	1	0.79	1.96	1.15	6.77	1.08	53.71	2.17	2.31
time (sec)	N/A	0.846	5.412	4.121	0.274	0.280	100.070	0.382	14.671

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	185	149	119	0	287	0	296	0
N.S.	1	0.93	0.75	0.60	0.00	1.45	0.00	1.49	0.00
time (sec)	N/A	1.033	4.164	7.458	0.000	0.273	0.000	0.457	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	148	123	103	0	243	0	262	0
N.S.	1	0.94	0.78	0.66	0.00	1.55	0.00	1.67	0.00
time (sec)	N/A	0.876	2.966	7.356	0.000	0.257	0.000	0.495	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	111	104	81	0	184	0	194	0
N.S.	1	0.96	0.90	0.70	0.00	1.59	0.00	1.67	0.00
time (sec)	N/A	0.680	2.451	1.601	0.000	0.272	0.000	0.374	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	71	191	63	0	130	0	118	0
N.S.	1	0.97	2.62	0.86	0.00	1.78	0.00	1.62	0.00
time (sec)	N/A	0.528	1.153	1.698	0.000	0.254	0.000	0.383	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	129	166	159	0	254	0	300	0
N.S.	1	1.06	1.36	1.30	0.00	2.08	0.00	2.46	0.00
time (sec)	N/A	0.746	2.941	1.879	0.000	0.281	0.000	0.330	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	123	157	227	0	318	0	390	0
N.S.	1	1.07	1.37	1.97	0.00	2.77	0.00	3.39	0.00
time (sec)	N/A	0.703	3.312	1.687	0.000	0.268	0.000	0.362	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	135	199	267	0	394	0	419	0
N.S.	1	1.07	1.58	2.12	0.00	3.13	0.00	3.33	0.00
time (sec)	N/A	0.731	4.284	2.642	0.000	0.273	0.000	0.379	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	168	217	352	0	490	0	628	0
N.S.	1	1.03	1.33	2.16	0.00	3.01	0.00	3.85	0.00
time (sec)	N/A	0.843	6.370	2.974	0.000	0.282	0.000	0.448	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	189	1355	121	0	358	0	373	0
N.S.	1	0.90	6.45	0.58	0.00	1.70	0.00	1.78	0.00
time (sec)	N/A	1.087	10.716	36.968	0.000	0.272	0.000	0.526	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	152	1173	105	0	313	0	340	0
N.S.	1	0.91	7.02	0.63	0.00	1.87	0.00	2.04	0.00
time (sec)	N/A	0.902	12.620	36.990	0.000	0.279	0.000	0.620	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	113	106	83	0	228	0	238	0
N.S.	1	0.94	0.88	0.69	0.00	1.90	0.00	1.98	0.00
time (sec)	N/A	0.754	9.196	2.052	0.000	0.264	0.000	0.651	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	77	89	65	0	193	0	200	0
N.S.	1	0.95	1.10	0.80	0.00	2.38	0.00	2.47	0.00
time (sec)	N/A	0.606	0.822	1.367	0.000	0.269	0.000	0.540	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F	B	F	B	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	161	160	0	197	0	310	0	480	0
N.S.	1	0.99	0.00	1.22	0.00	1.93	0.00	2.98	0.00
time (sec)	N/A	0.881	0.000	2.360	0.000	0.285	0.000	0.478	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	172	355	282	0	385	0	566	0
N.S.	1	0.98	2.02	1.60	0.00	2.19	0.00	3.22	0.00
time (sec)	N/A	0.940	11.348	2.579	0.000	0.275	0.000	0.520	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	169	344	386	0	449	0	601	0
N.S.	1	0.97	1.97	2.21	0.00	2.57	0.00	3.43	0.00
time (sec)	N/A	0.932	11.511	3.296	0.000	0.283	0.000	0.675	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	171	342	354	0	521	0	658	0
N.S.	1	0.98	1.95	2.02	0.00	2.98	0.00	3.76	0.00
time (sec)	N/A	0.948	11.883	3.256	0.000	0.271	0.000	0.738	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	214	357	440	0	654	0	745	0
N.S.	1	0.96	1.61	1.98	0.00	2.95	0.00	3.36	0.00
time (sec)	N/A	1.076	12.355	3.725	0.000	0.287	0.000	0.538	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	189	1569	121	0	405	0	458	0
N.S.	1	0.90	7.47	0.58	0.00	1.93	0.00	2.18	0.00
time (sec)	N/A	1.085	11.611	167.160	0.000	0.280	0.000	0.561	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	150	143	105	0	334	0	372	0
N.S.	1	0.93	0.89	0.65	0.00	2.07	0.00	2.31	0.00
time (sec)	N/A	0.905	10.180	168.774	0.000	0.275	0.000	0.591	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	115	1157	83	0	287	0	294	0
N.S.	1	0.93	9.33	0.67	0.00	2.31	0.00	2.37	0.00
time (sec)	N/A	0.792	12.547	2.448	0.000	0.277	0.000	0.488	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	77	89	65	0	232	0	253	0
N.S.	1	0.95	1.10	0.80	0.00	2.86	0.00	3.12	0.00
time (sec)	N/A	0.592	1.127	2.439	0.000	0.268	0.000	0.443	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	199	193	233	0	353	0	647	0
N.S.	1	1.00	0.96	1.16	0.00	1.76	0.00	3.24	0.00
time (sec)	N/A	1.079	11.554	2.968	0.000	0.279	0.000	0.389	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	211	444	354	0	430	0	730	0
N.S.	1	0.97	2.04	1.62	0.00	1.97	0.00	3.35	0.00
time (sec)	N/A	1.106	11.747	3.461	0.000	0.289	0.000	0.429	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	210	434	434	0	505	0	778	0
N.S.	1	0.93	1.93	1.93	0.00	2.24	0.00	3.46	0.00
time (sec)	N/A	1.117	12.106	3.984	0.000	0.285	0.000	0.449	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	210	422	524	0	554	0	719	0
N.S.	1	0.97	1.94	2.41	0.00	2.55	0.00	3.31	0.00
time (sec)	N/A	1.138	12.664	4.362	0.000	0.287	0.000	0.522	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	212	355	432	0	633	0	928	0
N.S.	1	0.98	1.64	1.99	0.00	2.92	0.00	4.28	0.00
time (sec)	N/A	1.130	13.414	4.518	0.000	0.296	0.000	0.492	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	255	409	526	0	760	0	990	0
N.S.	1	0.96	1.54	1.98	0.00	2.86	0.00	3.72	0.00
time (sec)	N/A	1.290	15.203	5.053	0.000	0.290	0.000	0.562	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	182	157	111	478	115	0	777	0
N.S.	1	0.91	0.78	0.56	2.39	0.58	0.00	3.88	0.00
time (sec)	N/A	0.943	14.124	3.194	0.344	0.275	0.000	0.515	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	147	134	95	386	95	0	573	0
N.S.	1	0.92	0.84	0.60	2.43	0.60	0.00	3.60	0.00
time (sec)	N/A	0.808	11.898	3.150	0.322	0.254	0.000	0.443	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	112	113	73	294	67	0	353	0
N.S.	1	0.95	0.96	0.62	2.49	0.57	0.00	2.99	0.00
time (sec)	N/A	0.694	9.755	0.826	0.331	0.262	0.000	0.420	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	174	44	0	175	128
N.S.	1	1.00	0.60	0.73	2.38	0.60	0.00	2.40	1.75
time (sec)	N/A	0.567	1.632	0.648	0.313	0.246	0.000	0.351	15.139

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	89	140	130	0	162	0	148	0
N.S.	1	0.98	1.54	1.43	0.00	1.78	0.00	1.63	0.00
time (sec)	N/A	0.570	1.353	0.875	0.000	0.281	0.000	0.348	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	130	284	225	0	231	0	405	0
N.S.	1	0.96	2.09	1.65	0.00	1.70	0.00	2.98	0.00
time (sec)	N/A	0.718	1.682	0.832	0.000	0.264	0.000	0.358	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	164	404	350	0	282	0	504	0
N.S.	1	0.91	2.24	1.94	0.00	1.57	0.00	2.80	0.00
time (sec)	N/A	0.877	2.038	1.014	0.000	0.270	0.000	0.618	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	217	176	143	762	153	0	978	0
N.S.	1	0.90	0.73	0.59	3.15	0.63	0.00	4.04	0.00
time (sec)	N/A	1.354	12.204	20.203	0.335	0.272	0.000	0.836	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	182	159	121	670	133	0	774	0
N.S.	1	0.91	0.79	0.60	3.33	0.66	0.00	3.85	0.00
time (sec)	N/A	1.134	8.236	20.181	0.339	0.263	0.000	0.747	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	145	130	105	577	110	0	293	0
N.S.	1	0.94	0.84	0.68	3.75	0.71	0.00	1.90	0.00
time (sec)	N/A	0.952	7.308	19.675	0.329	0.273	0.000	0.611	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	110	113	81	482	80	0	283	492
N.S.	1	0.96	0.98	0.70	4.19	0.70	0.00	2.46	4.28
time (sec)	N/A	0.760	6.740	0.796	0.351	0.251	0.000	0.545	17.888

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	87	63	343	60	0	223	137
N.S.	1	1.01	1.12	0.81	4.40	0.77	0.00	2.86	1.76
time (sec)	N/A	0.607	1.445	0.667	0.312	0.269	0.000	0.736	17.432

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	125	176	168	0	217	0	259	0
N.S.	1	0.93	1.30	1.24	0.00	1.61	0.00	1.92	0.00
time (sec)	N/A	0.708	1.764	0.855	0.000	0.280	0.000	0.410	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	164	300	280	0	206	0	499	0
N.S.	1	0.94	1.71	1.60	0.00	1.18	0.00	2.85	0.00
time (sec)	N/A	0.861	2.367	0.918	0.000	0.271	0.000	0.446	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	202	430	426	0	279	0	750	0
N.S.	1	0.90	1.91	1.89	0.00	1.24	0.00	3.33	0.00
time (sec)	N/A	1.030	2.807	0.918	0.000	0.279	0.000	0.574	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	221	176	143	945	166	0	395	0
N.S.	1	0.91	0.73	0.59	3.90	0.69	0.00	1.63	0.00
time (sec)	N/A	1.310	9.439	120.732	0.327	0.282	0.000	0.556	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	186	158	121	854	148	0	774	0
N.S.	1	0.89	0.76	0.58	4.09	0.71	0.00	3.70	0.00
time (sec)	N/A	1.127	8.766	121.349	0.330	0.265	0.000	0.560	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	149	132	105	761	125	0	449	904
N.S.	1	0.93	0.82	0.66	4.76	0.78	0.00	2.81	5.65
time (sec)	N/A	0.947	7.568	118.777	0.311	0.264	0.000	0.511	22.845

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	116	113	83	663	95	0	373	683
N.S.	1	0.96	0.93	0.69	5.48	0.79	0.00	3.08	5.64
time (sec)	N/A	0.789	6.828	0.687	0.310	0.262	0.000	0.449	19.599

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	83	89	65	505	77	0	368	479
N.S.	1	0.98	1.05	0.76	5.94	0.91	0.00	4.33	5.64
time (sec)	N/A	0.611	2.143	0.643	0.307	0.265	0.000	0.413	18.177

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	161	204	200	0	262	0	450	0
N.S.	1	0.93	1.17	1.15	0.00	1.51	0.00	2.59	0.00
time (sec)	N/A	0.907	2.003	0.781	0.000	0.272	0.000	0.456	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	202	357	308	0	277	0	650	0
N.S.	1	0.90	1.59	1.38	0.00	1.24	0.00	2.90	0.00
time (sec)	N/A	1.043	2.860	1.094	0.000	0.282	0.000	0.464	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	236	479	410	0	238	0	901	0
N.S.	1	0.91	1.86	1.59	0.00	0.92	0.00	3.49	0.00
time (sec)	N/A	1.227	3.571	0.990	0.000	0.288	0.000	0.561	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	118	131	0	140	0	150	173
N.S.	1	1.00	1.26	1.39	0.00	1.49	0.00	1.60	1.84
time (sec)	N/A	0.621	2.390	3.704	0.000	0.273	0.000	0.369	16.389

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	102	95	0	121	0	150	149
N.S.	1	1.00	1.09	1.01	0.00	1.29	0.00	1.60	1.59
time (sec)	N/A	0.600	2.106	3.587	0.000	0.273	0.000	0.380	15.011

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	71	0	92	0	144	122
N.S.	1	1.00	0.89	0.76	0.00	0.98	0.00	1.53	1.30
time (sec)	N/A	0.598	1.551	3.129	0.000	0.283	0.000	0.385	1.999

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	57	49	0	61	0	140	75
N.S.	1	1.00	0.62	0.53	0.00	0.66	0.00	1.52	0.82
time (sec)	N/A	0.576	0.096	2.884	0.000	0.266	0.000	0.363	1.030

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	120	300	175	0	0	141	0
N.S.	1	1.01	1.20	3.00	1.75	0.00	0.00	1.41	0.00
time (sec)	N/A	0.826	2.364	2.526	0.389	0.000	0.000	0.377	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	100	147	139	0	0	0	112	0
N.S.	1	1.01	1.48	1.40	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.854	2.570	2.761	0.000	0.000	0.000	0.375	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	101	71	0	87	0	112	0
N.S.	1	1.00	1.10	0.77	0.00	0.95	0.00	1.22	0.00
time (sec)	N/A	0.594	2.384	3.413	0.000	0.276	0.000	0.450	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	103	103	0	106	0	112	153
N.S.	1	1.00	1.10	1.10	0.00	1.13	0.00	1.19	1.63
time (sec)	N/A	0.612	3.226	3.555	0.000	0.266	0.000	0.433	18.328

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	144	205	135	0	148	0	247	323
N.S.	1	0.99	1.40	0.92	0.00	1.01	0.00	1.69	2.21
time (sec)	N/A	0.761	4.913	3.311	0.000	0.294	0.000	0.500	17.672

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	144	172	132	0	126	0	247	174
N.S.	1	0.99	1.18	0.90	0.00	0.86	0.00	1.69	1.19
time (sec)	N/A	0.747	4.539	3.198	0.000	0.284	0.000	0.497	16.597

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	135	99	80	0	83	0	237	103
N.S.	1	1.01	0.74	0.60	0.00	0.62	0.00	1.77	0.77
time (sec)	N/A	0.710	1.358	2.838	0.000	0.270	0.000	0.497	2.030

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	81	71	0	87	0	144	122
N.S.	1	1.00	0.84	0.74	0.00	0.91	0.00	1.50	1.27
time (sec)	N/A	0.590	1.394	3.026	0.000	0.267	0.000	0.431	14.136

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	424	0	0	0	243	0
N.S.	1	1.00	0.94	2.92	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.831	3.985	2.665	0.000	0.000	0.000	0.745	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	155	210	237	366	0	0	225	0
N.S.	1	0.98	1.33	1.50	2.32	0.00	0.00	1.42	0.00
time (sec)	N/A	0.854	8.556	2.676	0.330	0.000	0.000	0.398	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	150	198	216	0	0	0	205	0
N.S.	1	1.01	1.33	1.45	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.892	9.000	2.941	0.000	0.000	0.000	0.405	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	125	104	0	125	0	183	0
N.S.	1	1.00	1.30	1.08	0.00	1.30	0.00	1.91	0.00
time (sec)	N/A	0.556	9.781	3.348	0.000	0.270	0.000	0.415	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	123	127	0	134	0	183	245
N.S.	1	1.00	0.84	0.87	0.00	0.92	0.00	1.25	1.68
time (sec)	N/A	0.744	11.599	3.974	0.000	0.272	0.000	0.419	19.273

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	152	126	188	0	155	0	183	279
N.S.	1	0.99	0.82	1.22	0.00	1.01	0.00	1.19	1.81
time (sec)	N/A	0.770	11.913	4.242	0.000	0.282	0.000	0.407	21.015

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	193	223	200	0	160	0	355	383
N.S.	1	0.97	1.13	1.01	0.00	0.81	0.00	1.79	1.93
time (sec)	N/A	0.952	7.513	73.458	0.000	0.308	0.000	0.492	18.721

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	184	116	112	0	117	0	355	131
N.S.	1	1.02	0.64	0.62	0.00	0.65	0.00	1.97	0.73
time (sec)	N/A	0.949	2.561	72.761	0.000	0.274	0.000	0.436	16.763

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	165	132	0	119	0	341	174
N.S.	1	1.00	1.16	0.93	0.00	0.84	0.00	2.40	1.23
time (sec)	N/A	0.717	4.371	3.614	0.000	0.291	0.000	0.437	16.439

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	102	95	0	117	0	150	149
N.S.	1	1.00	1.06	0.99	0.00	1.22	0.00	1.56	1.55
time (sec)	N/A	0.642	1.861	3.399	0.000	0.274	0.000	0.427	3.030

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	192	177	493	0	0	0	303	0
N.S.	1	0.99	0.92	2.55	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	1.071	11.680	3.761	0.000	0.000	0.000	0.467	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	200	231	262	0	0	0	0	0
N.S.	1	0.95	1.10	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.129	11.768	3.568	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	202	207	368	506	0	0	297	0
N.S.	1	0.95	0.98	1.74	2.39	0.00	0.00	1.40	0.00
time (sec)	N/A	1.104	11.412	3.536	0.318	0.000	0.000	0.419	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	200	204	320	0	0	0	276	0
N.S.	1	1.02	1.04	1.63	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	1.105	11.384	4.024	0.000	0.000	0.000	0.421	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	145	150	0	165	0	268	0
N.S.	1	1.00	1.51	1.56	0.00	1.72	0.00	2.79	0.00
time (sec)	N/A	0.562	12.490	3.760	0.000	0.271	0.000	0.402	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	190	0	182	0	268	341
N.S.	1	1.00	1.00	1.30	0.00	1.25	0.00	1.84	2.34
time (sec)	N/A	0.765	13.159	3.471	0.000	0.275	0.000	0.439	21.066

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	144	217	0	196	0	268	357
N.S.	1	1.00	0.73	1.11	0.00	1.00	0.00	1.37	1.82
time (sec)	N/A	0.969	14.085	4.175	0.000	0.304	0.000	0.523	21.986

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	242	269	253	0	183	0	453	482
N.S.	1	0.97	1.08	1.01	0.00	0.73	0.00	1.81	1.93
time (sec)	N/A	1.237	11.512	74.961	0.000	0.303	0.000	0.549	19.642

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	233	138	140	0	134	0	453	384
N.S.	1	1.03	0.61	0.62	0.00	0.59	0.00	2.00	1.70
time (sec)	N/A	1.178	4.392	74.577	0.000	0.289	0.000	0.542	17.249

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	191	232	200	0	150	0	453	383
N.S.	1	0.99	1.21	1.04	0.00	0.78	0.00	2.36	1.99
time (sec)	N/A	0.953	5.897	75.322	0.000	0.295	0.000	0.563	17.662

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	212	135	0	142	0	247	321
N.S.	1	1.00	1.49	0.95	0.00	1.00	0.00	1.74	2.26
time (sec)	N/A	0.749	5.193	3.974	0.000	0.280	0.000	0.520	17.835

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	121	131	0	139	0	150	173
N.S.	1	1.00	1.26	1.36	0.00	1.45	0.00	1.56	1.80
time (sec)	N/A	0.676	2.107	2.992	0.000	0.278	0.000	0.431	15.871

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	183	549	0	0	0	469	0
N.S.	1	1.00	0.77	2.30	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	1.370	12.437	3.302	0.000	0.000	0.000	0.785	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	251	292	293	0	0	0	392	0
N.S.	1	0.93	1.08	1.08	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	1.392	12.864	3.754	0.000	0.000	0.000	0.464	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	249	251	406	0	0	0	413	0
N.S.	1	0.95	0.95	1.54	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	1.372	12.245	3.846	0.000	0.000	0.000	0.434	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	254	244	545	749	0	0	359	0
N.S.	1	0.96	0.92	2.06	2.84	0.00	0.00	1.36	0.00
time (sec)	N/A	1.414	12.492	4.484	0.363	0.000	0.000	0.457	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	250	238	409	0	0	0	338	0
N.S.	1	1.01	0.96	1.66	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	1.392	12.285	4.477	0.000	0.000	0.000	0.431	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	434	190	0	199	0	341	0
N.S.	1	1.00	4.52	1.98	0.00	2.07	0.00	3.55	0.00
time (sec)	N/A	0.562	17.068	4.772	0.000	0.297	0.000	0.418	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	442	206	0	214	0	341	406
N.S.	1	1.00	3.03	1.41	0.00	1.47	0.00	2.34	2.78
time (sec)	N/A	0.766	17.127	4.937	0.000	0.289	0.000	0.550	22.906

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	198	442	283	0	234	0	341	827
N.S.	1	0.98	2.19	1.40	0.00	1.16	0.00	1.69	4.09
time (sec)	N/A	0.991	17.202	4.968	0.000	0.300	0.000	0.506	25.378

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	436	309	0	243	0	341	841
N.S.	1	1.00	1.77	1.26	0.00	0.99	0.00	1.39	3.42
time (sec)	N/A	1.262	17.215	5.352	0.000	0.307	0.000	0.537	28.876

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	192	185	495	0	0	0	302	0
N.S.	1	0.97	0.94	2.51	0.00	0.00	0.00	1.53	0.00
time (sec)	N/A	1.101	11.561	4.173	0.000	0.000	0.000	0.462	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	145	146	426	0	0	0	247	0
N.S.	1	0.99	1.00	2.92	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.860	6.421	4.076	0.000	0.000	0.000	0.378	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	98	119	306	176	0	0	143	0
N.S.	1	1.02	1.24	3.19	1.83	0.00	0.00	1.49	0.00
time (sec)	N/A	0.815	2.144	3.391	0.362	0.000	0.000	0.385	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	116	83	131	0	0	0	0	0
N.S.	1	1.03	0.73	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.751	0.860	3.136	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	191	465	0	337	0	204	0
N.S.	1	1.00	1.85	4.51	0.00	3.27	0.00	1.98	0.00
time (sec)	N/A	0.640	2.334	3.121	0.000	0.293	0.000	0.357	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	222	773	0	424	0	229	0
N.S.	1	1.00	1.45	5.05	0.00	2.77	0.00	1.50	0.00
time (sec)	N/A	0.851	2.251	3.637	0.000	0.315	0.000	0.372	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	251	271	293	0	0	0	394	0
N.S.	1	0.93	1.00	1.08	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	1.371	12.839	2.967	0.000	0.000	0.000	0.397	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	200	212	262	0	0	0	253	0
N.S.	1	0.95	1.01	1.25	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	1.132	11.711	3.234	0.000	0.000	0.000	0.363	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	155	190	238	367	0	0	226	0
N.S.	1	0.97	1.19	1.50	2.31	0.00	0.00	1.42	0.00
time (sec)	N/A	0.877	11.327	2.781	0.320	0.000	0.000	0.359	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	102	143	141	0	0	0	114	0
N.S.	1	1.02	1.43	1.41	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.880	2.366	2.536	0.000	0.000	0.000	0.354	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	186	460	0	329	0	183	0
N.S.	1	1.00	1.81	4.47	0.00	3.19	0.00	1.78	0.00
time (sec)	N/A	0.624	2.211	2.685	0.000	0.287	0.000	0.389	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	148	142	111	0	272	0	0	0
N.S.	1	0.99	0.95	0.74	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.837	1.755	2.787	0.000	0.335	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	207	306	361	0	437	0	281	0
N.S.	1	0.95	1.41	1.66	0.00	2.01	0.00	1.29	0.00
time (sec)	N/A	1.128	2.829	3.135	0.000	0.329	0.000	0.419	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	299	286	445	0	0	0	453	0
N.S.	1	0.93	0.89	1.38	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	1.717	14.471	3.908	0.000	0.000	0.000	0.456	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	250	243	407	0	0	0	414	0
N.S.	1	0.95	0.92	1.55	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	1.365	12.192	3.134	0.000	0.000	0.000	0.457	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	203	199	368	504	0	0	298	0
N.S.	1	0.96	0.94	1.74	2.39	0.00	0.00	1.41	0.00
time (sec)	N/A	1.136	11.414	3.069	0.474	0.000	0.000	0.479	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	152	179	214	0	0	0	206	0
N.S.	1	1.02	1.20	1.44	0.00	0.00	0.00	1.38	0.00
time (sec)	N/A	0.865	11.379	3.248	0.000	0.000	0.000	0.456	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	99	70	0	85	0	111	156
N.S.	1	1.00	1.05	0.74	0.00	0.90	0.00	1.18	1.66
time (sec)	N/A	0.620	2.136	2.547	0.000	0.272	0.000	0.466	15.092

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	214	772	0	408	0	211	0
N.S.	1	1.01	1.42	5.11	0.00	2.70	0.00	1.40	0.00
time (sec)	N/A	0.865	2.265	2.938	0.000	0.329	0.000	0.514	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	202	305	361	0	419	0	276	0
N.S.	1	0.97	1.47	1.74	0.00	2.01	0.00	1.33	0.00
time (sec)	N/A	1.086	2.986	2.717	0.000	0.311	0.000	0.503	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	248	192	136	0	306	0	0	0
N.S.	1	1.01	0.78	0.56	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	1.361	3.577	3.035	0.000	0.327	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	174	211	0	0	0	0	0	0	0
N.S.	1	1.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	145	137	604	0	0	0	0	0	0
N.S.	1	0.94	4.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.652	26.946	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	145	137	443	0	0	0	0	0	0
N.S.	1	0.94	3.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	13.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	133	468	0	0	0	0	0	0
N.S.	1	0.96	3.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	3.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.304	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	123	122	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	138	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	148	138	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.652	21.883	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.653	21.974	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	231	271	0	725	562	0	0	749
N.S.	1	0.84	0.99	0.00	2.64	2.04	0.00	0.00	2.72
time (sec)	N/A	0.972	11.975	0.000	0.344	0.309	0.000	0.000	21.314

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	174	174	0	498	313	0	0	480
N.S.	1	1.05	1.05	0.00	3.00	1.89	0.00	0.00	2.89
time (sec)	N/A	0.754	4.090	0.000	0.345	0.299	0.000	0.000	19.073

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	323	165	0	0	105
N.S.	1	1.00	1.12	0.00	3.11	1.59	0.00	0.00	1.01
time (sec)	N/A	0.531	1.391	0.000	0.340	0.267	0.000	0.000	1.549

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.632	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	139	144	0	0	0	0	0	0
N.S.	1	1.04	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	25.657	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	139	159	0	0	0	0	0	0
N.S.	1	1.04	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.705	33.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	233	183	0	0	205	0	0	368
N.S.	1	0.87	0.69	0.00	0.00	0.77	0.00	0.00	1.38
time (sec)	N/A	0.966	0.608	0.000	0.000	0.277	0.000	0.000	20.859

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	176	141	0	0	137	0	0	239
N.S.	1	0.92	0.74	0.00	0.00	0.72	0.00	0.00	1.25
time (sec)	N/A	0.741	0.416	0.000	0.000	0.280	0.000	0.000	15.166

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	92	0	0	89	0	0	134
N.S.	1	1.00	0.81	0.00	0.00	0.78	0.00	0.00	1.18
time (sec)	N/A	0.515	0.327	0.000	0.000	0.277	0.000	0.000	14.180

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	201	310	0	0	0	0	0	0
N.S.	1	1.23	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	7.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	196	402	0	0	0	0	0	0
N.S.	1	1.24	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.628	6.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	170	209	0	0	0	0	0	0	0
N.S.	1	1.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	173	211	0	0	0	0	0	0	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	63	63	0	78	898	9586	64
N.S.	1	1.00	1.85	1.85	0.00	2.29	26.41	281.94	1.88
time (sec)	N/A	0.500	1.208	11.497	0.000	0.283	65.207	48.339	14.177

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	67	62	0	77	898	9587	61
N.S.	1	1.00	1.97	1.82	0.00	2.26	26.41	281.97	1.79
time (sec)	N/A	0.463	7.361	13.155	0.000	0.261	65.370	48.817	13.980

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	66	62	0	78	898	9586	61
N.S.	1	1.00	2.00	1.88	0.00	2.36	27.21	290.48	1.85
time (sec)	N/A	0.468	7.131	13.326	0.000	0.274	65.469	45.410	13.618

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	65	0	77	898	9587	64
N.S.	1	1.00	1.74	1.86	0.00	2.20	25.66	273.91	1.83
time (sec)	N/A	0.481	0.893	11.232	0.000	0.260	66.132	41.811	13.557

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	0	36	0	8947	36
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	248.53	1.00
time (sec)	N/A	0.311	1.866	9.819	0.000	0.263	0.000	53.146	12.739

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	0	37	0	8948	37
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	241.84	1.00
time (sec)	N/A	0.308	1.313	9.799	0.000	0.258	0.000	51.920	12.444

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	87	90	157	105	440	131	300
N.S.	1	1.00	0.62	0.64	1.12	0.75	3.14	0.94	2.14
time (sec)	N/A	0.405	0.370	2.191	0.243	0.254	0.481	0.305	14.600

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	77	79	138	91	359	113	256
N.S.	1	1.00	0.64	0.65	1.14	0.75	2.97	0.93	2.12
time (sec)	N/A	0.385	0.240	1.627	0.228	0.250	0.348	0.291	14.479

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	106	57	112	77	267	77	292
N.S.	1	1.00	1.10	0.59	1.17	0.80	2.78	0.80	3.04
time (sec)	N/A	0.320	0.723	1.359	0.239	0.250	0.249	0.287	14.730

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	57	86	63	196	77	250
N.S.	1	1.00	0.66	0.70	1.05	0.77	2.39	0.94	3.05
time (sec)	N/A	0.443	0.555	1.298	0.214	0.253	0.173	0.282	14.513

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	74	58	85	92	0	107	212
N.S.	1	1.00	0.97	0.76	1.12	1.21	0.00	1.41	2.79
time (sec)	N/A	0.314	0.242	0.962	0.235	0.262	0.000	0.296	12.643

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	80	83	111	0	153	226
N.S.	1	1.00	0.97	1.01	1.05	1.41	0.00	1.94	2.86
time (sec)	N/A	0.421	0.272	0.965	0.237	0.265	0.000	0.304	13.313

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	142	78	90	152	0	137	220
N.S.	1	1.00	1.82	1.00	1.15	1.95	0.00	1.76	2.82
time (sec)	N/A	0.330	0.135	1.210	0.216	0.300	0.000	0.308	13.776

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	141	99	117	175	0	150	245
N.S.	1	1.00	1.81	1.27	1.50	2.24	0.00	1.92	3.14
time (sec)	N/A	0.333	0.542	1.194	0.252	0.286	0.000	0.308	13.972

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	210	114	145	166	0	174	244
N.S.	1	1.00	2.44	1.33	1.69	1.93	0.00	2.02	2.84
time (sec)	N/A	0.349	0.233	1.284	0.218	0.285	0.000	0.322	13.202

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	107	268	121	175	201	0	174	244
N.S.	1	1.02	2.55	1.15	1.67	1.91	0.00	1.66	2.32
time (sec)	N/A	0.482	0.268	1.849	0.242	0.258	0.000	0.325	12.872

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	306	125	207	240	0	242	340
N.S.	1	1.00	2.35	0.96	1.59	1.85	0.00	1.86	2.62
time (sec)	N/A	0.402	0.290	1.886	0.219	0.281	0.000	0.320	12.866

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	211	154	715	248	3614	156	326
N.S.	1	1.00	1.64	1.19	5.54	1.92	28.02	1.21	2.53
time (sec)	N/A	0.419	3.802	1.044	0.352	0.264	22.200	0.319	16.441

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	155	115	543	225	2290	113	261
N.S.	1	1.00	1.50	1.12	5.27	2.18	22.23	1.10	2.53
time (sec)	N/A	0.392	2.187	0.934	0.323	0.269	12.943	0.312	16.239

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	112	85	392	204	1268	93	178
N.S.	1	1.00	1.26	0.96	4.40	2.29	14.25	1.04	2.00
time (sec)	N/A	0.380	2.119	0.660	0.340	0.269	7.409	0.316	14.394

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	107	62	348	156	461	63	110
N.S.	1	1.00	1.30	0.76	4.24	1.90	5.62	0.77	1.34
time (sec)	N/A	0.338	2.525	0.584	0.248	0.258	4.171	0.305	12.425

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	61	92	62	387	154	573	79	134
N.S.	1	1.05	1.59	1.07	6.67	2.66	9.88	1.36	2.31
time (sec)	N/A	0.386	1.549	0.558	0.232	0.247	2.323	0.303	12.336

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	313	95	433	310	0	99	199
N.S.	1	1.00	3.19	0.97	4.42	3.16	0.00	1.01	2.03
time (sec)	N/A	0.361	6.548	0.931	0.223	0.258	0.000	0.313	14.769

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	167	120	519	406	0	146	210
N.S.	1	1.00	1.48	1.06	4.59	3.59	0.00	1.29	1.86
time (sec)	N/A	0.615	2.327	1.125	0.246	0.276	0.000	0.321	15.918

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	245	148	622	498	0	180	288
N.S.	1	1.00	1.78	1.07	4.51	3.61	0.00	1.30	2.09
time (sec)	N/A	0.447	2.896	1.091	0.268	0.283	0.000	0.327	16.644

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	348	174	706	594	0	213	314
N.S.	1	1.00	2.27	1.14	4.61	3.88	0.00	1.39	2.05
time (sec)	N/A	0.464	6.595	1.170	0.286	0.269	0.000	0.324	15.980

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	336	267	244	406	239	996	309	830
N.S.	1	1.03	0.82	0.75	1.24	0.73	3.05	0.94	2.54
time (sec)	N/A	1.117	2.542	1.827	0.232	0.272	0.340	0.304	16.272

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	220	185	172	264	160	571	194	547
N.S.	1	1.03	0.87	0.81	1.24	0.75	2.68	0.91	2.57
time (sec)	N/A	0.754	1.636	1.125	0.231	0.269	0.228	0.301	15.703

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	123	104	94	143	84	277	98	134
N.S.	1	1.11	0.94	0.85	1.29	0.76	2.50	0.88	1.21
time (sec)	N/A	0.477	0.630	0.883	0.232	0.251	0.136	0.289	12.652

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	44	57	43	94	46	100
N.S.	1	1.00	0.94	0.92	1.19	0.90	1.96	0.96	2.08
time (sec)	N/A	0.213	0.143	0.438	0.241	0.258	0.095	0.281	12.550

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	109	196	120	0	292	5508	137	3074
N.S.	1	1.11	2.00	1.22	0.00	2.98	56.20	1.40	31.37
time (sec)	N/A	0.632	1.040	0.719	0.000	0.276	113.708	0.296	16.441

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	155	217	174	0	655	0	197	5102
N.S.	1	1.25	1.75	1.40	0.00	5.28	0.00	1.59	41.15
time (sec)	N/A	0.739	1.902	0.837	0.000	0.294	0.000	0.298	20.534

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	191	345	424	0	967	0	570	554
N.S.	1	1.09	1.96	2.41	0.00	5.49	0.00	3.24	3.15
time (sec)	N/A	0.817	2.991	1.100	0.000	0.304	0.000	0.327	15.752

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	480	437	324	724	364	1865	468	1291
N.S.	1	1.03	0.94	0.70	1.56	0.78	4.02	1.01	2.78
time (sec)	N/A	1.753	2.056	2.532	0.256	0.294	0.517	0.326	15.939

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	350	296	222	478	245	1129	306	765
N.S.	1	1.04	0.88	0.66	1.42	0.73	3.36	0.91	2.28
time (sec)	N/A	1.332	0.998	1.771	0.250	0.274	0.349	0.294	15.552

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	145	144	135	268	144	571	168	492
N.S.	1	0.87	0.87	0.81	1.61	0.87	3.44	1.01	2.96
time (sec)	N/A	0.615	0.426	1.158	0.240	0.261	0.229	0.284	14.991

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	83	106	70	114	70	199	85	91
N.S.	1	0.88	1.13	0.74	1.21	0.74	2.12	0.90	0.97
time (sec)	N/A	0.302	0.223	0.750	0.216	0.270	0.136	0.280	12.760

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	185	177	235	0	452	0	305	7371
N.S.	1	1.08	1.04	1.37	0.00	2.64	0.00	1.78	43.11
time (sec)	N/A	1.045	0.451	0.737	0.000	0.292	0.000	0.294	20.392

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	213	192	251	0	731	0	480	8706
N.S.	1	1.08	0.97	1.27	0.00	3.69	0.00	2.42	43.97
time (sec)	N/A	1.158	5.631	1.040	0.000	0.318	0.000	0.302	22.055

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	255	226	431	0	1483	0	678	8632
N.S.	1	1.19	1.05	2.00	0.00	6.90	0.00	3.15	40.15
time (sec)	N/A	1.242	5.895	1.020	0.000	0.331	0.000	0.328	22.607

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	628	528	384	1056	432	2878	559	1395
N.S.	1	1.04	0.87	0.64	1.75	0.72	4.76	0.93	2.31
time (sec)	N/A	2.561	3.053	2.718	0.228	0.298	0.716	0.335	16.311

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	485	355	268	704	299	1804	374	976
N.S.	1	1.05	0.77	0.58	1.52	0.65	3.90	0.81	2.11
time (sec)	N/A	2.029	1.589	2.096	0.254	0.288	0.504	0.316	16.123

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	163	156	167	398	178	960	212	550
N.S.	1	0.81	0.78	0.83	1.98	0.89	4.78	1.05	2.74
time (sec)	N/A	0.698	0.598	1.658	0.217	0.266	0.330	0.298	14.932

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	101	120	87	171	93	371	112	330
N.S.	1	0.80	0.94	0.69	1.35	0.73	2.92	0.88	2.60
time (sec)	N/A	0.362	0.344	1.086	0.288	0.268	0.199	0.279	14.608

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	269	233	378	0	627	0	597	10256
N.S.	1	1.09	0.95	1.54	0.00	2.55	0.00	2.43	41.69
time (sec)	N/A	1.694	1.764	1.041	0.000	0.300	0.000	0.307	22.535

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	295	244	406	0	1027	0	571	11993
N.S.	1	1.04	0.86	1.43	0.00	3.63	0.00	2.02	42.38
time (sec)	N/A	1.745	5.947	1.671	0.000	0.333	0.000	0.309	24.891

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	330	830	586	0	1670	0	953	13891
N.S.	1	1.08	2.72	1.92	0.00	5.48	0.00	3.12	45.54
time (sec)	N/A	1.833	7.176	2.056	0.000	0.343	0.000	0.345	25.541

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	788	337	1124	470	14644	460	839
N.S.	1	1.00	3.58	1.53	5.11	2.14	66.56	2.09	3.81
time (sec)	N/A	0.702	7.016	0.865	0.316	0.279	3.875	0.291	14.293

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	137	200	193	606	303	5763	214	297
N.S.	1	0.96	1.40	1.35	4.24	2.12	40.30	1.50	2.08
time (sec)	N/A	0.467	6.462	0.629	0.295	0.260	2.041	0.286	16.958

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	69	126	81	256	154	1307	151	122
N.S.	1	1.03	1.88	1.21	3.82	2.30	19.51	2.25	1.82
time (sec)	N/A	0.544	0.441	0.652	0.299	0.257	1.042	0.288	13.555

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	79	42	78	66	109	38	35
N.S.	1	1.00	2.26	1.20	2.23	1.89	3.11	1.09	1.00
time (sec)	N/A	0.265	0.134	0.346	0.276	0.258	0.557	0.294	13.269

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	148	94	0	595	0	110	154
N.S.	1	1.06	1.47	0.93	0.00	5.89	0.00	1.09	1.52
time (sec)	N/A	0.440	4.393	0.768	0.000	0.278	0.000	0.294	13.833

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	189	209	197	0	1538	0	425	437
N.S.	1	1.04	1.15	1.09	0.00	8.50	0.00	2.35	2.41
time (sec)	N/A	0.697	2.931	1.110	0.000	0.305	0.000	0.335	15.169

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	302	313	482	0	3303	0	727	1076
N.S.	1	1.07	1.11	1.70	0.00	11.67	0.00	2.57	3.80
time (sec)	N/A	1.076	3.278	2.488	0.000	0.366	0.000	0.382	17.950

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	234	547	340	1382	584	14612	472	663
N.S.	1	1.03	2.40	1.49	6.06	2.56	64.09	2.07	2.91
time (sec)	N/A	0.866	2.414	1.052	0.361	0.271	7.600	0.319	17.167

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	138	338	193	831	375	5358	264	365
N.S.	1	1.05	2.56	1.46	6.30	2.84	40.59	2.00	2.77
time (sec)	N/A	0.998	1.328	0.794	0.319	0.266	4.014	0.311	16.492

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	89	180	110	454	208	1062	133	94
N.S.	1	1.05	2.12	1.29	5.34	2.45	12.49	1.56	1.11
time (sec)	N/A	0.568	1.400	0.582	0.310	0.267	2.109	0.307	14.075

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	64	43	60	214	117	372	64	97
N.S.	1	0.98	0.66	0.92	3.29	1.80	5.72	0.98	1.49
time (sec)	N/A	0.288	0.041	0.472	0.211	0.249	1.121	0.290	13.836

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	165	229	159	0	1285	0	249	302
N.S.	1	1.09	1.51	1.05	0.00	8.45	0.00	1.64	1.99
time (sec)	N/A	0.723	2.632	1.104	0.000	0.296	0.000	0.310	14.739

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	288	313	263	0	3123	0	411	844
N.S.	1	1.05	1.14	0.96	0.00	11.36	0.00	1.49	3.07
time (sec)	N/A	1.162	7.005	1.867	0.000	0.363	0.000	0.337	16.731

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	417	1257	547	0	4997	0	911	1686
N.S.	1	1.08	3.26	1.42	0.00	12.95	0.00	2.36	4.37
time (sec)	N/A	1.645	10.899	4.518	0.000	0.453	0.000	0.371	17.877

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	237	366	349	1682	649	11456	567	593
N.S.	1	1.05	1.63	1.55	7.48	2.88	50.92	2.52	2.64
time (sec)	N/A	1.604	6.898	1.072	0.342	0.272	14.033	0.317	16.137

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	174	514	240	1132	432	3468	362	286
N.S.	1	1.06	3.13	1.46	6.90	2.63	21.15	2.21	1.74
time (sec)	N/A	1.025	0.812	0.734	0.320	0.261	7.853	0.312	16.842

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	176	139	733	271	1819	210	245
N.S.	1	1.02	1.39	1.09	5.77	2.13	14.32	1.65	1.93
time (sec)	N/A	0.608	1.718	0.655	0.226	0.251	4.366	0.315	14.016

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	99	63	95	387	190	1015	122	150
N.S.	1	0.97	0.62	0.93	3.79	1.86	9.95	1.20	1.47
time (sec)	N/A	0.391	0.058	0.565	0.302	0.239	2.371	0.291	13.020

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	258	502	252	0	2292	0	553	591
N.S.	1	1.13	2.19	1.10	0.00	10.01	0.00	2.41	2.58
time (sec)	N/A	1.207	4.747	1.516	0.000	0.333	0.000	0.317	17.222

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	409	1253	355	0	4486	0	743	1349
N.S.	1	1.07	3.29	0.93	0.00	11.77	0.00	1.95	3.54
time (sec)	N/A	1.789	12.275	2.897	0.000	0.421	0.000	0.347	17.708

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	551	548	639	0	7283	0	1224	2387
N.S.	1	1.08	1.08	1.26	0.00	14.34	0.00	2.41	4.70
time (sec)	N/A	2.341	8.637	6.115	0.000	0.559	0.000	0.433	19.582

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	235	305	242	0	467	0	550	0
N.S.	1	0.92	1.19	0.95	0.00	1.82	0.00	2.15	0.00
time (sec)	N/A	1.044	1.298	2.190	0.000	0.271	0.000	0.397	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	187	176	161	0	306	0	348	0
N.S.	1	0.97	0.92	0.84	0.00	1.59	0.00	1.81	0.00
time (sec)	N/A	0.771	0.841	1.763	0.000	0.277	0.000	0.345	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	129	117	102	0	175	0	186	0
N.S.	1	1.09	0.99	0.86	0.00	1.48	0.00	1.58	0.00
time (sec)	N/A	0.637	0.740	1.540	0.000	0.267	0.000	0.303	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	85	0
N.S.	1	1.00	1.32	0.94	0.00	1.37	0.00	1.37	0.00
time (sec)	N/A	0.292	0.147	0.931	0.000	0.272	0.000	0.291	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	651	0	125	0
N.S.	1	1.00	9.03	1.39	0.00	6.51	0.00	1.25	0.00
time (sec)	N/A	0.483	6.754	0.696	0.000	0.715	0.000	0.309	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1012	0	212	0
N.S.	1	1.00	7.15	2.17	0.00	8.03	0.00	1.68	0.00
time (sec)	N/A	0.533	7.018	0.838	0.000	0.848	0.000	0.318	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	186	967	628	0	1750	0	422	0
N.S.	1	0.97	5.04	3.27	0.00	9.11	0.00	2.20	0.00
time (sec)	N/A	0.743	8.527	1.091	0.000	1.291	0.000	0.339	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	316	390	312	0	637	0	755	0
N.S.	1	0.84	1.04	0.83	0.00	1.70	0.00	2.02	0.00
time (sec)	N/A	1.531	4.337	2.475	0.000	0.296	0.000	0.454	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	268	267	207	0	430	0	497	0
N.S.	1	0.91	0.91	0.70	0.00	1.46	0.00	1.69	0.00
time (sec)	N/A	1.242	2.915	2.044	0.000	0.280	0.000	0.384	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	163	144	150	0	257	0	285	0
N.S.	1	0.99	0.87	0.91	0.00	1.56	0.00	1.73	0.00
time (sec)	N/A	0.818	2.348	1.821	0.000	0.269	0.000	0.326	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	99	101	77	0	137	0	139	0
N.S.	1	0.98	1.00	0.76	0.00	1.36	0.00	1.38	0.00
time (sec)	N/A	0.395	1.087	1.217	0.000	0.255	0.000	0.318	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	159	898	292	0	880	0	274	0
N.S.	1	1.04	5.87	1.91	0.00	5.75	0.00	1.79	0.00
time (sec)	N/A	0.840	3.347	1.058	0.000	0.848	0.000	0.309	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	199	922	592	0	1428	0	364	0
N.S.	1	1.04	4.83	3.10	0.00	7.48	0.00	1.91	0.00
time (sec)	N/A	0.922	9.590	0.967	0.000	0.996	0.000	0.327	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	229	957	896	0	2208	0	624	0
N.S.	1	1.04	4.33	4.05	0.00	9.99	0.00	2.82	0.00
time (sec)	N/A	0.998	10.840	1.344	0.000	1.494	0.000	0.356	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	420	1565	374	0	863	0	1014	0
N.S.	1	0.79	2.93	0.70	0.00	1.62	0.00	1.90	0.00
time (sec)	N/A	2.162	8.546	168.510	0.000	0.308	0.000	0.553	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	371	328	257	0	593	0	684	0
N.S.	1	0.86	0.76	0.60	0.00	1.38	0.00	1.59	0.00
time (sec)	N/A	1.884	7.726	36.059	0.000	0.298	0.000	0.424	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	202	152	0	361	0	404	0
N.S.	1	0.93	0.95	0.72	0.00	1.70	0.00	1.91	0.00
time (sec)	N/A	0.904	4.732	7.572	0.000	0.269	0.000	0.360	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	133	119	99	0	191	0	202	0
N.S.	1	0.96	0.86	0.72	0.00	1.38	0.00	1.46	0.00
time (sec)	N/A	0.492	2.562	1.995	0.000	0.264	0.000	0.306	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	232	992	543	0	1314	0	523	0
N.S.	1	1.06	4.55	2.49	0.00	6.03	0.00	2.40	0.00
time (sec)	N/A	1.326	5.378	2.937	0.000	1.302	0.000	0.333	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	275	1002	932	0	2046	0	597	0
N.S.	1	1.04	3.78	3.52	0.00	7.72	0.00	2.25	0.00
time (sec)	N/A	1.448	10.427	10.175	0.000	1.487	0.000	0.361	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	308	322	1046	1587	0	3046	0	895	0
N.S.	1	1.05	3.40	5.15	0.00	9.89	0.00	2.91	0.00
time (sec)	N/A	1.522	12.857	63.259	0.000	1.731	0.000	0.408	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	B	F	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	0	375	560	0	629	0	555	0
N.S.	1	0.00	1.32	1.97	0.00	2.21	0.00	1.95	0.00
time (sec)	N/A	0.000	2.608	3.556	0.000	0.295	0.000	0.407	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	214	246	396	0	448	0	363	0
N.S.	1	1.07	1.23	1.98	0.00	2.24	0.00	1.82	0.00
time (sec)	N/A	1.175	2.263	2.617	0.000	0.272	0.000	0.350	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	137	135	232	0	303	0	221	0
N.S.	1	1.05	1.04	1.78	0.00	2.33	0.00	1.70	0.00
time (sec)	N/A	0.657	1.516	2.187	0.000	0.276	0.000	0.320	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	210	0	143	151
N.S.	1	1.00	1.34	1.62	0.00	2.66	0.00	1.81	1.91
time (sec)	N/A	0.305	0.281	2.230	0.000	0.261	0.000	0.293	1.114

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	619	199	0	744	0	252	0
N.S.	1	1.00	4.55	1.46	0.00	5.47	0.00	1.85	0.00
time (sec)	N/A	0.600	3.085	0.750	0.000	0.842	0.000	0.335	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	230	736	899	0	2159	0	492	0
N.S.	1	1.11	3.56	4.34	0.00	10.43	0.00	2.38	0.00
time (sec)	N/A	1.073	5.967	1.004	0.000	2.122	0.000	0.377	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	0	852	2275	0	4180	0	876	0
N.S.	1	0.00	2.76	7.36	0.00	13.53	0.00	2.83	0.00
time (sec)	N/A	0.000	9.961	1.589	0.000	4.283	0.000	0.479	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	299	684	817	0	784	0	650	0
N.S.	1	1.06	2.42	2.89	0.00	2.77	0.00	2.30	0.00
time (sec)	N/A	1.829	3.772	3.181	0.000	0.292	0.000	0.405	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	212	357	612	0	584	0	461	0
N.S.	1	1.04	1.76	3.01	0.00	2.88	0.00	2.27	0.00
time (sec)	N/A	1.177	2.731	2.592	0.000	0.275	0.000	0.347	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	137	246	389	0	407	0	0	0
N.S.	1	1.03	1.85	2.92	0.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.677	1.292	2.284	0.000	0.308	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	184	0
N.S.	1	1.00	1.72	2.02	0.00	3.37	0.00	2.11	0.00
time (sec)	N/A	0.330	0.325	1.464	0.000	0.270	0.000	0.310	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	200	781	624	0	1561	0	471	0
N.S.	1	1.07	4.18	3.34	0.00	8.35	0.00	2.52	0.00
time (sec)	N/A	1.009	3.963	0.984	0.000	1.814	0.000	0.357	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	319	904	2049	0	3403	0	866	0
N.S.	1	1.09	3.10	7.02	0.00	11.65	0.00	2.97	0.00
time (sec)	N/A	1.645	9.573	1.398	0.000	4.255	0.000	0.451	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	402	444	1757	4707	0	5864	0	1263	0
N.S.	1	1.10	4.37	11.71	0.00	14.59	0.00	3.14	0.00
time (sec)	N/A	2.421	12.801	2.070	0.000	8.457	0.000	0.598	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	324	523	1157	0	980	0	751	0
N.S.	1	1.05	1.70	3.76	0.00	3.18	0.00	2.44	0.00
time (sec)	N/A	1.920	6.775	3.994	0.000	0.302	0.000	0.554	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	229	544	852	0	744	0	530	0
N.S.	1	1.05	2.48	3.89	0.00	3.40	0.00	2.42	0.00
time (sec)	N/A	1.212	2.855	3.220	0.000	0.291	0.000	0.429	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	157	267	449	0	536	0	364	0
N.S.	1	1.04	1.77	2.97	0.00	3.55	0.00	2.41	0.00
time (sec)	N/A	0.688	1.577	2.586	0.000	0.273	0.000	0.353	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	124	227	279	0	392	0	236	0
N.S.	1	0.98	1.80	2.21	0.00	3.11	0.00	1.87	0.00
time (sec)	N/A	0.435	0.440	1.912	0.000	0.269	0.000	0.320	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	287	912	1418	0	2577	0	760	0
N.S.	1	1.10	3.49	5.43	0.00	9.87	0.00	2.91	0.00
time (sec)	N/A	1.538	6.737	1.319	0.000	3.693	0.000	0.425	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	395	438	1680	4092	0	5151	0	1099	0
N.S.	1	1.11	4.25	10.36	0.00	13.04	0.00	2.78	0.00
time (sec)	N/A	2.419	15.726	2.024	0.000	8.100	0.000	0.707	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	519	574	2465	7322	0	8555	0	2024	0
N.S.	1	1.11	4.75	14.11	0.00	16.48	0.00	3.90	0.00
time (sec)	N/A	3.305	16.006	3.448	0.000	24.729	0.000	1.001	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.606	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	427	426	245	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.851	11.789	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	220	222	244	0	0	0	0	0	0
N.S.	1	1.01	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	8.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	269	267	603	0	0	0	0	0	0
N.S.	1	0.99	2.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	16.631	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	351	349	854	0	0	0	0	0	0
N.S.	1	0.99	2.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.700	24.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	199	200	317	0	0	0	0	0	0
N.S.	1	1.01	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.856	22.750	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	191	205	0	0	0	0	0	0	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.725	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	293	310	0	0	0	0	0	0	0
N.S.	1	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.247	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	467	504	0	0	0	0	0	0	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.153	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	284	308	0	0	0	0	0	0	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.750	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	274	298	0	0	0	0	0	0	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.698	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	274	298	0	0	0	0	0	0	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	288	312	0	0	0	0	0	0	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.733	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	270	294	0	0	0	0	0	0	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	277	287	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	132	146	0	0	0	0	0	0	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	139	153	0	0	0	0	0	0	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	56	0	0	98
N.S.	1	1.00	1.00	0.00	0.00	1.44	0.00	0.00	2.51
time (sec)	N/A	0.335	2.555	0.000	0.000	0.317	0.000	0.000	16.044

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	57	0	0	99
N.S.	1	1.00	1.00	0.00	0.00	1.42	0.00	0.00	2.48
time (sec)	N/A	0.348	2.415	0.000	0.000	0.323	0.000	0.000	15.287

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	234	188	374	0	1308	0	749	16312
N.S.	1	1.18	0.94	1.88	0.00	6.57	0.00	3.76	81.97
time (sec)	N/A	1.087	2.631	1.453	0.000	0.356	0.000	0.497	28.885

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	840	851	2042	1442707	0	0	0	0	0
N.S.	1	1.01	2.43	1717.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.298	8.056	114.640	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	630	621	1901	637252	0	0	0	0	0
N.S.	1	0.99	3.02	1011.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.689	15.536	51.578	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	87094	0	0	0	0	0
N.S.	1	1.00	4.67	208.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.032	6.761	14.801	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	544	578	2266	174462	0	0	0	0	0
N.S.	1	1.06	4.17	320.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.733	7.243	18.711	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	858	903	2837	726985	0	0	0	0	0
N.S.	1	1.05	3.31	847.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.340	8.074	33.723	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	37	0	37	37
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.06
time (sec)	N/A	0.260	14.642	1.000	29.145	0.543	0.000	3.788	18.233

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [285] had the largest ratio of [.542857000000000034]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	12	0.98	33	0.364
2	A	10	10	0.98	33	0.303
3	A	8	8	0.98	31	0.258
4	A	6	6	1.00	33	0.182
5	A	9	9	1.01	33	0.273
6	A	11	11	1.04	33	0.333
7	A	13	12	0.96	35	0.343
8	A	10	9	0.98	35	0.257
9	A	7	6	0.96	35	0.171
10	A	14	13	1.00	35	0.371
11	A	17	16	1.02	35	0.457
12	A	10	9	1.00	33	0.273
13	A	6	5	1.00	34	0.147
14	A	2	2	1.00	43	0.047
15	A	2	2	1.00	37	0.054
16	A	11	10	1.24	31	0.323
17	A	13	13	0.87	34	0.382
18	A	11	11	0.87	34	0.324
19	A	9	9	0.89	34	0.265
20	A	7	7	0.96	32	0.219
21	A	5	5	1.09	34	0.147
22	A	9	9	1.07	34	0.265

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	9	1.03	34	0.265
24	A	11	11	1.01	34	0.324
25	A	13	13	0.99	34	0.382
26	A	15	15	0.83	36	0.417
27	A	13	13	0.81	36	0.361
28	A	11	11	0.79	36	0.306
29	A	9	9	0.87	36	0.250
30	A	9	9	0.85	34	0.265
31	A	9	9	0.92	36	0.250
32	A	9	9	0.97	36	0.250
33	A	9	9	1.04	36	0.250
34	A	6	6	1.00	36	0.167
35	A	8	8	0.98	36	0.222
36	A	10	10	0.96	36	0.278
37	A	12	12	0.94	36	0.333
38	A	17	17	0.81	36	0.472
39	A	15	15	0.81	36	0.417
40	A	13	13	0.78	36	0.361
41	A	11	11	0.88	36	0.306
42	A	11	11	0.82	36	0.306
43	A	11	11	0.87	34	0.324
44	A	11	11	0.84	36	0.306
45	A	11	11	0.91	36	0.306
46	A	11	11	0.95	36	0.306
47	A	11	11	1.02	36	0.306
48	A	6	6	0.97	36	0.167
49	A	8	8	0.96	36	0.222
50	A	10	10	0.96	36	0.278
51	A	12	12	0.94	36	0.333
52	A	13	13	0.90	36	0.361
53	A	11	11	0.84	36	0.306
54	A	9	9	0.92	36	0.250
55	A	5	5	1.11	34	0.147
56	A	8	7	0.86	36	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	8	7	0.95	36	0.194
58	A	10	9	0.92	36	0.250
59	A	12	11	0.90	36	0.306
60	A	15	15	0.87	36	0.417
61	A	13	13	0.92	36	0.361
62	A	11	11	0.90	36	0.306
63	A	9	9	0.97	36	0.250
64	A	8	8	1.08	34	0.235
65	A	8	7	0.95	36	0.194
66	A	8	7	0.82	36	0.194
67	A	8	7	0.83	36	0.194
68	A	10	9	0.84	36	0.250
69	A	12	11	0.85	36	0.306
70	A	15	15	0.85	36	0.417
71	A	13	13	0.92	36	0.361
72	A	11	11	0.93	36	0.306
73	A	9	9	1.05	36	0.250
74	A	8	8	1.03	34	0.235
75	A	10	9	0.92	36	0.250
76	A	8	7	0.84	36	0.194
77	A	8	7	0.75	36	0.194
78	A	8	7	0.74	36	0.194
79	A	10	9	0.77	36	0.250
80	A	12	11	0.79	36	0.306
81	A	12	12	0.93	36	0.333
82	A	10	10	0.94	36	0.278
83	A	8	8	0.96	36	0.222
84	A	6	6	0.97	36	0.167
85	A	11	10	1.06	36	0.278
86	A	11	10	1.07	36	0.278
87	A	11	10	1.07	36	0.278
88	A	13	12	1.03	36	0.333
89	A	12	12	0.90	38	0.316
90	A	10	10	0.91	38	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	8	8	0.94	38	0.211
92	A	6	6	0.95	38	0.158
93	A	12	11	0.99	38	0.289
94	A	12	11	0.98	38	0.289
95	A	12	11	0.97	38	0.289
96	A	12	11	0.98	38	0.289
97	A	14	13	0.96	38	0.342
98	A	12	12	0.90	38	0.316
99	A	10	10	0.93	38	0.263
100	A	8	8	0.93	38	0.211
101	A	6	6	0.95	38	0.158
102	A	14	13	1.00	38	0.342
103	A	14	13	0.97	38	0.342
104	A	14	13	0.93	38	0.342
105	A	14	13	0.97	38	0.342
106	A	14	13	0.98	38	0.342
107	A	16	15	0.96	38	0.395
108	A	12	12	0.91	38	0.316
109	A	10	10	0.92	38	0.263
110	A	8	8	0.95	38	0.211
111	A	6	6	1.00	38	0.158
112	A	8	7	0.98	38	0.184
113	A	10	9	0.96	38	0.237
114	A	12	11	0.91	38	0.289
115	A	14	14	0.90	38	0.368
116	A	12	12	0.91	38	0.316
117	A	10	10	0.94	38	0.263
118	A	8	8	0.96	38	0.211
119	A	6	6	1.01	38	0.158
120	A	10	9	0.93	38	0.237
121	A	12	11	0.94	38	0.289
122	A	14	13	0.90	38	0.342
123	A	14	14	0.91	38	0.368
124	A	12	12	0.89	38	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	10	10	0.93	38	0.263
126	A	8	8	0.96	38	0.211
127	A	6	6	0.98	38	0.158
128	A	12	11	0.93	38	0.289
129	A	14	13	0.90	38	0.342
130	A	16	15	0.91	38	0.395
131	A	4	4	1.00	40	0.100
132	A	4	4	1.00	40	0.100
133	A	4	4	1.00	40	0.100
134	A	4	4	1.00	40	0.100
135	A	9	8	1.01	40	0.200
136	A	9	8	1.01	40	0.200
137	A	4	4	1.00	40	0.100
138	A	4	4	1.00	40	0.100
139	A	6	6	0.99	40	0.150
140	A	6	6	0.99	40	0.150
141	A	6	6	1.01	40	0.150
142	A	4	4	1.00	40	0.100
143	A	10	9	1.00	40	0.225
144	A	10	9	0.98	40	0.225
145	A	10	9	1.01	40	0.225
146	A	4	4	1.00	40	0.100
147	A	6	6	1.00	40	0.150
148	A	6	6	0.99	40	0.150
149	A	8	8	0.97	40	0.200
150	A	8	8	1.02	40	0.200
151	A	6	6	1.00	40	0.150
152	A	4	4	1.00	40	0.100
153	A	12	11	0.99	40	0.275
154	A	12	11	0.95	40	0.275
155	A	12	11	0.95	40	0.275
156	A	12	11	1.02	40	0.275
157	A	4	4	1.00	40	0.100
158	A	6	6	1.00	40	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	8	8	1.00	40	0.200
160	A	10	10	0.97	40	0.250
161	A	10	10	1.03	40	0.250
162	A	8	8	0.99	40	0.200
163	A	6	6	1.00	40	0.150
164	A	4	4	1.00	40	0.100
165	A	14	13	1.00	40	0.325
166	A	14	13	0.93	40	0.325
167	A	14	13	0.95	40	0.325
168	A	14	13	0.96	40	0.325
169	A	14	13	1.01	40	0.325
170	A	4	4	1.00	40	0.100
171	A	6	6	1.00	40	0.150
172	A	8	8	0.98	40	0.200
173	A	10	10	1.00	40	0.250
174	A	12	11	0.97	40	0.275
175	A	10	9	0.99	40	0.225
176	A	9	8	1.02	40	0.200
177	A	8	7	1.03	40	0.175
178	A	6	6	1.00	40	0.150
179	A	8	8	1.00	40	0.200
180	A	14	13	0.93	40	0.325
181	A	12	11	0.95	40	0.275
182	A	10	9	0.97	40	0.225
183	A	9	8	1.02	40	0.200
184	A	6	6	1.00	40	0.150
185	A	8	8	0.99	40	0.200
186	A	10	10	0.95	40	0.250
187	A	16	15	0.93	40	0.375
188	A	14	13	0.95	40	0.325
189	A	12	11	0.96	40	0.275
190	A	10	9	1.02	40	0.225
191	A	4	4	1.00	40	0.100
192	A	8	8	1.01	40	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	10	10	0.97	40	0.250
194	A	12	12	1.01	40	0.300
195	A	9	8	1.21	36	0.222
196	A	9	8	0.94	36	0.222
197	A	9	8	0.94	36	0.222
198	A	9	8	0.96	34	0.235
199	A	6	6	1.00	23	0.261
200	A	9	8	0.99	36	0.222
201	A	9	8	0.93	36	0.222
202	A	9	8	0.93	36	0.222
203	A	8	7	1.04	38	0.184
204	A	8	7	1.04	38	0.184
205	A	8	8	0.84	38	0.211
206	A	6	6	1.05	38	0.158
207	A	4	4	1.00	38	0.105
208	A	8	7	1.04	38	0.184
209	A	8	7	1.04	38	0.184
210	A	8	7	1.04	38	0.184
211	A	8	8	0.87	40	0.200
212	A	6	6	0.92	40	0.150
213	A	4	4	1.00	40	0.100
214	A	9	8	1.23	40	0.200
215	A	9	8	1.24	38	0.211
216	A	9	8	1.23	40	0.200
217	A	9	8	1.22	40	0.200
218	A	4	4	1.00	46	0.087
219	A	4	4	1.00	45	0.089
220	A	5	5	1.00	44	0.114
221	A	5	5	1.00	43	0.116
222	A	2	2	1.00	47	0.043
223	A	2	2	1.00	46	0.043
224	A	3	3	1.00	32	0.094
225	A	3	3	1.00	32	0.094
226	A	3	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	9	9	1.00	24	0.375
228	A	3	3	1.00	30	0.100
229	A	5	5	1.00	32	0.156
230	A	3	3	1.00	32	0.094
231	A	3	3	1.00	32	0.094
232	A	3	3	1.00	32	0.094
233	A	5	5	1.02	32	0.156
234	A	3	3	1.00	32	0.094
235	A	3	3	1.00	32	0.094
236	A	3	3	1.00	32	0.094
237	A	3	3	1.00	32	0.094
238	A	3	3	1.00	30	0.100
239	A	6	6	1.05	24	0.250
240	A	3	3	1.00	30	0.100
241	A	5	5	1.00	32	0.156
242	A	3	3	1.00	32	0.094
243	A	3	3	1.00	32	0.094
244	A	10	10	1.03	33	0.303
245	A	8	8	1.03	33	0.242
246	A	6	6	1.11	31	0.194
247	A	2	2	1.00	21	0.095
248	A	11	10	1.11	33	0.303
249	A	12	11	1.25	33	0.333
250	A	14	13	1.09	33	0.394
251	A	14	14	1.03	35	0.400
252	A	12	12	1.04	35	0.343
253	A	8	8	0.87	33	0.242
254	A	4	4	0.88	23	0.174
255	A	13	12	1.08	35	0.343
256	A	14	13	1.08	35	0.371
257	A	14	13	1.19	35	0.371
258	A	16	16	1.04	35	0.457
259	A	14	14	1.05	35	0.400
260	A	9	9	0.81	33	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	5	5	0.80	23	0.217
262	A	16	15	1.09	35	0.429
263	A	16	15	1.04	35	0.429
264	A	17	16	1.08	35	0.457
265	A	6	6	1.00	35	0.171
266	A	4	4	0.96	35	0.114
267	A	8	8	1.03	33	0.242
268	A	4	4	1.00	23	0.174
269	A	9	8	1.06	35	0.229
270	A	11	10	1.04	35	0.286
271	A	14	13	1.07	35	0.371
272	A	6	6	1.03	35	0.171
273	A	10	10	1.05	35	0.286
274	A	9	9	1.05	33	0.273
275	A	4	4	0.98	23	0.174
276	A	11	10	1.09	35	0.286
277	A	13	12	1.05	35	0.343
278	A	15	14	1.08	35	0.400
279	A	12	12	1.05	35	0.343
280	A	11	11	1.06	35	0.314
281	A	9	9	1.02	33	0.273
282	A	6	6	0.97	23	0.261
283	A	14	13	1.13	35	0.371
284	A	17	16	1.07	35	0.457
285	A	20	19	1.08	35	0.543
286	A	11	11	0.92	37	0.297
287	A	9	9	0.97	37	0.243
288	A	9	9	1.09	35	0.257
289	A	4	4	1.00	25	0.160
290	A	6	5	1.00	37	0.135
291	A	6	5	1.00	37	0.135
292	A	8	7	0.97	37	0.189
293	A	14	14	0.84	37	0.378
294	A	12	12	0.91	37	0.324

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	11	11	0.99	35	0.314
296	A	6	6	0.98	25	0.240
297	A	9	8	1.04	37	0.216
298	A	9	8	1.04	37	0.216
299	A	9	8	1.04	37	0.216
300	A	17	17	0.79	37	0.459
301	A	15	15	0.86	37	0.405
302	A	13	13	0.93	35	0.371
303	A	8	8	0.96	25	0.320
304	A	12	11	1.06	37	0.297
305	A	12	11	1.04	37	0.297
306	A	12	11	1.05	37	0.297
307	F	0	0	N/A	0.000	N/A
308	A	14	13	1.07	37	0.351
309	A	11	10	1.05	35	0.286
310	A	6	5	1.00	25	0.200
311	A	8	7	1.00	37	0.189
312	A	11	10	1.11	37	0.270
313	F	0	0	N/A	0.000	N/A
314	A	17	16	1.06	37	0.432
315	A	14	13	1.04	37	0.351
316	A	11	10	1.03	35	0.286
317	A	6	5	1.00	25	0.200
318	A	11	10	1.07	37	0.270
319	A	14	13	1.09	37	0.351
320	A	17	16	1.10	37	0.432
321	A	17	16	1.05	37	0.432
322	A	14	13	1.05	37	0.351
323	A	11	10	1.04	35	0.286
324	A	8	7	0.98	25	0.280
325	A	14	13	1.10	37	0.351
326	A	17	16	1.11	37	0.432
327	A	20	19	1.11	37	0.514

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	7	6	1.00	35	0.171
329	A	12	11	1.00	33	0.333
330	A	10	9	1.00	35	0.257
331	A	7	6	1.00	35	0.171
332	A	16	15	1.00	37	0.405
333	A	7	6	1.00	37	0.162
334	A	11	10	1.01	37	0.270
335	A	8	7	0.99	37	0.189
336	A	12	12	0.99	35	0.343
337	A	10	10	1.01	33	0.303
338	A	6	6	1.00	23	0.261
339	A	11	10	1.07	35	0.286
340	A	14	13	1.06	35	0.371
341	A	16	15	1.08	35	0.429
342	A	9	8	1.08	37	0.216
343	A	9	8	1.09	37	0.216
344	A	9	8	1.09	37	0.216
345	A	9	8	1.08	37	0.216
346	A	9	8	1.09	35	0.229
347	A	10	9	1.04	39	0.231
348	A	7	6	1.11	36	0.167
349	A	7	6	1.10	40	0.150
350	A	2	2	1.00	55	0.036
351	A	2	2	1.00	51	0.039
352	A	12	11	1.18	35	0.314
353	A	15	15	1.01	39	0.385
354	A	8	8	0.99	39	0.205
355	A	5	5	1.00	39	0.128
356	A	8	8	1.06	39	0.205
357	A	11	11	1.05	39	0.282
358	N/A	2	0	1.00	35	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$ 139
3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$ 147
3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$ 154
3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$ 160
3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$ 166
3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$ 173
3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$ 180
3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$ 188
3.9 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$ 196
3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$ 202
3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$ 210
3.12 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$ 219
3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$ 226
3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$ 231
3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$ 236
3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$ 241
3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$ 250
3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$ 259
3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$ 267
3.20 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$ 274
3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$ 280
3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$ 287
3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$ 295
3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$ 303
3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$ 311
3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$ 320
3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$ 330

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$ 339
3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$ 348
3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$ 355
3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$ 362
3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$ 370
3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$ 378
3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$ 386
3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$ 393
3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$ 401
3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$ 410
3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$ 421
3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$ 431
3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$ 441
3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$ 450
3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$ 458
3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$ 467
3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$ 476
3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$ 485
3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$ 494
3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$ 503
3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$ 512
3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$ 520
3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$ 528
3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$ 538
3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$ 550
3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$ 561
3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$ 570
3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$ 578
3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$ 585
3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$ 591
3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$ 598
3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$ 606
3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$ 614
3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$ 629
3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$ 640

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$ . . . 649
3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$ . . . 657
3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$ . . . 665
3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$ . . . 672
3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$ . . . 678
3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$ . . . 686
3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$ . . . 694
3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$ . . . 703
3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$ . . . 718
3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$ . . . 730
3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$ . . . 739
3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$ . . . 747
3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$ . . . 755
3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$ . . . 763
3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$ . . . 771
3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$ . . . 778
3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$ . . . 786
3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$ . . . 794
3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ . . . 804
3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ . . . 811
3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ . . . 819
3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ . . . 826
3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ . . . 832
3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ . . . 840
3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ . . . 848
3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ . . . 856
3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ . . . 864
3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ . . . 872
3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ . . . 880
3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ . . . 887
3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ . . . 893
3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ . . . 901
3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ . . . 910
3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ . . . 919
3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$ . . . 928

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 938
3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 946
3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 953
3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 961
3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 967
3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 977
3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 987
3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ 998
3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1009
3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1020
3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$ 1032
3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$ 1040
3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$ 1047
3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$ 1053
3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$ 1059
3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$ 1065
3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$ 1072
3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$ 1080
3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$ 1088
3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$ 1096
3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$ 1104
3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$ 1111
3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$ 1117
3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$ 1124
3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$ 1132
3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$ 1141
3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$ 1149
3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$ 1157
3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$ 1166
3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$ 1173
3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$ 1180
3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$ 1188
3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$ 1196

3.131 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 1205
3.132 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1211
3.133 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1217
3.134 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1223
3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1229
3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1236
3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1242
3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1247
3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 1252
3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1259
3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1266
3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1273
3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1279
3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1286
3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1293
3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1300
3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1305
3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1311
3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 1317
3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1325
3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1332
3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1339
3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1345
3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1353
3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1360
3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1369
3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1377
3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1383
3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$ 1390
3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$ 1397
3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 1406
3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1414
3.163 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1422
3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1429
3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1435

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1444
3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1452
3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1461
3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1471
3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1480
3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$ 1487
3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$ 1495
3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$ 1504
3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1513
3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1521
3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1528
3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$ 1535
3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$ 1541
3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$ 1547
3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 1554
3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 1562
3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 1570
3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 1578
3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$ 1584
3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$ 1590
3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$ 1597
3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 1605
3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 1614
3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 1623
3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 1631
3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 1638
3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$ 1643
3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$ 1650
3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$ 1658
3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$ 1666
3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$ 1672
3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$ 1679

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$ 1686
3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$ 1693
3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$ 1698
3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$ 1704
3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$ 1710
3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1716
3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$ 1722
3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1728
3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1736
3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1743
3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1748
3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1754
3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1760
3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$ 1766
3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$ 1773
3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$ 1779
3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$ 1784
3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$ 1791
3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$ 1798
3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$ 1805
3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$ 1812
3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$ 1818
3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$ 1824
3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$ 1831
3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$ 1838
3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$ 1844
3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1850
3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1857
3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1863
3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1869
3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1876
3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1882
3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1888
3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1894
3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1900
3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1906
3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$ 1912
3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ 1919

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1927
3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1934
3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1941
3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$ . . . 1947
3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1954
3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1960
3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1967
3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$ . . . 1974
3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 1981
3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 1990
3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 1998
3.247 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$ . . . 2004
3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2009
3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2017
3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2025
3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 2035
3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 2047
3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 2058
3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$ . . . 2066
3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2072
3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2081
3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2091
3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 2102
3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 2115
3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 2126
3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$ . . . 2135
3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2142
3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2153
3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2164
3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$ . . . 2176
3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$ . . . 2186
3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$ . . . 2193
3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$ . . . 2200
3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$ . . . 2205
3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$ . . . 2212
3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$ . . . 2221

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$ . . . 2232
3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$ . . . 2242
3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$ . . . 2251
3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$ . . . 2259
3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$ . . . 2265
3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$ . . . 2273
3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$ . . . 2283
3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$ . . . 2295
3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$ . . . 2306
3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$ . . . 2316
3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$ . . . 2324
3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$ . . . 2331
3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$ . . . 2341
3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$ . . . 2352
3.286 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 2364
3.287 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 2373
3.288 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 2381
3.289 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$ . . . 2388
3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2393
3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2400
3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2408
3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 2417
3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 2427
3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 2436
3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$ . . . 2444
3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2450
3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2458
3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2467
3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ . . . 2476
3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ . . . 2488
3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ . . . 2498
3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$ . . . 2506
3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ . . . 2512
3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ . . . 2521
3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ . . . 2531

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2540
3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2552
3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2561
3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2568
3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$ 2574
3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$ 2582
3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$ 2591
3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2604
3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2615
3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2625
3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2632
3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$ 2637
3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$ 2646
3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$ 2656
3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2666
3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2677
3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2687
3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2694
3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$ 2700
3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$ 2710
3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$ 2720
3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2732
3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2738
3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$ 2745
3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$ 2752
3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2759
3.333 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2769
3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2775
3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2783
3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ 2790
3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$ 2798
3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$ 2805
3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$ 2810
3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ 2817

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$ 2825
3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$ 2834
3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$ 2841
3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2848
3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2855
3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2862
3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$ 2869
3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$ 2877
3.349 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$ 2883
3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$ 2889
3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$ 2894
3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$ 2899
3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$ 2909
3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$ 2920
3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$ 2928
3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$ 2935
3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$ 2943
3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 2953

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

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3.1.1 Optimal result

Integrand size = 33, antiderivative size = 373

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}$$
$$+\frac{a^3(B(15+19n+4n^2)+A(20+21n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx\right.}{df(1+n)(2+n)(4+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^3(B(9+4n)+A(11+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$

output

```
-a^3*(B*(2*n^2+14*n+27)+A*(2*n^2+15*n+28))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)
/d/f/(4+n)/(n^2+5*n+6)-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e)
)^2/d/f/(4+n)-(A*(4+n)+B*(6+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^3+a^3*s
in(f*x+e))/d/f/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*cos(f
*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^
(1+n)/d/f/(4+n)/(n^2+3*n+2)/(cos(f*x+e)^2)^(1/2)+a^3*(B*(9+4*n)+A*(11+4*n)
)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e
))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.66

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \frac{a^3\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(3A+B)\,\text{Hy}}{}\right.\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x
]
```

output
```
(a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/
2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A +
B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) +
 Sin[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin
[e + f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4
 + n)/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (
5 + n)/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Co
s[e + f*x]^2])
```

3.1.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3455, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$
$$\downarrow 3455$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^2(ad(B(n+1)+A(n+4))+ad(A(n+4)+B(n+6))\sin(e+fx))dx}{d(n+4)} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^2(ad(B(n+1)+A(n+4))+ad(A(n+4)+B(n+6))\sin(e+fx))dx}{d(n+4)} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3455$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)\left(a^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)d^2+a^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)d^2\right)dx}{d(n+3)} - \frac{(A(n+}{d(n+4)} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)\left(a^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)d^2+a^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)d^2\right)dx}{d(n+3)} - \frac{(A(n+}{d(n+4)} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3447$$

$$\frac{\int (d\sin(e+fx))^n\left(d^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin^2(e+fx)a^3+d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)a^3+\left(d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2\right.\right.\right.\right.}{d(n+3)} - \frac{d(n}{} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n\left(d^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)^2a^3+d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)a^3+\left(d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2\right.\right.\right.\right.}{d(n+3)} - \frac{d(n}{} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3502$$

$$\frac{\frac{\int (d\sin(e+fx))^n \left(a^3(n+3)\left(B\left(4n^2+19n+15\right)+A\left(4n^2+21n+20\right)\right)d^3+a^3(n+2)(n+4)(B(4n+9)+A(4n+11))\sin(e+fx)d^3\right)dx}{d(n+2)} - \frac{a^3 d\left(A\left(2n^2+15n+28\right)+B\left(2n^2+14\right.\right.}{f(n}}{d(n+3)}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (d\sin(e+fx))^n \left(a^3(n+3)\left(B\left(4n^2+19n+15\right)+A\left(4n^2+21n+20\right)\right)d^3+a^3(n+2)(n+4)(B(4n+9)+A(4n+11))\sin(e+fx)d^3\right)dx}{d(n+2)} - \frac{a^3 d\left(A\left(2n^2+15n+28\right)+B\left(2n^2+14\right.\right.}{f(n}}{d(n+3)}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3227$$

$$\frac{\frac{a^3d^3(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\int (d\sin(e+fx))^n dx+a^3d^2(n+2)(n+4)(A(4n+11)+B(4n+9))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{a^3 d\left(A\left(2n^2+15n+28\right)+B\right.}{d(n+3)}}{d(n+4)}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3042$$

$$\frac{\frac{a^3d^3(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\int (d\sin(e+fx))^n dx+a^3d^2(n+2)(n+4)(A(4n+11)+B(4n+9))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{a^3 d\left(A\left(2n^2+15n+28\right)+B\right.}{d(n+3)}}{d(n+4)}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\downarrow 3122$$

$$\frac{\frac{a^3d^2(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}} + \frac{a^3d(n+4)(A(4n+11)+B(4n+9))}{d(n+2)}}{d(n+3)}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output
```
-((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*
(4 + n))) + (-(((A*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
 n)*(a^3 + a^3*Sin[e + f*x]))/(f*(3 + n))) + (-((a^3*d*(B*(27 + 14*n + 2*n
^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(2
+ n))) + ((a^3*d^2*(3 + n)*(B*(15 + 19*n + 4*n^2) + A*(20 + 21*n + 4*n^2))
*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]
*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d*(4 +
n)*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n
)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*Sqrt[Cos[e +
f*x]^2]))/(d*(2 + n)))/(d*(3 + n)))/(d*(4 + n))
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.1.4 Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^3 (A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

3.1.5 Fricas [F]

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^3 (A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3 (d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*si
n(f*x + e))^n, x)
```

3.1.6 Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

3.1.7 Maxima [F]

$$\begin{aligned}&\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx \\ &= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx\end{aligned}$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n,
x)
```

3.1.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n,
x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\,(a+a\,\sin(e+f\,x))^3\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

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3.2.1 Optimal result

Integrand size = 33, antiderivative size = 277

$$
\begin{aligned}
&\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx \\
&= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} \\
&\quad + \frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}} \\
&\quad + \frac{a^2(2A(3+n)+B(5+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}} \\
&\quad - \frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\,(a^2+a^2\sin(e+fx))}{df(3+n)}
\end{aligned}
$$

output

```
-a^2*(A*(3+n)+B*(4+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)/(3+n)-B*c
os(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^2+a^2*sin(f*x+e))/d/f/(3+n)+a^2*(2*B*(1+
n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)
^2)*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a^2*(2*A*(3+
n)+B*(5+2*n))*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*
(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.74

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \frac{a^2\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(2A+B)\,\text{Hy}}{}\right.\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x
]
```

output
```
(a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/
2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A +
B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) +
 Sin[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin
[e + f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Si
n[e + f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

3.2.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$
$$\downarrow 3455$$
$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)(ad(B(n+1)+A(n+3))+ad(A(n+3)+B(n+4))\sin(e+fx))dx}{d(n+3)} -$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)(ad(B(n+1)+A(n+3))+ad(A(n+3)+B(n+4))\sin(e+fx))dx}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3447$$

$$\frac{\int (d\sin(e+fx))^n\left(d(A(n+3)+B(n+4))\sin^2(e+fx)a^2+d(B(n+1)+A(n+3))a^2+\left(d(B(n+1)+A(n+3)\right.\right.}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n\left(d(A(n+3)+B(n+4))\sin(e+fx)^2a^2+d(B(n+1)+A(n+3))a^2+\left(d(B(n+1)+A(n+3)\right.\right.}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3502$$

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^2(n+3)(2B(n+1)+A(2n+3))d^2+a^2(n+2)(2A(n+3)+B(2n+5))\sin(e+fx)d^2\right)dx}{d(n+2)} - \frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin(e}{f(n+2)}}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^2(n+3)(2B(n+1)+A(2n+3))d^2+a^2(n+2)(2A(n+3)+B(2n+5))\sin(e+fx)d^2\right)dx}{d(n+2)} - \frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin(e}{f(n+2)}}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3227$$

$$\frac{\frac{a^2d^2(n+3)(A(2n+3)+2B(n+1))\int (d\sin(e+fx))^n dx+a^2d(n+2)(2A(n+3)+B(2n+5))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{a^2(A(n+3)+B(n+4))\cos(e+fx}{f(n+2)}}{d(n+3)} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2d^2(n+3)(A(2n+3)+2B(n+1))\int(d\sin(e+fx))^n dx+a^2d(n+2)(2A(n+3)+B(2n+5))\int(d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{a^2(A(n+3)+B(n+4))\cos(e+fx}{f(n+2)}}{d(n+3)}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\downarrow 3122$$

$$\frac{\frac{a^2d(n+3)(A(2n+3)+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}} + \frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}}{f\sqrt{\cos^2(e}}}{d(n+2)}}{d(n+3)}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))) + (-((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^
(1 + n))/(f*(2 + n))) + ((a^2*d*(3 + n)*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin
[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) +
 B*(5 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*Sqrt[Cos[e + f*x]^2]))/(d*(2 +
n)))/(d*(3 + n))
```

3.2.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.2.4 Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^2 (A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

3.2.5 Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

3.2.6 Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

3.2.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x)
```

3.2.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n,
x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \int (d\sin(e+fx))^n(A+B\sin(e+fx))(a+a\sin(e+fx))^2\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

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3.3.1 Optimal result

Integrand size = 31, antiderivative size = 191

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}$$
$$+\frac{a(B(1+n)+A(2+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$

output

```
-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*cos(f*x
+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1
+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a*(A+B)*cos(f*x+e)*hypergeom([1/2
, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(cos(f
*x+e)^2)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \frac{a\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left((B(1+n)+A(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(\right.\right.}{f(1+n)(2+}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((B*(1 + n) + A*(2 + n))*H
ypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] - (1 + n)*(B*S
qrt[Cos[e + f*x]^2] - (A + B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
 Sin[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2])
```

3.3.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$
$$\downarrow 3447$$
$$\int (d\sin(e+fx))^n\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$
$$\downarrow 3042$$
$$\int (d\sin(e+fx))^n\left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$
$$\downarrow 3502$$

$$\frac{\int (d\sin(e+fx))^n (ad(B(n+1)+A(n+2))+a(A+B)d(n+2)\sin(e+fx))dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n (ad(B(n+1)+A(n+2))+a(A+B)d(n+2)\sin(e+fx))dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\downarrow 3227$$

$$\frac{ad(A(n+2)+B(n+1))\int (d\sin(e+fx))^n dx + a(n+2)(A+B)\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\downarrow 3042$$

$$\frac{ad(A(n+2)+B(n+1))\int (d\sin(e+fx))^n dx + a(n+2)(A+B)\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\downarrow 3122$$

$$\frac{\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}} + \frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hyperg}}{df\sqrt{\cos^2(e}}}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
-((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + ((a*(B*(1 +
 n) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]
) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, S
in[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d*f*Sqrt[Cos[e + f*x]^2]))/(d*(2
 + n))
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.3.4 Maple [F]

$$\int (d \sin (fx+e))^n (a + a \sin (fx+e)) (A + B \sin (fx+e)) \, dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

3.3.5 Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin
(f*x + e))^n, x)
```

3.3.6 Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

3.3.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

3.3.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\,(a+a\,\sin(e+f\,x))\,\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

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3.4.1 Optimal result

Integrand size = 33, antiderivative size = 202

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(B-An+Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{ad^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))}$$

output

```
(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))+(-A*n+B*n+B)*co
s(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e
))^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f*x+e)*hypergeom
([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a/d^2/f/(2+n)
/(cos(f*x+e)^2)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(d\sin(e+fx))^n\left(\frac{(B-An+Bn)\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\frac{(A-B)(1+n)\sqrt{\cos^2(e+fx)}\,\text{Hypergeome}}{af}\right.$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x
]
```

output
```
((d*Sin[e + f*x])^n*(((B - A*n + B*n)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2
F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + ((A - B)*(1 + n)*
Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
f*x]^2]*Sin[e + f*x])/(2 + n) + ((A - B)*Cos[e + f*x]^2)/(1 + Sin[e + f*x]
))*Tan[e + f*x])/(a*f)
```

3.4.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3457, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3457$$
$$\frac{\int (d\sin(e+fx))^n(ad(nB+B-An)+a(A-B)d(n+1)\sin(e+fx))dx}{a^2d}+$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$
$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n(ad(nB+B-An)+a(A-B)d(n+1)\sin(e+fx))dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$$\downarrow 3227$$

$$\frac{ad(-An+Bn+B)\int (d\sin(e+fx))^n dx+a(n+1)(A-B)\int (d\sin(e+fx))^{n+1}dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{ad(-An+Bn+B)\int (d\sin(e+fx))^n dx+a(n+1)(A-B)\int (d\sin(e+fx))^{n+1}dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$$\downarrow 3122$$

$$\frac{\frac{a(-An+Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}}+\frac{a(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,\text{Hype}}{df(n+2)\sqrt{\cos^2}}}{a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*Sin[e + f*x]))
 + ((a*(B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
 n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2
, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d*f*(2 + n)*Sqrt[C
os[e + f*x]^2]))/(a^2*d)
```

3.4.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.4.4 Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{a+a\sin(fx+e)}dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

3.4.5 Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

3.4.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

3.4.7 Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

3.4.8 Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)
```

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

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3.5.1 Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{n(A-2An+2B(1+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+}}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1+n)(B+2A(1-n)+2Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+}{3a^2d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2}$$

output

```
1/3*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+sin(f*x
+e))+1/3*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^2-1/3*
n*(A-2*A*n+2*B*(1+n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],si
n(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1/3*(1
+n)*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(
f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^2/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{(d\sin(e+fx))^n\left((A-B)\sin(2(e+fx)) - \frac{2(1+\sin(e+fx))\Big((1+n)(2+n)(2A(-1+n)-B(1+2n))\cos^2(e+fx)+\sqrt{\cos^2(e+fx)}(}{}\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2
,x]
```

output
```
((d*Sin[e + f*x])^n*((A - B)*Sin[2*(e + f*x)] - (2*(1 + Sin[e + f*x])*((1
+ n)*(2 + n)*(2*A*(-1 + n) - B*(1 + 2*n))*Cos[e + f*x]^2 + Sqrt[Cos[e + f*
x]^2]*(1 + Sin[e + f*x])*(n*(2 + n)*(A - 2*A*n + 2*B*(1 + n))*Hypergeometr
ic2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] + (1 + n)^2*(2*A*(-1 + n)
 - B*(1 + 2*n))*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^
2]*Sin[e + f*x]))*Tan[e + f*x])/((1 + n)*(2 + n))))/(6*a^2*f*(1 + Sin[e +
f*x])^2)
```

3.5.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3457, 3042, 3457, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}dx$$
$$\downarrow 3457$$
$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+2A+B+Bn)+a(A-B)dn\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+2A+B+Bn)+a(A-B)dn\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3457$$

$$\frac{\int -(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+fx)\right)dx}{a^2d}+\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+f}{f(\sin(e+fx)+1)}$$

$$\frac{3a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 25$$

$$\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{\int(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+fx\right.}{a^2d}$$

$$\frac{3a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{\int(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+fx\right.}{a^2d}$$

$$\frac{3a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3227$$

$$\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{a^2d^2n(-2An+A+2B(n+1))\int(d\sin(e+fx))^ndx-a^2d(n+1)(2A(1-n)+2Bn+B)\int(d\sin(e}{a^2d}$$

$$\frac{3a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{a^2d^2n(-2An+A+2B(n+1))\int(d\sin(e+fx))^ndx-a^2d(n+1)(2A(1-n)+2Bn+B)\int(d\sin(e}{a^2d}$$

$$\frac{3a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$$\downarrow 3122$$

$$\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)} - \frac{a^2dn(-2An+A+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx\right.}{f(n+1)\sqrt{\cos^2(e+fx)}}$$

$$3a^2d$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f*x]
)^2) + (((2*A*(1 - n) + B*(1 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)
)/(f*(1 + Sin[e + f*x])) - ((a^2*d*n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x
]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e +
f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) - (a^2*(1 + n)*(B + 2*A*(1
 - n) + 2*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, S
in[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*(2 + n)*Sqrt[Cos[e + f*x]^2]))
/(a^2*d))/(3*a^2*d)
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.5.4 Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{(a+a\sin(fx+e))^2}dx$$

input

```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output

```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

3.5.5 Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

input

```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output

```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*
a^2*sin(f*x + e) - 2*a^2), x)
```

3.5.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.5.7 Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2,
x)
```

3.5.8 Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2,
x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

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3.6.1 Optimal result

Integrand size = 33, antiderivative size = 362

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^n}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1-n)(1+n)(7A+3B-4An+4Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{15a^3d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3}$$
$$+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2}$$
$$+\frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\,(a^3+a^3\sin(e+fx))}$$

output

```
1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^3+1/15*(A*(
5-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f/(a+a*sin(f*x+e))^2+1/1
5*(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a^3+a^
3*sin(f*x+e))-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom
([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(
1+n)/(cos(f*x+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+
e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a
^3/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.80

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{(d\sin(e+fx))^n\left(\frac{3}{2}a^5(A-B)\sin(2(e+fx)) + \frac{a^5(1+\sin(e+fx))\Big((1+n)(2+n)(A(5-2n)+2Bn)\cos^2(e+fx)-(1+\sin(e+fx))\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3
,x]
```

output
```
((d*Sin[e + f*x])^n*((3*a^5*(A - B)*Sin[2*(e + f*x)])/2 + (a^5*(1 + Sin[e
+ f*x])*((1 + n)*(2 + n)*(A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]^2 - (1 + Sin[e
 + f*x])*(-((-1 + n)*(1 + n)*(2 + n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Cos[e +
f*x]^2) + Sqrt[Cos[e + f*x]^2]*(1 + Sin[e + f*x])*(n*(2 + n)*(A*(2 - 9*n +
 4*n^2) - B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sin[e + f*x]^2] - (-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hyperge
ometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])))*Tan[
e + f*x])/((1 + n)*(2 + n))))/(15*a^8*f*(1 + Sin[e + f*x])^3)
```

3.6.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^3}dx$$
$$\downarrow 3457$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+4A+B+Bn)-a(A-B)d(1-n)\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+4A+B+Bn)-a(A-B)d(1-n)\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3457$$

$$\frac{\int \frac{(d\sin(e+fx))^n\left(a^2\left(B\left(-2n^2+n+3\right)+A\left(2n^2-6n+7\right)\right)d^2+a^2n(A(5-2n)+2Bn)\sin(e+fx)d^2\right)}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{a(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3f(a\sin(e+fx)+a)^2}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(d\sin(e+fx))^n\left(a^2\left(B\left(-2n^2+n+3\right)+A\left(2n^2-6n+7\right)\right)d^2+a^2n(A(5-2n)+2Bn)\sin(e+fx)d^2\right)}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{a(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3f(a\sin(e+fx)+a)^2}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3457$$

$$\frac{\frac{\int -(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B+4Bn)\sin(e+fx)\right)dx}{a^2d}+\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+f}{f(a\sin(e+fx)+a)}}{3a^2d}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 25$$

$$\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{\int(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B+4}{a^2d}}{3a^2d}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{\int(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B+4\right.}{a^2d}}{3a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3227$$

$$\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{a^3d^3n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\int(d\sin(e+fx))^n dx-a^3d^2(1-n)(n+1)(-4An+7A+4Bn}{a^2d}}{3a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{a^3d^3n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\int(d\sin(e+fx))^n dx-a^3d^2(1-n)(n+1)(-4An+7A+4Bn}{a^2d}}{3a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\downarrow 3122$$

$$\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{a^3d^2n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\cos(e+fx)(d\sin(e+fx))^{n+1}\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{n+}{2}\right.}{f(n+1)\sqrt{\cos^2(e+fx)}}}{3a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e + f*x]
)^3) + ((a*(A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3
*f*(a + a*Sin[e + f*x])^2) + ((a^2*d*(1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*C
os[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(a + a*Sin[e + f*x])) - ((a^3*d^2
*n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*
(1 + n)*Sqrt[Cos[e + f*x]^2]) - (a^3*d*(1 - n)*(1 + n)*(7*A + 3*B - 4*A*n
+ 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
 f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*(2 + n)*Sqrt[Cos[e + f*x]^2]))/(a^2*
d))/(3*a^2*d))/(5*a^2*d)
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.6.4 Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{(a+a\sin(fx+e))^3}dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

3.6.5 Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 -
4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.6.7 Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3,
x)
```

3.6.8 Giac [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3,
x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

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3.7.1 Optimal result

Integrand size = 35, antiderivative size = 336

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx =$$

$$-\frac{2a^3(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\,\text{Hypergeometric2F1}}{f(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)}$$

$$-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)}$$

output

```
-2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(3/2)/d/f/(7+2*n)-
2*a^3*(2*B*(16*n^3+104*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*cos(f*
x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5
+2*n)/(7+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(2*B*(4*n^2+23*n
+35)+A*(8*n^2+50*n+77))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(5+2*n
)/(7+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(5+n)+A*(7+2*n))*cos(f*x+e)*(d
*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)/(7+2*n)
```

3.7.2 Mathematica [A] (warning: unable to verify)

Time = 25.42 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.77

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A$$

$$+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{5/2}\tan\left(\frac{1}{2}(e+$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x
]),x]
```

output
```
(2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2
)*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[
(e + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Ta
n[(e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/
2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((
5*A + 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)
/2]^2])/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 +
n)/2, 9/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]
*((5*(3*A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e
+ f*x)/2]^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1
[9/2 + n, (5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f
*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -T
an[(e + f*x)/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 +
 n)/2, (9 + n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n))))))))/(f
*Sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e +
f*x]^n)
```

3.7.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3255, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}(d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}\left(ad(2B(n+1)+2A(n+\frac{7}{2}))+ad(2B(n+5)+A(2n+7))\sin(e+f\right.}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 27$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}(ad(2B(n+1)+A(2n+7))+ad(2B(n+5)+A(2n+7))\sin(e+fx))dx}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}(ad(2B(n+1)+A(2n+7))+ad(2B(n+5)+A(2n+7))\sin(e+fx))dx}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}(d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)dx}{d(2n+5)}$$
$$d(2n+7)$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 27$$

$$\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)dx}{d(2n+5)} \quad -$$
$$d(2n+7)$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (d\sin(e+fx))^n \sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)dx}{d(2n+5)} - \cdots}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3460$$

$$\frac{\frac{a^2d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3} - \frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)\cos(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3} - \frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)\cos(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 3255$$

$$\frac{\frac{a^4d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\int \frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)\cos}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 77$$

$$\frac{\frac{a^4d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2a^3d\left(A\left(8n^2+50n+\right.\right.}{}}{d(2n+5)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\downarrow 75$$

$$\frac{-\frac{2a^3d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^3}{}}{d(2n+5)}$$

$$d(2n+$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2))/
(d*f*(7 + 2*n)) + ((-2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*(d*Sin
[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(f*(5 + 2*n)) + ((-2*a^3*d^2*
(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 + 32*n^3)
)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e
+ f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*
d*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*Cos[e + f*x]*(d*Sin[e
+ f*x])^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]))/(d*(5 + 2*n)))/(d
*(7 + 2*n))
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.7.4 Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{5}{2}} (A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

3.7.5 Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
 e))^n, x)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))
^n, x)
```

3.7.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))
^n, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (d\,\sin(e+f\,x))^n\,(A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

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3.8.1 Optimal result

Integrand size = 35, antiderivative size = 229

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx =$$
$$-\frac{2a^2(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)}$$

output

```
-2*a^2*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*cos(f*x+e)*hypergeom([1/2, -
n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(sin(f*x+e)^n)/(
a+a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(3+n)+A*(5+2*n))*cos(f*x+e)*(d*sin(f*x+e)
)^(1+n)/d/f/(3+2*n)/(5+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(d*sin
(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

3.8.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 478 vs. $2(229) = 458$.

Time $= 20.87$ (sec) , antiderivative size $= 478$, normalized size of antiderivative $= 2.09$

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A$$

$$+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{3/2}\tan\left(\frac{1}{2}(e+f\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x
]),x]
```

output
```
(2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2
)*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[
(e + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Ta
n[(e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric
2F1[(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e
+ f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2,
 -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeo
metric2F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + T
an[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n
)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)
/2, (8 + n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqr
t[Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]
^n)
```

3.8.3 Rubi [A] (verified)

Time $= 0.93$ (sec) , antiderivative size $= 225$, normalized size of antiderivative $= 0.98$, number of steps used $= 10$, number of rules used $= 9$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used $= \{3042, 3455, 27, 3042, 3460, 3042, 3255, 77, 75\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}(d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(ad\left(2B(n+1)+2A\left(n+\frac{5}{2}\right)\right)+ad(2B(n+3)+A(2n+5))\sin(e+fx)\right)}{d(2n+5)} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\downarrow 27$$

$$\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}(ad(2B(n+1)+A(2n+5))+ad(2B(n+3)+A(2n+5))\sin(e+fx))dx}{d(2n+5)} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\downarrow 3042$$

$$\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}(ad(2B(n+1)+A(2n+5))+ad(2B(n+3)+A(2n+5))\sin(e+fx))dx}{d(2n+5)} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\downarrow 3460$$

$$\frac{\frac{ad(A(8n^2+30n+25)+2B(4n^2+13n+9))\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3} - \frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\downarrow 3042$$

$$\frac{\frac{ad(A(8n^2+30n+25)+2B(4n^2+13n+9))\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3} - \frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\downarrow 3255$$

$$\frac{a^3d(A(8n^2+30n+25)+2B(4n^2+13n+9))\cos(e+fx)\int\frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow 77$$

$$\frac{a^3d(A(8n^2+30n+25)+2B(4n^2+13n+9))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+5)+2B(n+3))\cos}{f(2n+3)\sqrt{a\sin}}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow 75$$

$$-\frac{2a^2d(A(8n^2+30n+25)+2B(4n^2+13n+9))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\tfrac{1}{2},-n,\tfrac{3}{2},1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n-}{}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

input

```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output

```
(-2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d
*f*(5 + 2*n)) + ((-2*a^2*d*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2)
)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e
+ f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*
(2*B*(3 + n) + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(3 +
 2*n)*Sqrt[a + a*Sin[e + f*x]]))/(d*(5 + 2*n))
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.8.4 Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{3}{2}} (A+B\sin(fx+e))\,dx$$

input

```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

output

```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

3.8.5 Fricas [F]

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^{3/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input

```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output

```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

3.8.6 Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A + B\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}\,(d\sin(e+fx))^n\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n*(A + B*sin(e +
f*x)), x)
```

3.8.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))
^n, x)
```

3.8.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A + B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))
^n, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (d\,\sin(e+f\,x))^n\,(A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.9 $\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

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3.9.1 Optimal result

Integrand size = 35, antiderivative size = 137

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx =$$
$$-\frac{2a(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

output
```
-2*a*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+
e))*(d*sin(f*x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B
*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

3.9.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.99

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx =$$
$$-\frac{(1+i)2^{-2-n}e^{-\frac{3ie}{2}+ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\left(\frac{2Be^{-\frac{1}{2}if(3+2n)x}\,\text{Hypergeometric2F1}\left(\frac{1}{4}(\right.}{f(3+2n}$$

input
```
Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])
,x]
```

output
```
((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
 f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 -
 2*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E
^(I*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E
^((2*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*
e + f*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4,
 -n, (5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*H
ypergeometric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(
-3 + 2*n)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])])/((1
 - E^((2*I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*
x]^n)
```

3.9.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3460, 3042, 3255, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\downarrow 3460$$

$$\left(A+\frac{2B(n+1)}{2n+3}\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\left(A+\frac{2B(n+1)}{2n+3}\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3255$$

$$\frac{a^2\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\int\frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 77$$

$$\frac{a^2\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 75$$

$$-\frac{2a\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*(A + (2*B*(1 + n))/(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n
, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2
*n)*Sqrt[a + a*Sin[e + f*x]])
```

3.9.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.9.4 Maple [F]

$$\int (d\sin(fx+e))^n \sqrt{a+a\sin(fx+e)}\,(A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

3.9.5 Fricas [F]

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

3.9.6 Sympy [F]

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(d\sin\left(e+fx\right)\right)^n\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x
)), x)
```

3.9.7 Maxima [F]

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
, x)
```

3.9.8 Giac [F]

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int (d\sin(e+fx))^n (A+B\sin(e+fx))\sqrt{a+a\sin(e+fx)}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

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3.10.1 Optimal result

Integrand size = 35, antiderivative size = 152

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e}{f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
-(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(
d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*B*cos(f*x+e)*hyp
ergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+
a*sin(f*x+e))^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 12.00 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{\cos(e+fx)\sin^n(e+fx)(d\sin(e+fx))^n\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(4(}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x
]],x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]
*(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1
 + Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin
[e + f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])
/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1
))^n))/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 +
Sin[e + f*x])^(-1))^n)
```

3.10.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3466, 3042, 3255, 77, 75, 3266, 3042, 3265, 3042, 3264, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3466$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + \frac{B\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + \frac{B\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{a}$$

$$\downarrow 3255$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + \frac{aB\cos(e+fx)\int \frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 77$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + \frac{aB\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 75$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3266$$

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3265$$

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin(e+fx)^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3264$$

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)}d(1-\sin(e+fx))}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 148$$

$$-\frac{2(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sin(e+fx)+1}d\sqrt{1-\sin(e+fx)}}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 333$$

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/
2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f
*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x
]]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

3.10.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3466

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.10.4 Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{\sqrt{a+a\sin(fx+e)}}dx$$

input

```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

output

```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

3.10.5 Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input

```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

3.10.6 Sympy [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input

```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

3.10.7 Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a)
, x)
```

3.10.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x
)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
x)
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

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3.11.1 Optimal result

Integrand size = 35, antiderivative size = 226

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}}$$
$$-\frac{(A-4An+B(3+4n))\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+}{4af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-B)(1+2n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}$$

output

```
1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(
A-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-1/2*
(A-B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*
x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

3.11.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 523 vs. 2(226) = 452.

Time = 27.19 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.31

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sec(e+fx)(d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(a\,\text{Appell}\right.\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(
3/2),x]
```

output
```
(Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]
]*(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 +
 Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1
 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n
, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + S
in[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*A
ppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*
Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Si
n[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1
[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)
] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + S
in[e + f*x])^(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

3.11.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3457, 27, 3042, 3466, 3042, 3255, 77, 75, 3266, 3042, 3265, 3042, 3264, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\downarrow 3457$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3466$$

$$\frac{ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + d(2n+1)(A-B)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + d(2n+1)(A-B)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3255$$

$$\frac{\frac{a^2d(2n+1)(A-B)\cos(e+fx)\int \frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} + ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 77$$

$$\frac{\frac{a^2d(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} + ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx)}{\sqrt{\sin(e+fx)a}}}{4a^2d} + \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 75$$

$$\frac{ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{\frac{4a^2d}{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n \text{ Hypergeometric2F1}(\frac{1}{2}}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3266$$

$$\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n \text{ Hypergeometric2F1}(\frac{1}{2}}{f\sqrt{a\sin(e+fx)+a}}$$

$$\frac{4a^2d}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3042$$

$$\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n \text{ Hypergeometric2F1}(\frac{1}{2}}{f\sqrt{a\sin(e+fx)+a}}$$

$$\frac{4a^2d}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3265$$

$$\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+}{f\sqrt{a\sin(e}}$$

$$\frac{4a^2d}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3042$$

$$\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin(e+fx)^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+}{f\sqrt{a\sin(e}}$$

$$\frac{4a^2d}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3264$$

$$-\frac{ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)}d(1-\sin(e+fx))}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+f}{}$$

$$\frac{4a^2d}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 148$$

$$\frac{-\frac{2ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sin(e+fx)+1}d\sqrt{1-\sin(e+fx)}}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+f}{4a^2d}}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 333$$

$$\frac{-\frac{ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)\,\text{AppellF1}(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx)))(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}} - \frac{2ad(2n+1)(A-B)\co}{4a^2d}}{\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x
]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x]
)^(3/2)) + (-((a*d*(A + 3*B - 4*A*n + 4*B*n)*AppellF1[1/2, -n, 1, 3/2, 1 -
 Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*S
in[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])) - (2*a*(A - B)*d*(1 + 2*n)*Cos[e
+ f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^
n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))/(4*a^2*d)
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.11.4 Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{(a+a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

3.11.5 Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

3.11.6 Sympy [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

3.11.7 Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/
2), x)
```

3.11.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x
)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
x)
```

3.12 $\int (d\sin(e+fx))^n (a+a\sin(e+fx))^m (A+B\sin(e+fx))\,dx$

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3.12.1 Optimal result

Integrand size = 33, antiderivative size = 221

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^m (A+B\sin(e+fx))\,dx =$$

$$-\frac{2^{\frac{3}{2}+m} B\,\text{AppellF1}\left(\frac{1}{2},-n,-\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d}{f}$$

$$-\frac{2^{\frac{1}{2}+m}(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+f}{f}$$

output

```
-2^(3/2+m)*B*AppellF1(1/2,-n,-1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(s
in(f*x+e)^n)-2^(1/2+m)*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin
(f*x+e))^m/f/(sin(f*x+e)^n)
```

3.12.2 Mathematica [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x
]
```

output
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),
x]
```

3.12.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3466, 3042, 3266, 3042, 3265, 3042, 3264, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$
$$\downarrow 3466$$
$$(A-B)\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^m dx + \frac{B\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{m+1}dx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^m dx + \frac{B\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{m+1}dx}{a}$$
$$\downarrow 3266$$

$$(A-B)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^m dx+$$
$$B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^{m+1}dx$$

$$\downarrow 3042$$

$$(A-B)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^m dx+$$
$$B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^{m+1}dx$$

$$\downarrow 3265$$

$$(A-B)(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin^n(e+fx)(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin^n(e+fx)(\sin(e+fx)+1)^{m+1}dx$$

$$\downarrow 3042$$

$$(A-B)(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin(e+fx)^n(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin(e+fx)^n(\sin(e+fx)+1)^{m+1}dx$$

$$\downarrow 3264$$

$$-\frac{(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)(\sin(e+fx)+1)^m}{\sqrt{1-\sin(e+fx)}}}{f\sqrt{1-\sin(e+fx)}}$$
$$\frac{B\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)(\sin(e+fx)+1)^m}{\sqrt{1-\sin(e+fx)}}}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 150$$

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\,\text{AppellF1}\left(\right.}{f}$$
$$\frac{B2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\,\text{AppellF1}\left(\frac{1}{2},-n,-\right.}{f}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 -
 m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*App
ellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
 + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*
x])^m)/(f*Sin[e + f*x]^n)
```

3.12.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3466

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.12.4 Maple [F]

$$\int (d \sin (fx+e))^n (a+a \sin (fx+e))^m (A+B \sin (fx+e))\, dx$$

input

```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output

```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

3.12.5 Fricas [F]

$$\int (d \sin(e+fx))^n (a+a \sin(e+fx))^m (A+B \sin(e+fx))\, dx$$
$$= \int (B \sin (fx+e)+A)(a \sin (fx+e)+a)^m (d \sin (fx+e))^n \, dx$$

input

```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

3.12.6 Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (a(\sin(e+fx)+1))^m\,(d\sin(e+fx))^n\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)
), x)
```

3.12.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

3.12.8 Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

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3.13.1 Optimal result

Integrand size = 34, antiderivative size = 114

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(1+n,-\frac{1}{2},\frac{1}{2}-m,2+n,\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(d\sin(e+fx))^{1+n}(1+\sin(e+$$
$$df(1+n)\sqrt{1-\sin(e+fx)}$$

output
```
AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+
e))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(
1+n)/(1-sin(f*x+e))^(1/2)
```

3.13.2 Mathematica [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x
]
```

output
```
Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,
x]
```

3.13.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3487, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m (d\sin(e+fx))^n \, dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m (d\sin(e+fx))^n dx$$

$$\downarrow 3487$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int (d\sin(e+fx))^n \sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}} d\s}{f}$$

$$\downarrow 152$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \sqrt{1-\sin(e+fx)}(d\sin(e+fx))^n(\sin(e+fx)a+a)^{m-\frac{1}{2}} d\sin}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 152$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m \int \sqrt{1-\sin(e+fx)}(d\sin(e+fx))^n(\sin}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 150$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m (d\sin(e+fx))^{n+1} \operatorname{AppellF1}\left(n+1,-\frac{1}{2},\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

output
```
(AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e
+ f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

3.13.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.13.4 Maple [F]

$$\int (d\sin(fx+e))^n (a - a\sin(fx+e)) (a + a\sin(fx+e))^m \, dx$$

input
```
int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

output
```
int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

3.13.5 Fricas [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

3.13.6 Sympy [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= -a\left(\int \left(-(d\sin(e+fx))^n(a\sin(e+fx)+a)^m\right)\,dx + \int (d\sin(e+fx))^n(a\sin(e+fx)+a)^m\sin(e+fx)\,dx\right)$$

input
```
integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

output
```
-a*(Integral(-(d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m, x) + Integral((
d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

3.13.7 Maxima [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
 x)
```

3.13.8 Giac [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
 x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(a+a\,\sin(e+f\,x))^m\,(a-a\,\sin(e+f\,x))\,dx$$

input
```
int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)
```

output
```
int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

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3.14.1 Optimal result

Integrand size = 43, antiderivative size = 37

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{-2-n}}{d}$$

output
```
-cos(d*x+c)*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))^(-2-n)/d
```

3.14.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(37) = 74.

Time = 5.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{2^n \sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{4}(c+dx)\right)\left(-\sin\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{3}{4}(c+\right.\right.}{d}$$

input
```
Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*
Sin[c + d*x]),x]
```

output
```
-((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*
x)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Si
n[c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

3.14.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^n(c+dx)(-(-n-2)\sin(c+dx)-n-1)(a\sin(c+dx)+a)^{-n-2}\,dx$$

$$\downarrow 3042$$

$$\int \sin(c+dx)^n(-(-n-2)\sin(c+dx)-n-1)(a\sin(c+dx)+a)^{-n-2}dx$$

$$\downarrow 3453$$

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

input
```
Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c
+ d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

3.14.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

3.14.4 Maple [F]

$$\int (\sin^n(dx+c))\,(a+a\sin(dx+c))^{-2-n}\,(-1-n-(-2-n)\sin(dx+c))\,dx$$

input

```
int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

output

```
int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= -\frac{(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\cos(dx+c)\sin(dx+c)}{d}$$

input

```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="fricas")
```

output

```
-(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

3.14.6 Sympy [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int (a(\sin(c+dx)+1))^{-n-2}\,(n\sin(c+dx)-n+2\sin(c+dx)-1)\sin^n(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),
x)
```

output
```
Integral((a*(sin(c + d*x) + 1))**(-n - 2)*(n*sin(c + d*x) - n + 2*sin(c +
d*x) - 1)*sin(c + d*x)**n, x)
```

3.14.7 Maxima [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int ((n+2)\sin(dx+c)-n-1)(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\,dx$$

input
```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="maxima")
```

output
```
integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin
(d*x + c)^n, x)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9496 vs. $2(37) = 74$.

Time = 84.24 (sec) , antiderivative size = 9496, normalized size of antiderivative = 256.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="giac")
```

output
```
-8*(cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1
/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) +
 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4
*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*
e^(-n*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 +
 abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*
x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x +
 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2
+ 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x
 + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*...
```

3.14.9 Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= -\frac{\sin(c+dx)^n\,\sin(2c+2dx)}{a^2\,d\,(a\,(\sin(c+dx)+1))^n\,\left(2\sin(c+dx)^2+4\,\sin(c+dx)+2\right)}$$

input
```
int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^
(n + 2),x)
```

output
```
-(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(
c + d*x) + 2*sin(c + d*x)^2 + 2))
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

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3.15.1 Optimal result

Integrand size = 37, antiderivative size = 35

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

output
```
-cos(d*x+c)*sin(d*x+c)^(-1-m)*(a+a*sin(d*x+c))^m/d
```

3.15.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a(1+\sin(c+dx)))^m}{d}$$

input
```
Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c +
d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

3.15.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{-m-2}(c+dx)(m(-\sin(c+dx))+m+1)(a\sin(c+dx)+a)^m\,dx$$

$$\downarrow 3042$$

$$\int \sin(c+dx)^{-m-2}(m(-\sin(c+dx))+m+1)(a\sin(c+dx)+a)^m dx$$

$$\downarrow 3453$$

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

input
```
Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),
x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

3.15.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

3.15.4 Maple [F]

$$\int \left(\sin^{-2-m}(dx+c)\right)(a+a\sin(dx+c))^m(1+m-m\sin(dx+c))\,dx$$

input
```
int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= -\frac{(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\cos(dx+c)\sin(dx+c)}{d}$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="fricas")
```

output
```
-(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

3.15.6 Sympy [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= -\int \left(-(a\sin(c+dx)+a)^m\sin^{-m-2}(c+dx)\right)\,dx$$
$$-\int \left(-m(a\sin(c+dx)+a)^m\sin^{-m-2}(c+dx)\right)\,dx$$
$$-\int m(a\sin(c+dx)+a)^m\sin(c+dx)\sin^{-m-2}(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

output
```
-Integral(-(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(-
m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(m*(a*sin(c
 + d*x) + a)**m*sin(c + d*x)*sin(c + d*x)**(-m - 2), x)
```

3.15.7 Maxima [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= \int -(m\sin(dx+c)-m-1)(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\,dx$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="maxima")
```

output
```
-integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-
m - 2), x)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5502 vs. $2(35) = 70$.

Time $= 40.10$ (sec) , antiderivative size $= 5502$, normalized size of antiderivative $= 157.20$

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="giac")
```

output
```
-8*(cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*
tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*
pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 +
8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x +
 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1
/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/
2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/
2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2
+ tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 + 1)))*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*
a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(...
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\sin(2c+2dx)\,(a(\sin(c+dx)+1))^m}{2d\sin(c+dx)^{m+2}}$$

input
```
int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2)
,x)
```

output
```
-(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$

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3.16.1 Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f}$$
$$-\frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

output
```
(A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(
1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+
a^2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

3.16.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{(Ab-2aB)(e+fx) + \frac{2a\left(-a^2Ab+2Ab^3+2a^3B-3ab^2B\right)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - bB\cos(e+fx) + \frac{a^2b(Ab-aB)\cos(}{(a-b)(a+b)(a+b\sin}}{b^3f}$$

input
```
Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

output
```
((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*
B)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b
*B*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b
*Sin[e + f*x])))/(b^3*f)
```

3.16.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3467, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$

$$\downarrow 3467$$

$$\frac{\int \frac{b\left(a^2-b^2\right)B\sin^2(e+fx)+\left(a^2-b^2\right)(Ab-aB)\sin(e+fx)+ab(Ab-aB)}{a+b\sin(e+fx)}\,dx}{b^2\left(a^2-b^2\right)} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2 f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{b\left(a^2-b^2\right)B\sin(e+fx)^2+\left(a^2-b^2\right)(Ab-aB)\sin(e+fx)+ab(Ab-aB)}{a+b\sin(e+fx)}\,dx}{b^2\left(a^2-b^2\right)} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2 f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3502$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\sin(e+fx)b}{a+b\sin(e+fx)}\,dx}{b} - \frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2 f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\sin(e+fx)b}{a+b\sin(e+fx)}\,dx}{b} - \frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2 f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3214$$

$$\frac{\frac{x\left(a^2-b^2\right)(Ab-2aB)-a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int\frac{1}{a+b\sin(e+fx)}dx}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{x\left(a^2-b^2\right)(Ab-2aB)-a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int\frac{1}{a+b\sin(e+fx)}dx}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 3139$$

$$\frac{\frac{x\left(a^2-b^2\right)(Ab-2aB)-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int\frac{1}{a\tan^2\left(\frac{1}{2}(e+fx)\right)+2b\tan\left(\frac{1}{2}(e+fx)\right)+a}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 1083$$

$$\frac{\frac{\frac{4a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}+x\left(a^2-b^2\right)(Ab-2aB)}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\downarrow 217$$

$$\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{\frac{x\left(a^2-b^2\right)(Ab-2aB)-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(e+fx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}$$

input
```
Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

output
```
(((a^2 - b^2)*(A*b - 2*a*B)*x - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^
2*B)*ArcTan[(2*b + 2*a*Tan[(e + f*x)/2])/(2*Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
 b^2]*f))/b - ((a^2 - b^2)*B*Cos[e + f*x])/f)/(b^2*(a^2 - b^2)) + (a^2*(A*
b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

3.16.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.16.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(Aa^2b-2Ab^3-2Ba^3+3Bab^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}}{f}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2}\right.}}{f}$
default	$\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(Aa^2b-2Ab^3-2Ba^3+3Bab^2\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{b^3}}{f}+\frac{-\frac{2Bb}{1+\tan^2\left(\frac{fx}{2}\right.}}{f}$
risch	$\frac{xA}{b^2}-\frac{2xBa}{b^3}-\frac{B\,e^{i(fx+e)}}{2b^2f}-\frac{B\,e^{-i(fx+e)}}{2b^2f}+\frac{2ia^2(-Ab+Ba)\left(ib+a\,e^{i(fx+e)}\right)}{b^3(a^2-b^2)f\left(-ie^{2i(fx+e)}b+2a\,e^{i(fx+e)}+ib\right)}+\frac{a^3\ln\left(e^{i(fx+e)}+\frac{i\sqrt{}}{\sqrt{-a^2+b^2}\,(a-}\right.\right.$

input

```
int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/f*(-2*a/b^3*((-b^2*(A*b-B*a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)-b*a*(A*b-B*a)/
(a^2-b^2))/(tan(1/2*f*x+1/2*e)^2*a+2*b*tan(1/2*f*x+1/2*e)+a)+(A*a^2*b-2*A*
b^3-2*B*a^3+3*B*a*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+
2*b)/(a^2-b^2)^(1/2)))+2/b^3*(-B*b/(1+tan(1/2*f*x+1/2*e)^2)+(A*b-2*B*a)*ar
ctan(tan(1/2*f*x+1/2*e))))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(152) = 304$.

Time $= 0.33$ (sec) , antiderivative size $= 804$, normalized size of antiderivative $= 5.25$

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$

$$= \left[-\frac{2\,(2\,Ba^6 - Aa^5b - 4\,Ba^4b^2 + 2\,Aa^3b^3 + 2\,Ba^2b^4 - Aab^5)fx + (2\,Ba^5 - Aa^4b - 3\,Ba^3b^2 + 2\,Aa^2b^3 + (}{} \right.$$

$$\left. -\frac{(2\,Ba^6 - Aa^5b - 4\,Ba^4b^2 + 2\,Aa^3b^3 + 2\,Ba^2b^4 - Aab^5)fx + (2\,Ba^5 - Aa^4b - 3\,Ba^3b^2 + 2\,Aa^2b^3 + (2\,I}{} \right.$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
[-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*
a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b -
 A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log((
(2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f
*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)
^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3
*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B
*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4
 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f
*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4
*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B
*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)
*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^
2)*cos(f*x + e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a
*b^5)*cos(f*x + e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 +
 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*
sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a
^3*b^5 + a*b^7)*f)]
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.16.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(152) = 304$.

Time = 0.66 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.33

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{2\,(2\,Ba^4-Aa^3b-3\,Ba^2b^2+2\,Aab^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}} - \frac{2\left(Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\right.}{}$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="g
iac")
```

output
```
(2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((
a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*
b^2*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(
1/2*f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*
x + 1/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) +
 2*B*a^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x
 + 1/2*e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(
a^2*b^2 - b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

3.16.9 Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 3718, normalized size of antiderivative = 24.30

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)
```

output

```
((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*tan(e/2 + (f*x)/
2)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(2*B*b^2 - 3
*B*a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*
a^3 + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*ta
n(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) +
 (log(tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (log(tan(e/2 +
(f*x)/2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*atan(((a*(-(a + b)^3*(a - b
)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6
 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a
^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^1
0 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2
*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7
 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a
 + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b
^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a
^2*b^8 + a^4*b^6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^
6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^1
0 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^
14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(
-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/...
```

3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

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3.17.1 Optimal result

Integrand size = 34, antiderivative size = 182

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{7}{16}a(2A-B)c^4x + \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad - \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f} \\
&\quad + \frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}
\end{aligned}$$

output

```
7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*
cos(f*x+e)*sin(f*x+e)/f-1/6*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^3/f+1/10*a
*(2*A-B)*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/40*a*(2*A-B)*cos(f*x+e)^3
*(c^4-c^4*sin(f*x+e))/f
```

3.17.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$

$$= \frac{ac^4\cos(e+fx)\left(272A-176B-\frac{210(2A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+15(2A+7B)\sin(e+fx)-32(7A-B)\sin\right.}{240f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
,x]
```

output
```
(a*c^4*Cos[e + f*x]*(272*A - 176*B - (210*(2*A - B)*ArcSin[Sqrt[1 - Sin[e
+ f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(2*A + 7*B)*Sin[e + f*x] - 32*
(7*A - B)*Sin[e + f*x]^2 + 10*(18*A - 17*B)*Sin[e + f*x]^3 - 48*(A - 3*B)*
Sin[e + f*x]^4 - 40*B*Sin[e + f*x]^5))/(240*f)
```

3.17.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\downarrow 3339$$

$$ac\left(\frac{1}{2}(2A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{2}(2A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))^3dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}\right)$$

$$\downarrow 3157$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\int \cos^2(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)-\frac{B\cos^3(e+fx$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\int \cos(e+fx)^2(c-c\sin(e+fx))^2dx+\frac{c\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)-\frac{B\cos^3(e+fx$$

$$\downarrow 3157$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\int \cos^2(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos^3(e+$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\int \cos(e+fx)^2(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos^3(e+$$

$$\downarrow 3148$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\int \cos^2(e+fx)dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos^3($$

$$\downarrow 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\right.\right.$$

$$\downarrow 3115$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e\right.\right.\right.\right.$$

$$\downarrow 24$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{\cos^3(e+fx)\,(c^2-c^2\sin(e+fx))}{4f}+\frac{5}{4}c\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a*c*(-1/6*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^3)/f + ((2*A - B)*((c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^2)/(5*f) + (7*c*((Cos[e + f*x]^3*(c^2 - c
^2*Sin[e + f*x]))/(4*f) + (5*c*((c*Cos[e + f*x]^3)/(3*f) + c*(x/2 + (Cos[e
 + f*x]*Sin[e + f*x])/(2*f))))/4))/5))/2)
```

3.17.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.17.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{c^4 a\left(\left(-\frac{65A}{3}+\frac{35B}{3}\right)\cos(3fx+3e)+(A-3B)\cos(5fx+5e)+\left(-20A-\frac{5B}{4}\right)\sin(2fx+2e)+\left(\frac{15A}{2}-\frac{35B}{4}\right)\sin(4fx+4e)+\frac{5}{80f}$
risch	$\frac{7a\,c^4xA}{8}-\frac{7a\,c^4xB}{16}+\frac{7c^4a\cos(fx+e)A}{8f}-\frac{5c^4a\cos(fx+e)B}{8f}-\frac{B\,c^4a\sin(6fx+6e)}{192f}-\frac{c^4a\cos(5fx+5e)A}{80f}+\frac{3c}{}$
parts	$-\frac{(-3A\,c^4a+B\,c^4a)\cos(fx+e)}{f}+\frac{(-3A\,c^4a+2B\,c^4a)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(A\,c^4a}{}$
derivativedivides	$-\frac{A\,c^4a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3A\,c^4a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^4a}{}$
default	$-\frac{A\,c^4a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-3A\,c^4a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^4a}{}$
norman	$\frac{(\frac{7}{8}A\,c^4a-\frac{7}{16}B\,c^4a)x+(\frac{7}{8}A\,c^4a-\frac{7}{16}B\,c^4a)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{21}{4}A\,c^4a-\frac{21}{8}B\,c^4a)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{21}{4}A\,c^4a-\frac{21}{8}B$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETURNV
ERBOSE)
```

output
```
-1/80*c^4*a*((-65/3*A+35/3*B)*cos(3*f*x+3*e)+(A-3*B)*cos(5*f*x+5*e)+(-20*A
-5/4*B)*sin(2*f*x+2*e)+(15/2*A-35/4*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e
)+(-70*A+50*B)*cos(f*x+e)-70*f*x*A+35*f*x*B-272/3*A+176/3*B)/f
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx =$$
$$-\frac{48\,(A - 3\,B)ac^4\cos(fx + e)^5 - 320\,(A - B)ac^4\cos(fx + e)^3 - 105\,(2\,A - B)ac^4 fx + 5\,(8\,Bac^4\cos(f}{240\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
-1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)
^3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*
B)*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(163) = 326$.

Time = 0.44 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.69

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx$$
$$= \begin{cases} -\frac{9Aac^4x\sin^4(e+fx)}{8} - \frac{9Aac^4x\sin^2(e+fx)\cos^2(e+fx)}{4} + Aac^4x\sin^2(e + fx) - \frac{9Aac^4x\cos^4(e+fx)}{8} + Aac^4x\cos^2(e + \\ x(A + B\sin(e))\,(a\sin(e) + a)\,(-c\sin(e) + c)^4 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output

```
Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)
**4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4
*cos(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c
**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos
(e + f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin
(e + f*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*
cos(e + f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f
*x)**6/16 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*
x*sin(e + f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3
*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)*
*2/2 + 5*B*a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 -
3*B*a*c**4*x*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 3*B*a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f
) + 4*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)
**2*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*
a*c**4*sin(e + f*x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e
+ f*x)/(2*f) + 8*B*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)*
*3/(3*f) - B*a*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin...
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx =$$

$$-\frac{64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aac^4 - 640\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)A$$

input

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="maxima")
```

output
```
-1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^
4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*
x + 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos
(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e
) - 48*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) -
8*sin(2*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4
 - 2880*A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\begin{aligned}\int &(a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\ &= -\frac{Bac^4\sin(6fx+6e)}{192f} + \frac{7}{16}\left(2Aac^4 - Bac^4\right)x - \frac{(Aac^4 - 3Bac^4)\cos(5fx+5e)}{80f} \\ &\quad + \frac{(13Aac^4 - 7Bac^4)\cos(3fx+3e)}{48f} + \frac{(7Aac^4 - 5Bac^4)\cos(fx+e)}{8f} \\ &\quad - \frac{(6Aac^4 - 7Bac^4)\sin(4fx+4e)}{64f} + \frac{(16Aac^4 + Bac^4)\sin(2fx+2e)}{64f}\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="giac")
```

output
```
-1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A
*a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos
(3*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a
*c^4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f
*x + 2*e)/f
```

3.17.9 Mupad [B] (verification not implemented)

Time = 15.30 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{A\,a\,c^4}{4}+\frac{7\,B\,a\,c^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(6\,A\,a\,c^4-2\,B\,a\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(12\,A\,a\,c^4-4\,B\,a\,c^4\right)}{}$$

$$+\frac{7\,a\,c^4\,\mathrm{atan}\left(\frac{7\,a\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-B)}{8\left(\frac{7\,A\,a\,c^4}{4}-\frac{7\,B\,a\,c^4}{8}\right)}\right)(2\,A-B)}{8\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*
(6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) -
tan(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8
*(22*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*
a*c^4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e
/2 + (f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*(
(68*A*a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 -
(73*B*a*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24)
 + (34*A*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(
e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*t
an(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^
4*tan(e/2 + (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A
- B))/(8*f)
```

3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

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3.18.1 Optimal result

Integrand size = 34, antiderivative size = 142

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{1}{8}a(5A-2B)c^3x + \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} + \frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f}$$
$$- \frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} + \frac{a(5A-2B)\cos^3(e+fx)\,(c^3-c^3\sin(e+fx))}{20f}$$

output
```
1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*
c^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/
20*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

3.18.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{ac^3\cos(e+fx)\left(80A-56B-\frac{30(5A-2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+15(3A+2B)\sin(e+fx)+(-80A+32B)\right.}{120f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
,x]
```

output
```
(a*c^3*Cos[e + f*x]*(80*A - 56*B - (30*(5*A - 2*B)*ArcSin[Sqrt[1 - Sin[e +
 f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(3*A + 2*B)*Sin[e + f*x] + (-80
*A + 32*B)*Sin[e + f*x]^2 + 30*(A - 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e + f*x
]^4))/(120*f)
```

3.18.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\downarrow 3339$$

$$ac\left(\frac{1}{5}(5A-2B)\int \cos^2(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\int \cos(e+fx)^2(c-c\sin(e+fx))^2dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)$$

$$\downarrow 3157$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\int\cos^2(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(e+f}{}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\int\cos(e+fx)^2(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(e+f}{}\right.$$

$$\downarrow 3148$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\int\cos^2(e+fx)dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(e}{}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B}{}\right.$$

$$\downarrow 3115$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+f}{4f}\right.\right.\right.$$

$$\downarrow 24$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}+\frac{5}{4}c\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a*c*(-1/5*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^2)/f + ((5*A - 2*B)*((Cos
[e + f*x]^3*(c^2 - c^2*Sin[e + f*x]))/(4*f) + (5*c*((c*Cos[e + f*x]^3)/(3*
f) + c*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4))/5)
```

3.18.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.18.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{\left(\left(\frac{2A}{3}-\frac{5B}{12}\right)\cos(3fx+3e)+\left(-\frac{A}{8}+\frac{B}{4}\right)\sin(4fx+4e)+\frac{\cos(5fx+5e)B}{20}+A\sin(2fx+2e)+\left(2A-\frac{3B}{2}\right)\cos(fx+e)+\frac{5fxA}{2}-fx\right.}{4f}$
risch	$\frac{5a\,c^3xA}{8}-\frac{a\,c^3xB}{4}+\frac{c^3a\cos(fx+e)A}{2f}-\frac{3c^3a\cos(fx+e)B}{8f}+\frac{Ba\,c^3\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)A\,c^3a}{32f}+\frac{\sin(4}{}$
parts	$-\frac{(-2A\,c^3a+B\,c^3a)\cos(fx+e)}{f}+\frac{(-A\,c^3a+2B\,c^3a)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+a\,c^3x$
derivativedivides	$\frac{-A\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\cos(fx+e)a\,c^3+\frac{B\,c^3}{}}{}$
default	$\frac{-A\,c^3a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^3a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2A\cos(fx+e)a\,c^3+\frac{B\,c^3}{}}{}$
norman	$\frac{(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x+(\frac{5}{8}A\,c^3a-\frac{1}{4}B\,c^3a)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3a)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}A\,c^3a-\frac{5}{2}B\,c^3$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/4*((2/3*A-5/12*B)*cos(3*f*x+3*e)+(-1/8*A+1/4*B)*sin(4*f*x+4*e)+1/20*cos(
5*f*x+5*e)*B+A*sin(2*f*x+2*e)+(2*A-3/2*B)*cos(f*x+e)+5/2*f*x*A-f*x*B+8/3*A
-28/15*B)*c^3*a/f
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{24\,Bac^3\cos(fx+e)^5+80\,(A-B)ac^3\cos(fx+e)^3+15\,(5\,A-2\,B)ac^3fx-15\,\left(2\,(A-2\,B)ac^3\cos(fx\right.}{120\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5
*A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a
*c^3*cos(f*x + e))*sin(f*x + e))/f
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(129) = 258$.

Time $= 0.30$ (sec) , antiderivative size $= 486$, normalized size of antiderivative $= 3.42$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx$$
$$= \begin{cases} -\frac{3Aac^3x\sin^4(e+fx)}{8} - \frac{3Aac^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aac^3x\cos^4(e+fx)}{8} + Aac^3x + \frac{5Aac^3\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{2Aac}{} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c)^3 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e +
 f*x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e
 + f*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**
4/4 + 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e +
f*x)**2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*
a*c**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**
3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/
f + 8*B*a*c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0))
, (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))
```

3.18.7 Maxima [A] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 200$, normalized size of antiderivative $= 1.41$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx$$
$$= \frac{320\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aac^3 - 15\left(12fx + 12e + \sin(4fx+4e) - 8\sin(2fx+2e)\right)Aac^3 -}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 +
32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*
x + 2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c
^3*cos(f*x + e))/f
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx \\
&= \frac{Bac^3\cos(5fx+5e)}{80f} + \frac{Aac^3\sin(2fx+2e)}{4f} \\
&\quad + \frac{1}{8}\left(5\,Aac^3 - 2\,Bac^3\right)x + \frac{(8\,Aac^3 - 5\,Bac^3)\cos(3fx+3e)}{48f} \\
&\quad + \frac{(4\,Aac^3 - 3\,Bac^3)\cos(fx+e)}{8f} - \frac{(Aac^3 - 2\,Bac^3)\sin(4fx+4e)}{32f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*
A*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f +
 1/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*s
in(4*f*x + 4*e)/f
```

3.18.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aac^3}{4}+\frac{Bac^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(4Aac^3-2Bac^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{7Aac^3}{2}-3Bac^3\right)-}{f\left(\tan\right.}$$

$$+\frac{ac^3\operatorname{atan}\left(\frac{ac^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(5A-2B)}{4\left(\frac{5Aac^3}{4}-\frac{Bac^3}{2}\right)}\right)(5A-2B)}{4f}$$

$$-\frac{ac^3(5A-2B)\left(\operatorname{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

output
```
(tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^8*(
4*A*a*c^3 - 2*B*a*c^3) + tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3)
- tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - tan(e/2 + (f*x)/2)^9*
((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^
3) + tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + tan(e/2 + (f*x
)/2)^4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)
/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)
/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) + (a*c^3*atan
((a*c^3*tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))
*(5*A - 2*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2))/(4*f)
```

3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

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3.19.1 Optimal result

Integrand size = 34, antiderivative size = 97

$$\begin{aligned}&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\&=\frac{1}{8}a(4A-B)c^2x+\frac{a(A-B)c^2\cos^3(e+fx)}{3f}\\&\quad+\frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{aBc^2\cos^3(e+fx)\sin(e+fx)}{4f}\end{aligned}$$

output
```
1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*cos(f
*x+e)*sin(f*x+e)/f+1/4*a*B*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

3.19.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\&=\frac{ac^2\cos(e+fx)\left(8A-8B-\frac{6(4A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+3(4A+B)\sin(e+fx)-8(A-B)\sin^2(e+fx)\right.}{24f}\end{aligned}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
,x]
```

output
```
(a*c^2*Cos[e + f*x]*(8*A - 8*B - (6*(4*A - B)*ArcSin[Sqrt[1 - Sin[e + f*x]
]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 3*(4*A + B)*Sin[e + f*x] - 8*(A - B)*Si
n[e + f*x]^2 - 6*B*Sin[e + f*x]^3))/(24*f)
```

3.19.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\downarrow 3339$$

$$ac\left(\frac{1}{4}(4A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))dx - \frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{4}(4A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))dx - \frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\downarrow 3148$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\int \cos^2(e+fx)dx + \frac{c\cos^3(e+fx)}{3f}\right) - \frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{c\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\downarrow 3115$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\downarrow 24$$

$$ac\left(\frac{1}{4}(4A-B)\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a*c*(-1/4*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x]))/f + ((4*A - B)*((c*Cos[e
 + f*x]^3)/(3*f) + c*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4)
```

3.19.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.19.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{c^2\left(\frac{(A-B)\cos(3fx+3e)}{3}+A\sin(2fx+2e)+\frac{B\sin(4fx+4e)}{8}+(A-B)\cos(fx+e)+2fxA-\frac{fxB}{2}+\frac{4A}{3}-\frac{4B}{3}\right)a}{4f}$
risch	$\frac{a\,c^2xA}{2}-\frac{a\,c^2xB}{8}+\frac{c^2a\cos(fx+e)A}{4f}-\frac{c^2a\cos(fx+e)B}{4f}+\frac{B\,c^2a\sin(4fx+4e)}{32f}+\frac{c^2a\cos(3fx+3e)A}{12f}-\frac{c^2a\cos$
parts	$\frac{(-A\,c^2a-B\,c^2a)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-A\,c^2a+B\,c^2a)\cos(fx+e)}{f}-\frac{(A\,c^2a-B\,c^2a)(2+\sin^2(fx+e))}{3f}$
derivativedivides	$-\frac{A\,c^2a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-A\,c^2a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\cos(fx+e)a\,c^2+B\,c^2a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin}{\cdots}\right.\right.$
default	$-\frac{A\,c^2a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-A\,c^2a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+A\cos(fx+e)a\,c^2+B\,c^2a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin}{\cdots}\right.\right.$
norman	$\left(\frac{1}{2}A\,c^2a-\frac{1}{8}B\,c^2a\right)x+\left(2A\,c^2a-\frac{1}{2}B\,c^2a\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2A\,c^2a-\frac{1}{2}B\,c^2a\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3A\,c^2a-\frac{3}{4}B\,c^2a\right)x\left($

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/4*c^2*(1/3*(A-B)*cos(3*f*x+3*e)+A*sin(2*f*x+2*e)+1/8*B*sin(4*f*x+4*e)+(A
-B)*cos(f*x+e)+2*f*x*A-1/2*f*x*B+4/3*A-4/3*B)*a/f
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \frac{8\,(A - B)ac^2\cos(fx + e)^3 + 3\,(4\,A - B)ac^2 fx + 3\left(2\,Bac^2\cos(fx + e)^3 + (4\,A - B)ac^2\cos(fx + e)\right)\text{si}}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^
2*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(85) = 170.

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.08

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \begin{cases} -\frac{Aac^2 x\sin^2(e+fx)}{2} - \frac{Aac^2 x\cos^2(e+fx)}{2} + Aac^2 x - \frac{Aac^2\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aac^2\cos^3(e}{3f} \\ x(A + B\sin(e))\,(a\sin(e) + a)\,(-c\sin(e) + c)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 +
A*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*
x)*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e
+ f*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x
)**4/8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2
*B*a*c**2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \frac{32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aac^2 - 24\left(2fx + 2e - \sin(2fx+2e)\right)Aac^2 + 96(fx+e)Aac^2 - 32}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2
*e))*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*co
s(f*x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \frac{Bac^2\sin(4fx + 4e)}{32f} + \frac{Aac^2\sin(2fx + 2e)}{4f} + \frac{1}{8}\left(4Aac^2 - Bac^2\right)x$$
$$+ \frac{(Aac^2 - Bac^2)\cos(3fx + 3e)}{12f} + \frac{(Aac^2 - Bac^2)\cos(fx + e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*
A*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(
A*a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

3.19.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.56

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(A\,a\,c^2+\frac{B\,a\,c^2}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(2\,A\,a\,c^2-2\,B\,a\,c^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(2\,A\,a\,c^2-2\,B\,a\,c^2)+}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

$$+\frac{a\,c^2\,\mathrm{atan}\left(\frac{a\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4\,A-B)}{4\left(A\,a\,c^2-\frac{B\,a\,c^2}{4}\right)}\right)(4\,A-B)}{4\,f}-\frac{a\,c^2\,(4\,A-B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

output
```
(tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*
c^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2
+ (f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c
^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e
/2 + (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)
/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(
4*A - B))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)
*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.20 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$

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3.20.1 Optimal result

Integrand size = 32, antiderivative size = 49

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx = \frac{1}{2}aAcx - \frac{aBc\cos^3(e + fx)}{3f} + \frac{aAc\cos(e + fx)\sin(e + fx)}{2f}$$

output
```
1/2*a*A*c*x-1/3*a*B*c*cos(f*x+e)^3/f+1/2*a*A*c*cos(f*x+e)*sin(f*x+e)/f
```

3.20.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx = -\frac{ac(3B\cos(e + fx) + B\cos(3(e + fx)) - 3A(-2e + 2fx + \sin(2(e + fx))))}{12f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x
]
```

output
```
-1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Si
n[2*(e + f*x)])))/f
```

3.20.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))dx$$

$$\downarrow 3148$$

$$ac\left(A\int \cos^2(e+fx)dx-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\downarrow 3042$$

$$ac\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\downarrow 3115$$

$$ac\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\downarrow 24$$

$$ac\left(A\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)-\frac{B\cos^3(e+fx)}{3f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/3*(B*Cos[e + f*x]^3)/f + A*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))
```

3.20.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.20.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{ac(6fxA+3A\sin(2fx+2e)-3\cos(fx+e)B-\cos(3fx+3e)B-4B)}{12f}$
risch	$\frac{aAcx}{2}-\frac{Bac\cos(fx+e)}{4f}-\frac{Bac\cos(3fx+3e)}{12f}+\frac{Aac\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
default	$\frac{\frac{Bac\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
parts	$aAcx-\frac{Bac\cos(fx+e)}{f}-\frac{Aac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{Bac(2+\sin^2(fx+e))\cos(fx+e)}{3f}$
norman	$\frac{\frac{Aac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2Bac}{3f}-\frac{2Bac\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{aAcx}{2}-\frac{Aac\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3aAcx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3aAcx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
1/12*a*c*(6*f*x*A+3*A*sin(2*f*x+2*e)-3*cos(f*x+e)*B-cos(3*f*x+3*e)*B-4*B)/
f
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=-\frac{2\,Bac\cos(fx+e)^3-3\,Aacfx-3\,Aac\cos(fx+e)\sin(fx+e)}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x
+ e))/f
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(46) = 92$.

Time $= 0.13$ (sec) , antiderivative size $= 138$, normalized size of antiderivative $= 2.82$

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{Aacx\sin^2(e+fx)}{2} - \frac{Aacx\cos^2(e+fx)}{2} + Aacx + \frac{Aac\sin(e+fx)\cos(e+fx)}{2f} + \frac{Bac\sin^2(e+fx)\cos(e+fx)}{f} + \frac{2Bac\cos^3(e+fx)}{3f} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*
x + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e +
f*x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)),
(x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

3.20.7 Maxima [A] (verification not implemented)

Time $= 0.23$ (sec) , antiderivative size $= 73$, normalized size of antiderivative $= 1.49$

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx =$$
$$-\frac{3(2fx + 2e - \sin(2fx + 2e))Aac - 12(fx + e)Aac + 4\left(\cos(fx + e)^3 - 3\cos(fx + e)\right)Bac + 12B}{12f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(
cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \frac{1}{2}Aacx - \frac{Bac\cos(3fx+3e)}{12f} - \frac{Bac\cos(fx+e)}{4f} + \frac{Aac\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1
/4*A*a*c*sin(2*f*x + 2*e)/f
```

3.20.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx = \frac{Aacx}{2}$$
$$- \frac{Aac\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5 + \left(\frac{ac(12B-9A(e+fx))}{6} + \frac{3Aac(e+fx)}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 - Aac\tan\left(\frac{e}{2}+\frac{fx}{2}\right) + \frac{ac(4B-3A}{6}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)^3}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

output
```
(A*a*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A
*a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*tan(e/2 + (f*x)
/2) + (A*a*c*(e + f*x))/2 + A*a*c*tan(e/2 + (f*x)/2)^5)/(f*(tan(e/2 + (f*x
)/2)^2 + 1)^3)
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

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3.21.1 Optimal result

Integrand size = 34, antiderivative size = 56

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{a(A+2B)x}{c}+\frac{aB\cos(e+fx)}{cf} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}$$

output
```
-a*(A+2*B)*x/c+a*B*cos(f*x+e)/c/f+2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

3.21.2 Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a\cos(e+fx)\left(-2(A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)\sqrt{1-\sin(e+fx)}+\sqrt{1+\sin(e+fx)}(-2A-3B+B\right.}{cf(-1+\sin(e+fx))\sqrt{1+\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
,x]
```

output
```
(a*Cos[e + f*x]*(-2*(A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*Sqrt[
1 - Sin[e + f*x]] + Sqrt[1 + Sin[e + f*x]]*(-2*A - 3*B + B*Sin[e + f*x])))
/(c*f*(-1 + Sin[e + f*x])*Sqrt[1 + Sin[e + f*x]])
```

3.21.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3446, 3042, 3336, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int(-((A+2B)c)-B\sin(e+fx)c)dx}{c^3}+\frac{2(A+B)\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}\right)$$

$$\downarrow 2009$$

$$ac\left(\frac{\frac{Bc\cos(e+fx)}{f}-cx(A+2B)}{c^3}+\frac{2(A+B)\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a*c*((-((A + 2*B)*c*x) + (B*c*Cos[e + f*x])/f)/c^3 + (2*(A + B)*Cos[e + f*
x])/(f*(c^2 - c^2*Sin[e + f*x])))
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.21.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
default	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
parallelrisch	$\frac{2\left(\frac{B\cos(2fx+2e)}{4}+\left(-\frac{1}{2}fxA-fxB+A+\frac{3}{2}B\right)\cos(fx+e)+(A+B)\sin(fx+e)+A+\frac{5B}{4}\right)a}{cf\cos(fx+e)}$
risch	$-\frac{axA}{c}-\frac{2axB}{c}+\frac{Ba\,e^{i(fx+e)}}{2cf}+\frac{Ba\,e^{-i(fx+e)}}{2cf}+\frac{4aA}{fc\left(e^{i(fx+e)}-i\right)}+\frac{4aB}{fc\left(e^{i(fx+e)}-i\right)}$
norman	$\frac{\frac{a(A+2B)x}{c}+\frac{a(A+2B)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{4aA+4Ba}{cf}-\frac{2Ba\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2Ba\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{(4aA+2Ba)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(1+\tan^2\left(\right.\right.}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*a/c*(-(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)+B/(1+tan(1/2*f*x+1/2*e)^2)-(A+2
*B)*arctan(tan(1/2*f*x+1/2*e)))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(57) = 114$.

Time $= 0.27$ (sec) , antiderivative size $= 116$, normalized size of antiderivative $= 2.07$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = -\frac{(A + 2B)afx - Ba\cos(fx + e)^2 - 2(A + B)a + ((A + 2B)afx - (2A + 3B)a)\cos(fx + e) - ((A -}{cf\cos(fx + e) - cf\sin(fx + e) + cf}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x -
(2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A +
 B)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(48) = 96$.

Time $= 1.03$ (sec) , antiderivative size $= 828$, normalized size of antiderivative $= 14.79$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output

```
Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan
(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)*
*2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/
2) - c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/
2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2
)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e
/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*ta
n(e/2 + f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x
/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*
tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f
) + 2*B*a*f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
 f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f
*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*
f) + 2*B*a*f*x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*ta
n(e/2 + f*x/2) - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3
 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 +
 f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)
/(-c*sin(e) + c), True))
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(57) = 114$.

Time $= 0.30$ (sec) , antiderivative size $= 265$, normalized size of antiderivative $= 4.73$

$$\int \frac{(a + a\sin(e+fx))(A + B\sin(e+fx))}{c - c\sin(e+fx)}\,dx =$$

$$-\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2}{c - \frac{c\sin(fx+e)}{\cos(fx+e)+1} + \frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right) + Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} - \frac{1}{c - \frac{c\sin(fx+e)}{\cos(fx+e)}}\right.\right.}{f}$$

input

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x
 + e)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/
(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time $= 0.31$ (sec) , antiderivative size $= 117$, normalized size of antiderivative $= 2.09$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx$$
$$= -\frac{\frac{(Aa+2Ba)(fx+e)}{c} + \frac{2\left(2Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Aa+3Ba\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)c}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(
1/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f
*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

3.21.9 Mupad [B] (verification not implemented)

Time $= 12.74$ (sec) , antiderivative size $= 111$, normalized size of antiderivative $= 1.98$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx$$
$$= \frac{(4\,A\,a + 4\,B\,a)\,\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 2\,B\,a\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 4\,A\,a + 6\,B\,a}{f\left(-c\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + c\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - c\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + c\right)} - \frac{A\,a\,f\,x + 2\,B\,a\,f\,x}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)
```

output
```
(4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f
*x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2
+ (f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

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3.22.1 Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{aBx}{c^2} - \frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))} + \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}$$

output
```
a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*
x+e)/f/(c-c*sin(f*x+e))^2
```

3.22.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.

Time = 6.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.22

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{a\left(-9Bfx\cos\left(\frac{fx}{2}\right) - 6(A+3B)\cos\left(e+\frac{fx}{2}\right) + 2A\cos\left(e+\frac{3fx}{2}\right) + 14B\cos\left(e+\frac{3fx}{2}\right) + 3Bfx\cos\left(2e+\right.\right.}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^2,x]
```

output
```
-1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e
+ (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24
*B*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/
(c^2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

3.22.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}\,dx}{3c^3}+\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\int \frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}\,dx}{3c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\int \frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}\,dx}{3c^3}\right)$$

$$\downarrow 3214$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{c(A+7B)\int\frac{1}{c-c\sin(e+fx)}dx-3Bx}{3c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{c(A+7B)\int\frac{1}{c-c\sin(e+fx)}dx-3Bx}{3c^3}\right)$$

$$\downarrow 3127$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\frac{c(A+7B)\cos(e+fx)}{f(c-c\sin(e+fx))}-3Bx}{3c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(3*c*f*(c - c*Sin[e + f*x])^2) - (-3*B*x + (
(A + 7*B)*c*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])))/(3*c^3))
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3336

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.22.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
risch	$\frac{aBx}{c^2} - \frac{2\left(3Aa\,e^{2i(fx+e)}-12iBa\,e^{i(fx+e)}+9Ba\,e^{2i(fx+e)}-aA-7Ba\right)}{3\left(e^{i(fx+e)}-i\right)^3 f c^2}$
derivativedivides	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}\right)}{f c^2}$
default	$\frac{2a\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}\right)}{f c^2}$
parallelrisch	$-\frac{2\left(-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{3}{2}fxB+A-B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(-\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}-\frac{5B}{3}\right)a}{f c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
norman	$\frac{axB\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{2aA-10Ba}{3cf}-\frac{axB}{c}-\frac{16Ba\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8Ba\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{8Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}-\frac{(2aA-2Ba)\left(\tan^6($

input

```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
a*B*x/c^2-2/3*(3*A*a*exp(2*I*(f*x+e))-12*I*B*a*exp(I*(f*x+e))+9*B*a*exp(2*
I*(f*x+e))-a*A-7*B*a)/(exp(I*(f*x+e))-I)^3/f/c^2
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.25

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx =$$
$$-\frac{6\,Bafx - (3\,Bafx + (A + 7\,B)a)\cos(fx + e)^2 + 2\,(A + B)a + (3\,Bafx + (A - 5\,B)a)\cos(fx + e) -}{3\left(c^2 f\cos(fx + e)^2 - c^2 f\cos(fx + e) - 2\,c^2 f + (c^2 f\cos\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
 (3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*
a*f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c
^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(65) = 130$.

Time = 2.03 (sec) , antiderivative size = 700, normalized size of antiderivative = 9.72

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx$$
$$= \begin{cases} -\frac{6Aa\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2 f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2 f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2 f} - \frac{2Aa}{3c^2 f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2 f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2 f} \\ \frac{x(A+B\sin(e))(a\sin(e)+a)}{(-c\sin(e)+c)^2} \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c*
*2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(
3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan
(e/2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/
2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
 3*c**2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 -
 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*
B*a*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*
tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*
x/2) - 3*c**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3
 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) -
24*B*a*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 +
 f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(
e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2)
 - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)*
*2, True))
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(67) = 134$.

Time $= 0.32$ (sec) , antiderivative size $= 456$, normalized size of antiderivative $= 6.33$

$$\int \frac{(a + a\sin(e+fx))(A + B\sin(e+fx))}{(c - c\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(Ba\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 4}{c^2 - \frac{3c^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right) - \frac{Aa\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2\right)}{c^2 - \frac{3c^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2\sin(fx+e)}{(\cos(fx+e)}}$$

$$3f$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(co
s(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + B*a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(
cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3))/f
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\frac{3\,(fx+e)Ba}{c^2} - \frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa-5\,Ba\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2
*f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2
*f*x + 1/2*e) - 1)^3))/f
```

3.22.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{B\,a\,x}{c^2} - \frac{\left(\frac{a\,(6\,A-6\,B+9\,B\,(e+f\,x))}{3}-3\,B\,a\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\left(\frac{a\,(24\,B-9\,B\,(e+f\,x))}{3}+3\,B\,a\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\right.}{c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

output
```
(B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*(
(a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*
((a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(
tan(e/2 + (f*x)/2) - 1)^3)
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

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3.23.1 Optimal result

Integrand size = 34, antiderivative size = 104

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)c\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}$$

output
```
2/5*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^3-1/15*a*(A+11*B)*c*cos(f*x+e)/f
/(c^2-c^2*sin(f*x+e))^2-1/15*a*(A-4*B)*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

3.23.2 Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+25B\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)+\right.}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f
*x)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

3.23.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}\,dx}{5c^3}+\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\int \frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}\,dx}{5c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\int \frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}\,dx}{5c^3}\right)$$

$$\downarrow 3229$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{1}{3}(A-4B)\int\frac{1}{c-c\sin(e+fx)}dx+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{1}{3}(A-4B)\int\frac{1}{c-c\sin(e+fx)}dx+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

$$\downarrow 3127$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{(A-4B)\cos(e+fx)}{3f(c-c\sin(e+fx))}+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(5*c*f*(c - c*Sin[e + f*x])^3) - (((A + 11*B
)*c*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2) + ((A - 4*B)*Cos[e + f*x])/
(3*f*(c - c*Sin[e + f*x])))/(5*c^3))
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.23.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(5A+B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}-\frac{B}{15}\right)a}{f c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
derivativedivides	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2B+6A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f c^3}$
default	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2B+6A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f c^3}$
risch	$\frac{-\frac{10Ba\,e^{2i(fx+e)}}{3}-2iBa\,e^{3i(fx+e)}+\frac{2iBa\,e^{i(fx+e)}}{3}+2Ba\,e^{4i(fx+e)}-\frac{2iAa\,e^{i(fx+e)}}{3}-\frac{2Aa\,e^{2i(fx+e)}}{3}-\frac{2aA}{15}+\frac{8Ba}{15}+2iAa\,e^{3i}}{\left(e^{i(fx+e)}-i\right)^5 f c^3}$
norman	$\frac{-\frac{8aA-2Ba}{15cf}-\frac{2aA\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{10(aA-Ba)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2(aA-Ba)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(2aA-2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}-\frac{2(11}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2 c^2\left(t\right.}$

input

```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^4+(-A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A+B)*tan(1/2
*f*x+1/2*e)^2+1/3*(-A+B)*tan(1/2*f*x+1/2*e)+4/15*A-1/15*B)*a/f/c^3/(tan(1/
2*f*x+1/2*e)-1)^5
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{(A-4\,B)a\cos(fx+e)^3-(2\,A+7\,B)a\cos(fx+e)^2+3\,(A+B)a\cos(fx+e)+6\,(A+B)a+\left((A-\right.}{15\left(c^3f\cos(fx+e)^3+3\,c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)-4\,c^3f-\left(c^3f\cos\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A +
B)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*
a*cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*
f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2
- 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(92) = 184.

Time = 4.34 (sec) , antiderivative size = 1035, normalized size of antiderivative = 9.95

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*
tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan
(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 +
75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c
**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan
(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*
x/2) - 15*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5
 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c*
*3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a
/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3
*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/
2 + f*x/2) - 15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 +
f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3
 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f
) - 10*B*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*
tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f
*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + ...
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(101) = 202$.

Time $= 0.23$ (sec) , antiderivative size $= 737$, normalized size of antiderivative $= 7.09$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) - 3*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*si
n(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1
)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{15\,c^3 f\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*
B*a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2
*f*x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e)
+ 4*A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{11\,A\,a\cos(e+f\,x)}{2}-\frac{B\,a}{4}-\frac{41\,A\,a}{4}+\frac{B\,a\cos(e+f\,x)}{2}+5\,A\,a\sin(e+f\,x)-5\,B\,a\sin(e+f\,x)+\frac{3}{}\right.}{15\,c^3\,f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}-\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}}{}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((11*A*a*cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B
*a*cos(e + f*x))/2 + 5*A*a*sin(e + f*x) - 5*B*a*sin(e + f*x) + (3*A*a*cos(
2*e + 2*f*x))/4 + (3*B*a*cos(2*e + 2*f*x))/4 - (5*A*a*sin(2*e + 2*f*x))/4
+ (5*B*a*sin(2*e + 2*f*x))/4))/(15*c^3*f*((5*2^(1/2)*cos((3*e)/2 - pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4))
```

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

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3.24.1 Optimal result

Integrand size = 34, antiderivative size = 142

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}$$

output
```
2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f
/(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-
1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

3.24.2 Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)-5B\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right)\right.}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^4,x]
```

output
```
(a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e
+ (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f
*x)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin
[2*e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7)
```

3.24.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}+\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\int \frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\int\frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}\right)$$

$$\downarrow 3229$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\int\frac{1}{(c-c\sin(e+fx))^2}dx+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\int\frac{1}{(c-c\sin(e+fx))^2}dx+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$$\downarrow 3129$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$$\downarrow 3127$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}+\frac{1}{5}(2A-5B)\left(\frac{\cos(e+fx)}{3cf(c-c\sin(e+fx))}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{7c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(7*c*f*(c - c*Sin[e + f*x])^4) - (((A + 15*B
)*c*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) + ((2*A - 5*B)*(Cos[e + f*x
]/(3*f*(c - c*Sin[e + f*x])^2) + Cos[e + f*x]/(3*c*f*(c - c*Sin[e + f*x]))
))/5)/(7*c^3))
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.24.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(13A-B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{2(-5A+2B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{13A\left(\tan^2\left(\frac{f}{}\right.\right.\right.}{fc^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$
risch	$-\frac{2ia\left(140iA\,e^{4i(fx+e)}-35iB\,e^{4i(fx+e)}+105B\,e^{5i(fx+e)}-42iA\,e^{2i(fx+e)}-70A\,e^{3i(fx+e)}-140B\,e^{3i(fx+e)}+2iA-14A\,e\right.}{105fc^4\left(e^{i(fx+e)}-i\right)^7}$
derivativedivides	$\frac{2a\left(-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}\right.}\right.}{fc^4}$
default	$\frac{2a\left(-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}\right.}\right.}{fc^4}$
norman	$\frac{(4aA-2Ba)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{46aA-10Ba}{105cf}-\frac{2aA\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{(16aA-10Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}+\frac{2(22aA-10Ba)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{3cf}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^6+(B-2*A)*tan(1/2*f*x+1/2*e)^5+1/3*(13*A-B)*tan(1
/2*f*x+1/2*e)^4+2/3*(-5*A+2*B)*tan(1/2*f*x+1/2*e)^3+13/5*A*tan(1/2*f*x+1/2
*e)^2+1/3*(-8/5*A+B)*tan(1/2*f*x+1/2*e)+23/105*A-1/21*B)*a/f/c^4/(tan(1/2*
f*x+1/2*e)-1)^7
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$=\frac{(2A-5B)a\cos(fx+e)^4+4(2A-5B)a\cos(fx+e)^3-3(3A+10B)a\cos(fx+e)^2+15(A+B)a}{105\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+4\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(
3*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a -
((2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B
)*a*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c
^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e
) - 8*c^4*f)*sin(f*x + e))
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1831 vs. $2(124) = 248$.

Time $= 8.62$ (sec) , antiderivative size $= 1831$, normalized size of antiderivative $= 12.89$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 -
735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c*
*4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*t
an(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*A*a*t
an(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 +
f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)
**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 +
735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 910*A*a*tan(e/2 + f*x/2)**4/(1
05*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4
*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan
(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 +
f*x/2) - 105*c**4*f) + 700*A*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f
*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**
5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 22
05*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f)
- 546*A*a*tan(e/2 + f*x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f
*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f
*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f
*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 ...
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. 2(138) = 276.

Time = 0.24 (sec) , antiderivative size = 1080, normalized size of antiderivative = 7.61

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

output
```
2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^...
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.24

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{2\left(105\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-210\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+105\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+455\,Aa\tan\left(\frac{1}{2}\,fx\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

output
```
-2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 + 105*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*tan(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f*x + 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*e) + 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{15Ba}{4}-\frac{171Aa}{2}+\frac{353Aa\cos(e+fx)}{8}+\frac{5Ba\cos(e+fx)}{4}+\frac{595Aa\sin(e+fx)}{8}-35Ba\sin(e+fx)\right.}{105c^4f\left(\frac{35\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{8}-\frac{21}{}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*cos(e + f*x))/8 + (5*B*a*cos(e + f*x))/4 + (595*A*a*sin(e + f*x))/8 - 35*B*a*sin(e + f*x) + (43*A*a*cos(2*e + 2*f*x))/2 - (25*A*a*cos(3*e + 3*f*x))/8 - (5*B*a*cos(2*e + 2*f*x))/4 + (5*B*a*cos(3*e + 3*f*x))/4 - (77*A*a*sin(2*e + 2*f*x))/4 - (21*A*a*sin(3*e + 3*f*x))/8 + (35*B*a*sin(2*e + 2*f*x))/4))/(105*c^4*f*((35*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/8))
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

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3.25.1 Optimal result

Integrand size = 34, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)c\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3} - \frac{2a(A-2B)c\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2} - \frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}$$

output

```
2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f
/(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-
2/315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(
f*x+e)/f/(c^5-c^5*sin(f*x+e))
```

3.25.2 Mathematica [A] (verified)

Time = 6.47 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a\left(315A\cos\left(e+\frac{fx}{2}\right)-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right)-18B\cos\left(3e+\frac{7fx}{2}\right)+189A\sin\left(\frac{f}{2}\right.\right.}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^5,x]
```

output
```
(a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
 + (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin
[(f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*
Sin[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))
/(1260*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
)
```

3.25.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$\downarrow 3446$$
$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}+\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\int \frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\int \frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}\right)$$

$$\downarrow 3229$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\int \frac{1}{(c-c\sin(e+fx))^3}dx+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\int \frac{1}{(c-c\sin(e+fx))^3}dx+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\downarrow 3129$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\int \frac{1}{(c-c\sin(e+fx))^2}dx}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\int \frac{1}{(c-c\sin(e+fx))^2}dx}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\downarrow 3129$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos}{7f(c-c\sin(e-}}{9c^3}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos}{7f(c-c\sin(e-}}{9c^3}\right.$$

$$\downarrow 3127$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}+\frac{3}{7}(A-2B)\left(\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}+\frac{2\left(\frac{\cos(e+fx)}{3cf(c-c\sin(e+fx))}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx)}\right.\right.\right.}{9c^3}\right.$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(9*c*f*(c - c*Sin[e + f*x])^5) - (((A + 19*B
)*c*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) + (3*(A - 2*B)*(Cos[e + f*x
]/(5*f*(c - c*Sin[e + f*x])^3) + (2*(Cos[e + f*x]/(3*f*(c - c*Sin[e + f*x]
)^2) + Cos[e + f*x]/(3*c*f*(c - c*Sin[e + f*x]))))/(5*c)))/7)/(9*c^3))
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.25.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{4\left(-42iBa\,e^{3i(fx+e)}+210Ba\,e^{6i(fx+e)}+9iAa\,e^{i(fx+e)}-2Ba-18iBa\,e^{i(fx+e)}+aA-84iAa\,e^{3i(fx+e)}-36Aa\,e^{2i(fx+e)}\right.}{315\left(e^{i(fx+e)}-i\right)^9 f c^5}$
parallelrisch	$-\frac{2a\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{25A}{3}-B\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-11A+3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(6}{f c^5\left(\tan\right.}$
derivativedivides	$\frac{2a\left(-\frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{296A+248B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32A+32}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.}{f c^5}$
default	$\frac{2a\left(-\frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{296A+248B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{32A+32}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.}{f c^5}$
norman	$\frac{(34aA-10Ba)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{116aA-22Ba}{315cf}-\frac{2aA\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2(3aA-Ba)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2(31aA-3Ba)\left(\tan^{10}\left(\frac{fx}{2}\right.\right.}{3cf}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNV
ERBOSE)
```

output
```
-4/315*(-42*I*B*a*exp(3*I*(f*x+e))+210*B*a*exp(6*I*(f*x+e))+9*I*A*a*exp(I*
(f*x+e))-2*B*a-18*I*B*a*exp(I*(f*x+e))+a*A-84*I*A*a*exp(3*I*(f*x+e))-36*A*
a*exp(2*I*(f*x+e))+72*B*a*exp(2*I*(f*x+e))+315*I*A*a*exp(5*I*(f*x+e))-189*
A*a*exp(4*I*(f*x+e))-252*B*a*exp(4*I*(f*x+e)))/(exp(I*(f*x+e))-I)^9/f/c^5
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.73

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{2\,(A-2\,B)a\cos(fx+e)^5-8\,(A-2\,B)a\cos(fx+e)^4-25\,(A-2\,B)a\cos(fx+e)^3+5\,(4\,A+13\,B}{315\left(c^5 f\cos(fx+e)^5+5\,c^5 f\cos(fx+e)^4-8\,c^5 f\cos(fx+\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="fricas")
```

output
```
-1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(
A - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a
*cos(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B
)*a*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x
+ e) - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x
 + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f
*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c
^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3232 vs. $2(160) = 320$.

Time = 15.82 (sec) , antiderivative size = 3232, normalized size of antiderivative = 18.36

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 -
2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460
*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**
5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*
tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*A*
a*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 +
f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/
2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**
2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*A*a*tan(e/2 + f*x/2)
**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11
340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*
c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5
*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*ta
n(e/2 + f*x/2) - 315*c**5*f) + 6930*A*a*tan(e/2 + f*x/2)**5/(315*c**5*f*ta
n(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2
 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 3
15*c**5*f) - 7686*A*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)*...
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. 2(171) = 342.

Time = 0.26 (sec) , antiderivative size = 1425, normalized size of antiderivative = 8.10

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="maxima")
```

output
```
-2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*si
n(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(co
s(f*x + e) + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c
^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*si...
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-945\,Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+315\,Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+2625\,Aa\tan\left(\frac{1}{2}f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="giac")
```

output
```
-2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 +
315*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a
*tan(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/
2*f*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x +
 1/2*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)
^3 + 1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207
*A*a*tan(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)
/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1357\,A\,a}{4}-\frac{461\,B\,a}{16}-\frac{635\,A\,a\cos(e+fx)}{4}+\frac{5\,B\,a\cos(e+fx)}{2}-\frac{1575\,A\,a\sin(e+fx)}{4}+\frac{945\,B\,a\sin(e+fx)}{8}-\frac{62}{}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x)
)/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e
+ f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 +
(7*A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e +
 3*f*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (14
1*A*a*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e
 + 2*f*x))/8 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16)
)/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((
3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2)
)/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/
2 + pi/4 + (9*f*x)/2))/16))
```

3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

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3.26.1 Optimal result

Integrand size = 36, antiderivative size = 229

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{9}{128}a^2(8A-3B)c^5x + \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} \\
&\quad + \frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f} + \frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f} \\
&\quad + \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} \\
&\quad - \frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} \\
&\quad + \frac{3a^2(8A-3B)\cos^5(e+fx)\,(c^5-c^5\sin(e+fx))}{112f}
\end{aligned}$$

output

```
9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*cos(f*x+e)^5/f+9/128*a^2*
(8*A-3*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*cos(f*x+e)^3*
sin(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^2/f-1/8*
a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*cos(f*x+e)
^5*(c^5-c^5*sin(f*x+e))/f
```

3.26.2 Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$= \frac{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(27A-17B)\cos(e+fx)+560(1$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^5,x]
```

output
```
((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
 + 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 11
2*(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A
- 3*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*S
in[6*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

3.26.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$
$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\downarrow 3339$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\int \cos^4(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\int \cos(e+fx)^4(c-c\sin(e+fx))^3dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\right)$$

$$\downarrow 3157$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\int \cos^4(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)-\frac{B\cos^5(e+$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\int \cos(e+fx)^4(c-c\sin(e+fx))^2dx+\frac{c\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)-\frac{B\cos^5(e+$$

$$\downarrow 3157$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\int \cos^4(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{c\cos^5($$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\int \cos(e+fx)^4(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{c\cos^5($$

$$\downarrow 3148$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\int \cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{cc}{}\right.\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)\right.\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos}{}\right.\right.\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+\right.\right.\right.\right.$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}+\frac{7}{6}c\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

output
```
a^2*c^2*(-1/8*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^3)/f + ((8*A - 3*B)*(
(c*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (9*c*((Cos[e + f*x]^5*(c
^2 - c^2*Sin[e + f*x]))/(6*f) + (7*c*((c*Cos[e + f*x]^5)/(5*f) + c*((Cos[e
 + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))/4)))/6))/7))/8)
```

3.26.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a  Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))  Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))  Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m  Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.26.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{11c^5\left(\frac{5(13A-7B)\cos(3fx+3e)}{11}+\left(A-\frac{B}{11}\right)\cos(5fx+5e)+\frac{5(-A+3B)\cos(7fx+7e)}{77}+\frac{5(19A-3B)\sin(2fx+2e)}{11}+\frac{5\left(-A+\frac{7B}{2}\right)\sin(}{11}\right.}{320f}$
risch	$\frac{9a^2c^5xA}{16}-\frac{27a^2c^5xB}{128}+\frac{27c^5a^2\cos(fx+e)A}{64f}-\frac{17c^5a^2\cos(fx+e)B}{64f}-\frac{Ba^2c^5\sin(8fx+8e)}{1024f}-\frac{c^5a^2\cos(7fx+7e}{448f}$
parts	$\frac{(-5Aa^2c^5+5Ba^2c^5)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(-3Aa^2c^5+Ba^2c^5)\cos(fx+e)}{f}-\frac{(-A}{}$
derivativedivides	$Aa^2c^5(fx+e)+\frac{Aa^2c^5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3Aa^2c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx-\right.\right.}{4}\right.}{}$
default	$Aa^2c^5(fx+e)+\frac{Aa^2c^5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{7}+3Aa^2c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx-\right.\right.}{4}\right.}{}$
norman	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
11/320*c^5*(5/11*(13*A-7*B)*cos(3*f*x+3*e)+(A-1/11*B)*cos(5*f*x+5*e)+5/77*
(-A+3*B)*cos(7*f*x+7*e)+5/11*(19*A-3*B)*sin(2*f*x+2*e)+5/11*(-A+7/2*B)*sin
(4*f*x+4*e)+5/11*(-A+B)*sin(6*f*x+6*e)-5/176*B*sin(8*f*x+8*e)+5/11*(27*A-1
7*B)*cos(f*x+e)+180/11*f*x*A-135/22*f*x*B+1472/77*A-832/77*B)*a^2/f
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx =$$
$$-\frac{640\,(A-3\,B)a^2c^5\cos(fx+e)^7-3584\,(A-B)a^2c^5\cos(fx+e)^5-315\,(8\,A-3\,B)a^2c^5fx+35\,(16\,B$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f
*x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7
+ 8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x +
e)^3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(218) = 436$.

Time $= 0.87$ (sec) , antiderivative size $= 1586$, normalized size of antiderivative $= 6.93$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^5\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**
2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*
x*cos(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*
cos(e + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*
x)/f - 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*
sin(e + f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*
x)/f - 5*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**
5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e
 + f*x)**5/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5
*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*
cos(e + f*x)**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f)
 - A*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*
x)**7/(35*f) + 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e
 + f*x)**3/(3*f) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e +
 f*x)**8/128 - 35*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a
**2*c**5*x*sin(e + f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e +
f*x)**4/64 - 15*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a*
*2*c**5*x*sin(e + f*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + ...
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(218) = 436$.

Time $= 0.23$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 2.49$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^5\,dx =$$

$$-\frac{3072\left(5\cos(fx+e)^7 - 21\cos(fx+e)^5 + 35\cos(fx+e)^3 - 35\cos(fx+e)\right)Aa^2c^5 - 7168\left(3\cos(f$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3
 - 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
 + 15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*
a^2*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
 - 48*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e)
  - 8*sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*
A*a^2*c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f
*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(
f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f
*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2
*e))*B*a^2*c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x +
 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x
+ 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x +
2*e))*B*a^2*c^5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x
 + e))/f
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.18

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= -\frac{Ba^2c^5\sin(8fx+8e)}{1024f} + \frac{9}{128}\left(8Aa^2c^5-3Ba^2c^5\right)x - \frac{(Aa^2c^5-3Ba^2c^5)\cos(7fx+7e)}{448f} \\
&\quad + \frac{(11Aa^2c^5-Ba^2c^5)\cos(5fx+5e)}{320f} + \frac{(13Aa^2c^5-7Ba^2c^5)\cos(3fx+3e)}{64f} \\
&\quad + \frac{(27Aa^2c^5-17Ba^2c^5)\cos(fx+e)}{64f} - \frac{(Aa^2c^5-Ba^2c^5)\sin(6fx+6e)}{64f} \\
&\quad - \frac{(2Aa^2c^5-7Ba^2c^5)\sin(4fx+4e)}{128f} + \frac{(19Aa^2c^5-3Ba^2c^5)\sin(2fx+2e)}{64f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
 - 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^
5 - B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(
3*f*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*
(A*a^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*
c^5)*sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*
e)/f
```

3.26.9 Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.89

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}\left(6Aa^2c^5-2Ba^2c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(30Aa^2c^5-10Ba^2c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(22Aa^2c^{\cdots}}{\cdots} \\
&\quad + \frac{9a^2c^5\,\mathrm{atan}\left(\frac{9a^2c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8A-3B)}{64\left(\frac{9Aa^2c^5}{8}-\frac{27Ba^2c^5}{64}\right)}\right)(8A-3B)}{64f}
\end{aligned}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

output
```
(tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10
*(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*
B*a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2
+ (f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15
*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*
c^5)/35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 -
(158*B*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c
^5)/64) - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) +
tan(e/2 + (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (
f*x)/2)^11*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*
((13*A*a^2*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*
c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B
*a^2*c^5)/64) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*
x)/2)^2 + 28*tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 +
 (f*x)/2)^8 + 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(
e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c
^5*tan(e/2 + (f*x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/6
4)))*(8*A - 3*B))/(64*f)
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

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3.27.1 Optimal result

Integrand size = 36, antiderivative size = 189

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{1}{16}a^2(7A-2B)c^4x + \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} \\
&\quad + \frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f} + \frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad + \frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}
\end{aligned}$$

output

```
1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7
*A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*si
n(f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2
*B)*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f
```

3.27.2 Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{a^2c^4(2940Ae-840Be+2940Afx-840Bfx+105(16A-11B)\cos(e+fx)+105(8A-5B)\cos(3(e+$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^4,x]
```

output
```
(a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*
Cos[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] -
 63*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] -
 210*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)]
- 35*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

3.27.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$
$$\downarrow 3042$$
$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\downarrow 3339$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\int \cos^4(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\int \cos(e+fx)^4(c-c\sin(e+fx))^2dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)$$

$$\downarrow 3157$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\int \cos^4(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos^5(e+\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\int \cos(e+fx)^4(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos^5(e+\right.$$

$$\downarrow 3148$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\int \cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos^5(\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{I}{}\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)}{}\right.\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+\right.\right.\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right.\right.\right.$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}+\frac{7}{6}c\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a^2*c^2*(-1/7*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/f + ((7*A - 2*B)*(
(Cos[e + f*x]^5*(c^2 - c^2*Sin[e + f*x]))/(6*f) + (7*c*((c*Cos[e + f*x]^5)
/(5*f) + c*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*
Sin[e + f*x])/(2*f)))/4)))/6))/7)
```

3.27.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a  Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.27.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{\left(5\left(A-\frac{5B}{8}\right)\cos(3fx+3e)+\left(A-\frac{3B}{8}\right)\cos(5fx+5e)+\frac{5\left(\frac{17A}{2}-B\right)\sin(2fx+2e)}{4}+\frac{5\left(\frac{A}{2}+B\right)\sin(4fx+4e)}{4}+\frac{5\left(-\frac{A}{2}+B\right)\sin(6fx+}{12}}{40f}$
risch	$\frac{7a^2c^4xA}{16}-\frac{a^2c^4xB}{8}+\frac{a^2c^4\cos(fx+e)A}{4f}-\frac{11a^2c^4\cos(fx+e)B}{64f}+\frac{Ba^2c^4\cos(7fx+7e)}{448f}-\frac{\sin(6fx+6e)Aa^2c^4}{192f}-$
parts	$-\frac{\left(-2Aa^2c^4-Ba^2c^4\right)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}-\frac{\left(-2Aa^2c^4+Ba^2c^4\right)\cos(fx+e)}{f}+\frac{(-Aa^2c^4}{}$
derivativedivides	$Aa^2c^4(fx+e)+\frac{Ba^2c^4\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2Aa^2c^4\cos(fx+e)-2Ba^2c^4\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Aa^2c^4$
default	$Aa^2c^4(fx+e)+\frac{Ba^2c^4\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2Aa^2c^4\cos(fx+e)-2Ba^2c^4\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Aa^2c^4$
norman	$\left(\frac{7}{16}Aa^2c^4-\frac{1}{8}Ba^2c^4\right)x+\left(\frac{7}{16}Aa^2c^4-\frac{1}{8}Ba^2c^4\right)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{49}{16}Aa^2c^4-\frac{7}{8}Ba^2c^4\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{49}{16}Aa^2c$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
1/40*(5*(A-5/8*B)*cos(3*f*x+3*e)+(A-3/8*B)*cos(5*f*x+5*e)+5/4*(17/2*A-B)*s
in(2*f*x+2*e)+5/4*(1/2*A+B)*sin(4*f*x+4*e)+5/12*(-1/2*A+B)*sin(6*f*x+6*e)+
5/56*cos(7*f*x+7*e)*B+5*(2*A-11/8*B)*cos(f*x+e)+35/2*f*x*A-5*f*x*B+16*A-72
/7*B)*c^4*a^2/f
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{240\,Ba^2c^4\cos(fx+e)^7+672\,(A-B)a^2c^4\cos(fx+e)^5+105\,(7A-2B)a^2c^4fx-35\,\left(8\,(A-2B)a^2c^4\right.}{1680\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5
+ 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2
*(7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*
sin(f*x + e))/f
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(172) = 344$.

Time $= 0.62$ (sec) , antiderivative size $= 1210$, normalized size of antiderivative $= 6.40$

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*
c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*co
s(e + f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e
 + f*x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)
/(16*f) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin
(e + f*x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a*
*2*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e
+ f*x)**5/(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a*
*2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(
15*f) - 8*A*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f
 - 5*B*a**2*c**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*co
s(e + f*x)**2/8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2 - B*a**2*c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8
 + 3*B*a**2*c**4*x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a
**2*c**4*sin(e + f*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*c
os(e + f*x)/(8*f) - 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B...
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(179) = 358.

Time = 0.24 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.43

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx$$
$$= \frac{896\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aa^2c^4 + 8960\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2
*c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x
+ 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2
*c^4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c
^4 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^
2*c^4 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35
*cos(f*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*
cos(f*x + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4
 - 70*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(
2*f*x + 2*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 +
13440*A*a^2*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25

$$\begin{aligned}
&\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx \\
&= \frac{Ba^2c^4\cos(7fx+7e)}{448f} + \frac{1}{16}\left(7Aa^2c^4 - 2Ba^2c^4\right)x \\
&\quad + \frac{\left(8Aa^2c^4 - 3Ba^2c^4\right)\cos(5fx+5e)}{320f} + \frac{\left(8Aa^2c^4 - 5Ba^2c^4\right)\cos(3fx+3e)}{64f} \\
&\quad + \frac{\left(16Aa^2c^4 - 11Ba^2c^4\right)\cos(fx+e)}{64f} - \frac{\left(Aa^2c^4 - 2Ba^2c^4\right)\sin(6fx+6e)}{192f} \\
&\quad + \frac{\left(Aa^2c^4 + 2Ba^2c^4\right)\sin(4fx+4e)}{64f} + \frac{\left(17Aa^2c^4 - 2Ba^2c^4\right)\sin(2fx+2e)}{64f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x +
1/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 -
 5*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(
f*x + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*
a^2*c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c
^4)*sin(2*f*x + 2*e)/f
```

3.27.9 Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.93

$$\int (a + a\sin(e+fx))^2 (A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(4Aa^2c^4 - 2Ba^2c^4\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}\left(12Aa^2c^4 - 2Ba^2c^4\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(8Aa^2c^4 -\right.}{}$$

$$+ \frac{a^2c^4\,\mathrm{atan}\left(\frac{a^2c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(7A-2B)}{8\left(\frac{7Aa^2c^4}{8}-\frac{Ba^2c^4}{4}\right)}\right)(7A-2B)}{8f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*
(12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^
2*c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/
2 + (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(
16*A*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (1
1*B*a^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)
/3) + tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2
 + (f*x)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)
^9*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*
c^4)/8 + (B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e
/2 + (f*x)/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*t
an(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 +
 tan(e/2 + (f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(
7*A - 2*B))/(8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

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3.28.1 Optimal result

Integrand size = 36, antiderivative size = 147

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{1}{16}a^2(6A-B)c^3x + \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f} + \frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f}$$
$$+ \frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a^2B\cos^5(e+fx)\,(c^3-c^3\sin(e+fx))}{6f}$$

output
```
1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B
)*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)
/f-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f
```

3.28.2 Mathematica [A] (verified)

Time = 7.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a^2c^3(360Ae-60Be+360Afx-60Bfx+120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+12A\,c}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*
x] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e +
 f*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e +
f*x)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

3.28.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\downarrow 3339$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\int \cos^4(e+fx)(c-c\sin(e+fx))dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\int \cos(e+fx)^4(c-c\sin(e+fx))dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$$\downarrow 3148$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\int\cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\int\sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c-}{6f}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+f}{}\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{}{}\right.$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)\right)-\frac{B\cos}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^2*c^2*(-1/6*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x]))/f + ((6*A - B)*((c*C
os[e + f*x]^5)/(5*f) + c*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 +
(Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)))/6)
```

3.28.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.28.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{c^3\left(5(A-B)\cos(3fx+3e)+(A-B)\cos(5fx+5e)+5\left(4A-\frac{B}{4}\right)\sin(2fx+2e)+\frac{5\left(A+\frac{B}{2}\right)\sin(4fx+4e)}{2}+\frac{5B\sin(6fx+6e)}{12}+10(A\right.}{80f}$
risch	$\frac{3a^2c^3xA}{8}-\frac{a^2c^3xB}{16}+\frac{c^3a^2\cos(fx+e)A}{8f}-\frac{c^3a^2\cos(fx+e)B}{8f}+\frac{Ba^2c^3\sin(6fx+6e)}{192f}+\frac{c^3a^2\cos(5fx+5e)A}{80f}-$
parts	$\frac{(-2Aa^2c^3-Ba^2c^3)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-Aa^2c^3+Ba^2c^3)\cos(fx+e)}{f}-\frac{(-Aa^2c^3+Ba^2c^3)\left(\frac{8}{3}+si\right.}{}$
derivativedivides	$\frac{Aa^2c^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+Aa^2c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2Aa^2c^3}{}$
default	$\frac{Aa^2c^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+Aa^2c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2Aa^2c^3}{}$
norman	$(\frac{3}{8}Aa^2c^3-\frac{1}{16}Ba^2c^3)x+(\frac{3}{8}Aa^2c^3-\frac{1}{16}Ba^2c^3)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}Aa^2c^3-\frac{3}{8}Ba^2c^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}Aa^2c^3-$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/80*c^3*(5*(A-B)*cos(3*f*x+3*e)+(A-B)*cos(5*f*x+5*e)+5*(4*A-1/4*B)*sin(2*
f*x+2*e)+5/2*(A+1/2*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e)+10*(A-B)*cos(f
*x+e)+30*f*x*A-5*f*x*B+16*A-16*B)*a^2/f
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{48\,(A-B)a^2c^3\cos(fx+e)^5+15\,(6\,A-B)a^2c^3fx+5\,\left(8\,Ba^2c^3\cos(fx+e)^5+2\,(6\,A-B)a^2c^3\cos(fx\right.}{240\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8
*B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A -
B)*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.

Time $= 0.44$ (sec) , antiderivative size $= 910$, normalized size of antiderivative $= 6.19$

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx$$
$$= \begin{cases} \frac{3Aa^2c^3x\sin^4(e+fx)}{8} + \frac{3Aa^2c^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^3x\sin^2(e+fx) + \frac{3Aa^2c^3x\cos^4(e+fx)}{8} - Aa^2c^3x\cos^2(e \\ x(A + B\sin(e))(a\sin(e) + a)^2(-c\sin(e) + c)^3 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*c
os(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*
c**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e
+ f*x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a
**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3
*cos(e + f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**
3*cos(e + f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*s
in(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15
*B*a**2*c**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e
+ f*x)**2*cos(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c
**3*x*cos(e + f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3
*x*cos(e + f*x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f)
- B*a**2*c**3*sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*
*3*cos(e + f*x)**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f
) - 4*B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*si
n(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/
(16*f) - 3*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*si
n(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + ...
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(138) = 276$.

Time $= 0.23$ (sec) , antiderivative size $= 360$, normalized size of antiderivative $= 2.45$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx$$
$$= \frac{64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aa^2c^3 + 640\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)A}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c
^3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5
 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin
(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x
 + 2*e))*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x +
e))/f
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\begin{aligned}
&\int (a + a\sin(e+fx))^2 (A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx \\
&= \frac{Ba^2c^3\sin(6fx+6e)}{192f} + \frac{1}{16}\left(6Aa^2c^3 - Ba^2c^3\right)x + \frac{(Aa^2c^3 - Ba^2c^3)\cos(5fx+5e)}{80f} \\
&\quad + \frac{(Aa^2c^3 - Ba^2c^3)\cos(3fx+3e)}{16f} + \frac{(Aa^2c^3 - Ba^2c^3)\cos(fx+e)}{8f} \\
&\quad + \frac{(2Aa^2c^3 + Ba^2c^3)\sin(4fx+4e)}{64f} + \frac{(16Aa^2c^3 - Ba^2c^3)\sin(2fx+2e)}{64f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/
80*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^
3)*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*
(2*A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*
c^3)*sin(2*f*x + 2*e)/f
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.69

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(4Aa^2c^3-4Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2Aa^2c^3-2Ba^2c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(4Aa^2c^3-4A\right.}{}$$

$$+\frac{a^2c^3\,\mathrm{atan}\left(\frac{a^2c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(6A-B)}{8\left(\frac{3Aa^2c^3}{4}-\frac{Ba^2c^3}{8}\right)}\right)(6A-B)}{8f}$$

$$-\frac{a^2c^3(6A-B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{8f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

output
```
(tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(
2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c
^3) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/
2)^2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^
3)/2 + (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2
*c^3)/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e
/2 + (f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)
^9*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^
3)/4 + (B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 +
 (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e
/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) +
(a^2*c^3*atan((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 -
 (B*a^2*c^3)/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f
*x)/2)) - (f*x)/2))/(8*f)
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

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3.29.1 Optimal result

Integrand size = 36, antiderivative size = 89

$$\begin{aligned}&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\&=\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f}\\&\quad+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}\end{aligned}$$

output
```
3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*cos(f*x+e)^5/f+3/8*a^2*A*c^2*cos(f*x+e)*sin(
f*x+e)/f+1/4*a^2*A*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

3.29.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\begin{aligned}&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\&=\frac{a^2c^2(-32B\cos^5(e+fx)+5A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx))))}{160f}\end{aligned}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
(a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] +
Sin[4*(e + f*x)])))/(160*f)
```

3.29.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))dx$$

$$\downarrow 3148$$

$$a^2c^2\left(A\int \cos^4(e+fx)dx-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^4 dx-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\downarrow 3115$$

$$a^2c^2\left(A\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(A\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\downarrow 3115$$

$$a^2c^2\left(A\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(A\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/5*(B*Cos[e + f*x]^5)/f + A*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f
) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))
```

3.29.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a  Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.29.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{a^2c^2(60fxA+5\sin(4fx+4e)A+40A\sin(2fx+2e)-20\cos(fx+e)B-2\cos(5fx+5e)B-10\cos(3fx+3e)B-32B)}{160f}$
risch	$\frac{3a^2Ac^2x}{8}-\frac{Ba^2c^2\cos(fx+e)}{8f}-\frac{Ba^2c^2\cos(5fx+5e)}{80f}+\frac{Aa^2c^2\sin(4fx+4e)}{32f}-\frac{Ba^2c^2\cos(3fx+3e)}{16f}+\frac{Aa^2c^2s}{}$
derivativedivides	$\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(}{}}{f}$
default	$\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(}{}}{f}$
parts	$a^2Ac^2x+\frac{Aa^2c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{Ba^2c^2\cos(fx+e)}{f}-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4(f\right.}{}$
norman	$-\frac{2Ba^2c^2}{5f}-\frac{4Ba^2c^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2Ba^2c^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2Ac^2x}{8}+\frac{5Aa^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{Aa^2c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-\frac{A}{}$

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output

```
1/160*a^2*c^2*(60*f*x*A+5*sin(4*f*x+4*e)*A+40*A*sin(2*f*x+2*e)-20*cos(f*x+
e)*B-2*cos(5*f*x+5*e)*B-10*cos(3*f*x+3*e)*B-32*B)/f
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = -\frac{8\,Ba^2c^2\cos(fx+e)^5 - 15\,Aa^2c^2fx - 5\left(2\,Aa^2c^2\cos(fx+e)^3 + 3\,Aa^2c^2\cos(fx+e)\right)\sin(fx+e)}{40\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(
f*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.18

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \begin{cases} \frac{3Aa^2c^2x\sin^4(e+fx)}{8} + \frac{3Aa^2c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^2x\sin^2(e+fx) + \frac{3Aa^2c^2x\cos^4(e+fx)}{8} - Aa^2c^2x\cos^2(e \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*c
os(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**
2*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos
(e + f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2
*sin(e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*
x)**3/(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2
*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**
2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e
) + c)**2, True))
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(81) = 162$.

Time $= 0.22$ (sec) , antiderivative size $= 164$, normalized size of antiderivative $= 1.84$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= \frac{15\,(12\,fx + 12\,e + \sin(4\,fx + 4\,e) - 8\,\sin(2\,fx + 2\,e))Aa^2c^2 - 240\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2c^2 +}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^
2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c
^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2
 - 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x
 + e))/f
```

3.29.8 Giac [A] (verification not implemented)

Time $= 0.33$ (sec) , antiderivative size $= 113$, normalized size of antiderivative $= 1.27$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= \frac{3}{8}\,Aa^2c^2x - \frac{Ba^2c^2\cos(5\,fx + 5\,e)}{80\,f} - \frac{Ba^2c^2\cos(3\,fx + 3\,e)}{16\,f}$$
$$- \frac{Ba^2c^2\cos(fx + e)}{8\,f} + \frac{Aa^2c^2\sin(4\,fx + 4\,e)}{32\,f} + \frac{Aa^2c^2\sin(2\,fx + 2\,e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3
*f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x +
4*e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

3.29.9 Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.67

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{3\,A\,a^2\,c^2\,x}{8}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\left(\frac{a^2\,c^2\,(80\,B-75\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2\,c^2\,(e+f\,x)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\,c^2\,(160\,B-150\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2\,c^2}{4}\right.}{f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

output
```
(3*A*a^2*c^2*x)/8 - (tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x)
))/40 + (15*A*a^2*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*
B - 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B -
15*A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*tan(e/2 + (f*
x)/2)^3)/2 + (A*a^2*c^2*tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*tan(e/2 + (
f*x)/2)^9)/4 - (5*A*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f*(tan(e/2 + (f*x)/2)^
2 + 1)^5)
```

3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

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3.30.1 Optimal result

Integrand size = 34, antiderivative size = 98

$$\begin{aligned}&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx\\&=\frac{1}{8}a^2(4A+B)cx-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}\\&\quad+\frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{Bc\cos^3(e+fx)\,(a^2+a^2\sin(e+fx))}{4f}\end{aligned}$$

output
```
1/8*a^2*(4*A+B)*c*x-1/12*a^2*(4*A+B)*c*cos(f*x+e)^3/f+1/8*a^2*(4*A+B)*c*co
s(f*x+e)*sin(f*x+e)/f-1/4*B*c*cos(f*x+e)^3*(a^2+a^2*sin(f*x+e))/f
```

3.30.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx=$$
$$-\frac{a^2c\cos(e+fx)\left(12(4A+B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(8A+8B+8(A+B)\cos(2(e+f$$
$$48f\sqrt{\cos^2(e+fx)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
-1/48*(a^2*c*Cos[e + f*x]*(12*(4*A + B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt
[2]] + Sqrt[Cos[e + f*x]^2]*(8*A + 8*B + 8*(A + B)*Cos[2*(e + f*x)] - 3*(8
*A + B)*Sin[e + f*x] + 3*B*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

3.30.3 Rubi [A] (verified)

Time $= 0.48$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 0.85$, number of steps used $= 9$, number of rules used $= 9$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used $= \{3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$ac\left(\frac{1}{4}(4A+B)\int \cos^2(e+fx)(\sin(e+fx)a+a)dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{4}(4A+B)\int \cos(e+fx)^2(\sin(e+fx)a+a)dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\downarrow 3148$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\int \cos^2(e+fx)dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\downarrow 3115$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\downarrow 24$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/4*(B*Cos[e + f*x]^3*(a + a*Sin[e + f*x]))/f + ((4*A + B)*(-1/3*(a*
Cos[e + f*x]^3)/f + a*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4)
```

3.30.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.30.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{c\left(\frac{(-A-B)\cos(3fx+3e)}{3}+A\sin(2fx+2e)-\frac{B\sin(4fx+4e)}{8}+(-A-B)\cos(fx+e)+2fxA+\frac{fxB}{2}-\frac{4A}{3}-\frac{4B}{3}\right)a^2}{4f}$
risch	$\frac{a^2cxA}{2}+\frac{a^2cxB}{8}-\frac{a^2c\cos(fx+e)A}{4f}-\frac{a^2c\cos(fx+e)B}{4f}-\frac{Ba^2c\sin(4fx+4e)}{32f}-\frac{a^2c\cos(3fx+3e)A}{12f}-\frac{a^2c\cos($
parts	$-\frac{(-Aa^2c-Ba^2c)(2+\sin^2(fx+e))\cos(fx+e)}{3f}+\frac{(-Aa^2c+Ba^2c)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(Aa^2c+Ba^2$
derivativedivides	$\frac{Aa^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aa^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Ba^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+$
default	$\frac{Aa^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-Aa^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Ba^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+$
norman	$\left(\frac{1}{2}Aa^2c+\frac{1}{8}Ba^2c\right)x+\left(2Aa^2c+\frac{1}{2}Ba^2c\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2Aa^2c+\frac{1}{2}Ba^2c\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3Aa^2c+\frac{3}{4}Ba^2c\right)x\left($

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*c*(1/3*(-A-B)*cos(3*f*x+3*e)+A*sin(2*f*x+2*e)-1/8*B*sin(4*f*x+4*e)+(-A
-B)*cos(f*x+e)+2*f*x*A+1/2*f*x*B-4/3*A-4/3*B)*a^2/f
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx =$$
$$-\frac{8\,(A+B)a^2c\cos(fx+e)^3 - 3\,(4\,A+B)a^2cfx + 3\left(2\,Ba^2c\cos(fx+e)^3 - (4\,A+B)a^2c\cos(fx+e)\right)}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2
*c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(90) = 180.

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.04

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{Aa^2cx\sin^2(e+fx)}{2} - \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx + \frac{Aa^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2Aa^2c\cos^3(e} \\ x(A + B\sin(e))\,(a\sin(e) + a)^2\,(-c\sin(e) + c) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 +
A*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*
x)*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e
+ f*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x
)**4/8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2
*B*a**2*c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx =$$
$$-\frac{32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^2c + 24\left(2fx + 2e - \sin(2fx+2e)\right)Aa^2c - 96(fx+e)Aa^2c +}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*co
s(f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*c
os(f*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= -\frac{Ba^2c\sin(4fx+4e)}{32f} + \frac{Aa^2c\sin(2fx+2e)}{4f} + \frac{1}{8}\left(4Aa^2c + Ba^2c\right)x$$
$$-\frac{(Aa^2c + Ba^2c)\cos(3fx+3e)}{12f} - \frac{(Aa^2c + Ba^2c)\cos(fx+e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4
*A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*
(A*a^2*c + B*a^2*c)*cos(f*x + e)/f
```

3.30.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.46

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$

$$= \frac{a^2\,c\,\mathrm{atan}\left(\frac{a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A+B)}{4\left(A\,a^2\,c+\frac{B\,a^2\,c}{4}\right)}\right)\,(4\,A+B)}{4\,f} - \frac{a^2\,c\,(4\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,A\,a^2\,c+2\,B\,a^2\,c)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,a^2\,c-\frac{B\,a^2\,c}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(2\,A\,a^2\,c+2\,B\,a^2\,c)}{f\,\left(\tan\left(\frac{e}{2}+\right.\right.}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

output
```
(a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4
)))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/
2))/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x
)/2)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c
) + tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)
/2)^7*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c
)/4) + tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2
*B*a^2*c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e
/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

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3.31.1 Optimal result

Integrand size = 36, antiderivative size = 117

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx\\ &=-\frac{3a^2(2A+3B)x}{2c}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}\\ &\quad+\frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}\end{aligned}$$

output
```
-3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*cos(f*x+e)/c/f+a^2*(A+B)*c^2*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))
```

3.31.2 Mathematica [A] (verified)

Time = 9.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.63

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx\\ &=\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A+\right.}{4cf\left(\cos\left(\right.\right.}\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e +
f*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e +
 f*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x)
 - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

3.31.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^2}dx}{c}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$$\downarrow 3161$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) - ((2*A + 3*B
)*((3*(x/c - Cos[e + f*x]/(c*f)))/(2*c) - Cos[e + f*x]^3/(2*f*(c^2 - c^2*S
in[e + f*x]))))/c)
```

3.31.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.31.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{4\left(\frac{(A+3B)\cos(2fx+2e)}{8}+\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA-\frac{9}{2}fxB+5A+7B\right)\cos(fx+e)}{4}+\left(A+\frac{33B}{32}\right)\sin(fx+e)+\frac{9A}{8}+\frac{11B}{8}\right)a^2}{cf\cos(fx+e)}$
derivativedivides	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
default	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
risch	$-\frac{3a^2xA}{c}-\frac{9a^2xB}{2c}+\frac{a^2e^{i(fx+e)}A}{2cf}+\frac{3a^2e^{i(fx+e)}B}{2cf}+\frac{a^2e^{-i(fx+e)}A}{2cf}+\frac{3a^2e^{-i(fx+e)}B}{2cf}+\frac{8a^2A}{fc\left(e^{i(fx+e)}-i\right)}+$
norman	$\frac{-\frac{2Aa^2+5Ba^2}{cf}+\frac{3a^2(2A+3B)x}{2c}-\frac{\left(2Aa^2+3Ba^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(4Aa^2+8Ba^2\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(6Aa^2+4Ba^2\right)\tan\left(\frac{fx}{2}+\right.}{cf}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
4*(1/8*(A+3*B)*cos(2*f*x+2*e)+1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A-9/2*f*x*
B+5*A+7*B)*cos(f*x+e)+(A+33/32*B)*sin(f*x+e)+9/8*A+11/8*B)*a^2/c/f/cos(f*x
+e)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{Ba^2\cos(fx+e)^3-3(2A+3B)a^2fx+2(A+3B)a^2\cos(fx+e)^2+8(A+B)a^2-(3(2A+3B)a^2f}{2(cfc}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*
x + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos
(f*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^
2*cos(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(104) = 208$.

Time $= 1.98$ (sec) , antiderivative size $= 2365$, normalized size of antiderivative $= 20.21$

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 -
2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)*
*4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2
+ f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f)
- 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan
(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*
f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)
**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a*
*2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2
)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e
/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan
(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**4/(2*c*f*ta
n(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
- 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*t
an(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 +
f*x/2) - 2*c*f) - 36*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)...
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 624, normalized size of antiderivative = 5.33

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{2\,Aa^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+4\,Ba^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c}{(\cos}\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*a^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{\frac{3\left(2\,Aa^2+3\,Ba^2\right)(fx+e)}{c}+\frac{16\left(Aa^2+Ba^2\right)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c}}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x
+ 1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) -
 2*A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

3.31.9 Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{10\,A\,a^2 - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,A\,a^2+5\,B\,a^2) + 14\,B\,a^2 - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(2\,A\,a^2+7\,B\,a^2) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(8\,A}{f\left(-c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5 + c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 - 2\,c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3 + 2\,c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right.}$$
$$- \frac{3\,a^2\,\mathrm{atan}\left(\frac{3\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,A+3\,B)}{6\,A\,a^2+9\,B\,a^2}\right)\,(2\,A+3\,B)}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x)),x)
```

output
```
(10*A*a^2 - tan(e/2 + (f*x)/2)*(2*A*a^2 + 5*B*a^2) + 14*B*a^2 - tan(e/2 +
(f*x)/2)^3*(2*A*a^2 + 7*B*a^2) + tan(e/2 + (f*x)/2)^4*(8*A*a^2 + 9*B*a^2)
+ tan(e/2 + (f*x)/2)^2*(18*A*a^2 + 21*B*a^2))/(f*(c - c*tan(e/2 + (f*x)/2)
 + 2*c*tan(e/2 + (f*x)/2)^2 - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)
/2)^4 - c*tan(e/2 + (f*x)/2)^5)) - (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(
2*A + 3*B))/(6*A*a^2 + 9*B*a^2))*(2*A + 3*B))/(c*f)
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

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3.32.1 Optimal result

Integrand size = 36, antiderivative size = 109

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx\\ &=\frac{a^2(A+4B)x}{c^2}-\frac{a^2(A+4B)\cos(e+fx)}{c^2 f}\\ &\quad+\frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}\end{aligned}$$

output
```
a^2*(A+4*B)*x/c^2-a^2*(A+4*B)*cos(f*x+e)/c^2/f+1/3*a^2*(A+B)*c^2*cos(f*x+e
)^5/f/(c-c*sin(f*x+e))^4-2/3*a^2*(A+4*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(109) = 218.

Time = 11.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.18

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx\\ &=\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3(A+4B)(e+\right.}{}\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A
+ B)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^2*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{3c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^3}dx}{3c}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c}\right)$$

$$\downarrow 3161$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - ((A + 4*B
)*((-3*(x/c - Cos[e + f*x]/(c*f)))/c^2 + (2*Cos[e + f*x]^3)/(c*f*(c - c*Si
n[e + f*x])^2)))/(3*c))
```

3.32.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.32.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
default	$\frac{2a^2\left(-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
risch	$\frac{a^2xA}{c^2}+\frac{4a^2xB}{c^2}-\frac{B\,a^2\mathrm{e}^{i(fx+e)}}{2c^2f}-\frac{B\,a^2\mathrm{e}^{-i(fx+e)}}{2c^2f}-\frac{8\left(-3iA\,a^2\mathrm{e}^{i(fx+e)}+3A\,a^2\mathrm{e}^{2i(fx+e)}-9iB\,a^2\mathrm{e}^{i(fx+e)}+6B\,a^2\right.}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3f\,c^2}$
parallelrisch	$\frac{3\left(\left(4(-fx+2)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(\frac{11}{6}+4fx\right)B+fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+(fxA+4fxB-\frac{4}{3}A-\frac{14}{3}B)\sin\left(\frac{fx}{2}-\right.\right.}{f\,c^2\left(-3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right)-\right.}$
norman	$\frac{8B\,a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{a^2(A+4B)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{8A\,a^2+38B\,a^2}{3cf}-\frac{a^2(A+4B)x}{c}-\frac{2\left(4A\,a^2+13B\,a^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(4A\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2/c^2*(-B/(1+tan(1/2*f*x+1/2*e)^2)+(A+4*B)*arctan(tan(1/2*f*x+1/2*e)
)-1/3*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(8*A+8*B)/(tan(1/2*f*x+1/2*e)
-1)^2+4*B/(tan(1/2*f*x+1/2*e)-1))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx =$$
$$-\frac{3\,Ba^2\cos(fx+e)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2)\cos(fx+}{3\,(c^2}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
 + 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x
- 2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*
x + e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(
f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2
*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(100) = 200$.

Time = 3.93 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.70

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5
- 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*
x*tan(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f
*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 +
 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3
/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*
tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f
*x/2) - 3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 +
f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 -
 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9
*A*a**2*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(
e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan
(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 24*A*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 +
f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 ...
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(107) = 214.

Time = 0.31 (sec) , antiderivative size = 839, normalized size of antiderivative = 7.70

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2)
 + A*a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3
*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(
c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) ...
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(Aa^2+4\,Ba^2)(fx+e)}{c^2} - \frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2} + \frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2
+ 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*
e) - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1
/2*e) - 1)^3))/f
```

3.32.9 Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.26

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{2a^2\operatorname{atan}\left(\frac{2a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A+4B)}{2Aa^2+8Ba^2}\right)(A+4B)}{c^2 f}$$
$$-\frac{\frac{8Aa^2}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8Aa^2+30Ba^2)+\frac{38Ba^2}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(8Aa^2+26Ba^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{8}{}\right.}{f\left(-c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-4c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-3\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^2,x
)
```

output
```
(2*a^2*atan((2*a^2*tan(e/2 + (f*x)/2)*(A + 4*B))/(2*A*a^2 + 8*B*a^2))*(A +
 4*B))/(c^2*f) - ((8*A*a^2)/3 - tan(e/2 + (f*x)/2)*(8*A*a^2 + 30*B*a^2) +
(38*B*a^2)/3 - tan(e/2 + (f*x)/2)^3*(8*A*a^2 + 26*B*a^2) + tan(e/2 + (f*x)
/2)^2*((8*A*a^2)/3 + (74*B*a^2)/3) + 8*B*a^2*tan(e/2 + (f*x)/2)^4)/(f*(4*c
^2*tan(e/2 + (f*x)/2)^2 - 4*c^2*tan(e/2 + (f*x)/2)^3 + 3*c^2*tan(e/2 + (f*
x)/2)^4 - c^2*tan(e/2 + (f*x)/2)^5 + c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

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3.33.1 Optimal result

Integrand size = 36, antiderivative size = 112

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}$$

output
```
-a^2*B*x/c^3+1/5*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^2*B
*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3+2*a^2*B*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e
))
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. $2(112) = 224$.

Time = 11.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(3A+8B)\left(c\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin
[(e + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[
(e + f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[
(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4*(c - c*Sin[e + f*x])^3)
```

3.33.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}dx}{c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^4}dx}{c}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{c}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{\int 1dx}{c^2}}{c^2}\right)}{c}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{x}{c^2}}{c^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) - (B*((2*Co
s[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^3) - (-(x/c^2) + (2*Cos[e + f*x]
)/(f*(c^2 - c^2*Sin[e + f*x])))/c^2))/c)
```

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.33.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{}{(\tan($
default	$\frac{2a^2\left(-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{}{(\tan($
parallelrisch	$-\frac{2\left(\frac{B\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(-\frac{5}{2}fxB+A+B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B(5fx-4)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-5fxB+2A+\frac{34}{3}B\right)\left(\tan^2\right.\right.}{fc^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
risch	$-\frac{a^2Bx}{c^3}+\frac{-4Aa^2e^{2i(fx+e)}+2Aa^2e^{4i(fx+e)}-\frac{100Ba^2e^{2i(fx+e)}}{3}-24iBa^2e^{3i(fx+e)}+\frac{56iBa^2e^{i(fx+e)}}{3}+10Ba^2e^{4i(fx+}}{\left(e^{i(fx+e)}-i\right)^5fc^3}$
norman	$\frac{a^2xB}{c}+\frac{8Ba^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{48Ba^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{6Aa^2+46Ba^2}{15cf}+\frac{40Ba^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3cf}+\frac{64Ba^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{112B}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2/c^3*(-B*arctan(tan(1/2*f*x+1/2*e))-(A+B)/(tan(1/2*f*x+1/2*e)-1)-1/
5*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(24*A+16*B)/(tan(1/2*f*x+1/2*e)
-1)^3-1/4*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^4-4*A/(tan(1/2*f*x+1/2*e)-1)^
2)
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(111) = 222.

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{60\,Ba^2fx-(15\,Ba^2fx-(3\,A+43\,B)a^2)\cos(fx+e)^3-12\,(A+B)a^2-(45\,Ba^2fx-(9\,A-11\,B)a^2)c}{15\left(c^3f\cos(fx+e)^3+3c\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12
*(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a
^2*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (
15*B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*
B)*a^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*
f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

3.33.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 7.78 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.71

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 -
 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a*
*2*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2
 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)
**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*A*a**2/(15*c**3*f*tan(e/
2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2
)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
*3*f) - 15*B*a**2*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 -
 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*B*a*
*2*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(
e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*
x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*B*a**2*f*x*tan(e/2
 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 150*B*a**2*f*x*tan(e/2 + f*x/2)**2/(
15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f
*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2
 + f*x/2) - 15*c**3*f) - 75*B*a**2*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e...
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. 2(111) = 222.

Time = 0.33 (sec) , antiderivative size = 1139, normalized size of antiderivative = 10.17

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/c^3) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5) - 3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*...
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\,(fx+e)Ba^2}{c^3} + \frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+170\,Ba^2\tan\left(\frac{1}{2}\,f\right.\right.}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*
a^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*ta
n(1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/
2*f*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.33.9 Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{B\,a^2\,x}{c^3}$$
$$-\frac{\frac{a^2\,(6\,A+46\,B-15\,B\,(e+f\,x))}{15}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^2\,(120\,B-150\,B\,(e+f\,x))}{15}+10\,B\,a^2\,(e+f\,x)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\,(3}{}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^3,x
)
```

output
```
- (B*a^2*x)/c^3 - ((a^2*(6*A + 46*B - 15*B*(e + f*x)))/15 - tan(e/2 + (f*x
)/2)^3*((a^2*(120*B - 150*B*(e + f*x)))/15 + 10*B*a^2*(e + f*x)) + tan(e/2
 + (f*x)/2)^4*((a^2*(30*A + 30*B - 75*B*(e + f*x)))/15 + 5*B*a^2*(e + f*x)
) + tan(e/2 + (f*x)/2)^2*((a^2*(60*A + 340*B - 150*B*(e + f*x)))/15 + 10*B
*a^2*(e + f*x)) - tan(e/2 + (f*x)/2)*((a^2*(200*B - 75*B*(e + f*x)))/15 +
5*B*a^2*(e + f*x)) + B*a^2*(e + f*x))/(c^3*f*(tan(e/2 + (f*x)/2) - 1)^5)
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

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3.34.1 Optimal result

Integrand size = 36, antiderivative size = 75

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6} + \frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

output
```
1/7*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(75) = 150.

Time = 8.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = -\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)+35B\cos\left(\frac{5}{2}(e+fx)\right)+A\cos\left(\frac{7}{2}(e\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^4,x]
```

output
```
-1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f
*x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*
(e + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(
3*(e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7
*B*Sin[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

3.34.3 Rubi [A] (verified)

Time $= 0.52$ (sec) , antiderivative size $= 75$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used $= \{3042, 3446, 3042, 3338, 3042, 3150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$\downarrow 3446$$
$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$\downarrow 3042$$
$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$\downarrow 3338$$
$$a^2c^2\left(\frac{(A-6B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx}{7c}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)$$
$$\downarrow 3042$$
$$a^2c^2\left(\frac{(A-6B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}\,dx}{7c}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)$$
$$\downarrow 3150$$

$$a^2c^2\left(\frac{(A-6B)\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + ((A - 6*B
)*Cos[e + f*x]^5)/(35*c*f*(c - c*Sin[e + f*x])^5))
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.34.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.77

method	result
parallelrisch	$-\frac{2a^2\left(A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A+B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2(-A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(13A+2$
derivativedivides	$\frac{2a^2\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128A+}{5\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}$
default	$\frac{2a^2\left(-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128A+}{5\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}$
risch	$-\frac{2\left(14Aa^2e^{2i(fx+e)}+Aa^2+7iAa^2e^{i(fx+e)}-7iBa^2e^{i(fx+e)}-6Ba^2+91Ba^2e^{2i(fx+e)}-70iAa^2e^{3i(fx+e)}-35Aa^2e^{4i(}\right.}{35\left(e^{i(fx+e)}-i\right)^7f}$
norman	$\frac{\left(10Aa^2-10Ba^2\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{12Aa^2-2Ba^2}{35cf}-\frac{2Aa^2\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(Aa^2-Ba^2\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2Aa^2-2Ba^2\right)}{5c}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^2*(A*tan(1/2*f*x+1/2*e)^6+(-A+B)*tan(1/2*f*x+1/2*e)^5+(4*A+B)*tan(1/2
*f*x+1/2*e)^4+2*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/5*(13*A+2*B)*tan(1/2*f*x+1/2
*e)^2+1/5*(-A+B)*tan(1/2*f*x+1/2*e)+6/35*A-1/35*B)/f/c^4/(tan(1/2*f*x+1/2*
e)-1)^7
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{(A-6B)a^2\cos(fx+e)^4+(4A+11B)a^2\cos(fx+e)^3+(13A+27B)a^2\cos(fx+e)^2-10(A+B}{35\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (1
3*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*
a^2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10
*(A + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x +
 e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x
+ e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*
cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(66) = 132$.

Time = 14.43 (sec) , antiderivative size = 2008, normalized size of antiderivative = 26.77

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 -
 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c*
*4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*ta
n(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*t
an(e/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f
*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**
4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245
*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35
*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*
tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/
2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/
2) - 35*c**4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 -
1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c*
*4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*
A*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan
(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 +
f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)*
*2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)
/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*...
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 1571, normalized size of antiderivative = 20.95

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
3)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) +
1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f...
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(73) = 146.

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.89

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+35\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+140\,Aa^2\tan\left(\frac{1}{2}\,fx\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
 35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B
*a^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*t
an(1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2
*f*x + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2
*e) + 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{109\,A\,a^2}{4}+\frac{11\,B\,a^2}{4}-\frac{27\,A\,a^2\cos(2e+2fx)}{4}+\frac{5\,A\,a^2\cos(3e+3fx)}{8}-\frac{13\,B\,a^2\cos(2e+2fx)}{4}+\frac{5\,B\,a^2\cos(3e+3}{8}\right.}{35\,c^4\,f\left(\frac{35\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{}{}\right.}{8}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e +
2*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 +
 (5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*si
n(3*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*
x))/8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2
*sin(e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos
(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8
 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 -
pi/4 + (7*f*x)/2))/8))
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

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3.35.1 Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

output
```
1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*s
in(f*x+e))^5
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(115) = 230.

Time = 8.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.27

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+1\right.$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^5,x]
```

output
```
-1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(3
15*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 3
15*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f
*x))/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e
 + f*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*
B*Sin[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/
2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

3.35.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(2A-7B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(2A-7B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(2A-7B)\left(\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(2A-7B)\left(\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\downarrow 3150$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{(2A-7B)\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + ((2*A - 7
*B)*(Cos[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])^6) + Cos[e + f*x]^5/(35*c*f*
(c - c*Sin[e + f*x])^5)))/(9*c))
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.35.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(22A+B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+(-8A+3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(54A+B)(t}{f c^5\left(\tan\left(\frac{fx}{2}+\right.\right.}$
risch	$\frac{2ia^2\left(420iA\,e^{6i(fx+e)}+105iB\,e^{6i(fx+e)}+315B\,e^{7i(fx+e)}-882iA\,e^{4i(fx+e)}-630A\,e^{5i(fx+e)}-63iB\,e^{4i(fx+e)}-945B\,e^{5i(}{315f\,c^5\left(e^{i(fx}$
derivativedivides	$\frac{2a^2\left(-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{200A+104B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1}{2\left(\tan\right.}$
default	$\frac{2a^2\left(-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{200A+104B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1}{2\left(\tan\right.}$
norman	$\frac{\left(4A\,a^2-2B\,a^2\right)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(28A\,a^2-12B\,a^2\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{94A\,a^2-14B\,a^2}{315cf}-\frac{2A\,a^2\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(24A\,a^2-14\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^8+(B-2*A)*tan(1/2*f*x+1/2*e)^7+1/3*(22*A+B)*tan(1
/2*f*x+1/2*e)^6+(-8*A+3*B)*tan(1/2*f*x+1/2*e)^5+1/5*(54*A+B)*tan(1/2*f*x+1
/2*e)^4+1/5*(-26*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/5*(118/7*A+B)*tan(1/2*f*x+
1/2*e)^2+1/5*(-12/7*A+B)*tan(1/2*f*x+1/2*e)+47/315*A-1/45*B)*a^2/f/c^5/(ta
n(1/2*f*x+1/2*e)-1)^9
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(112) = 224$.

Time $= 0.26$ (sec) , antiderivative size $= 335$, normalized size of antiderivative $= 2.91$

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx$$
$$= \frac{(2\,A - 7\,B)a^2\cos(fx + e)^5 - 4\,(2\,A - 7\,B)a^2\cos(fx + e)^4 - 5\,(5\,A + 14\,B)a^2\cos(fx + e)^3 - 5\,(17\,A +}{315\left(c^5 f\cos(fx + e)^5 + 5\,c^5 f\cos(fx + e)^4 - 8\,c^5 f\cos\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
 5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 +
70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x +
 e)^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2
 + 70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos
(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*c
os(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 -
4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) +
16*c^5*f)*sin(f*x + e))
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(102) = 204$.

Time = 25.84 (sec) , antiderivative size = 3262, normalized size of antiderivative = 28.37

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9
 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26
460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*
c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5
*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260
*A*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*
tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(
e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2
+ f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*
x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*A*a**2*tan(e/2
 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2
)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6
 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2
6460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*
c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 5040*A*a**2*tan(e/2 + f*x/2)**5/(3
15*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
*5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f
*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan
(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2
+ f*x/2) - 315*c**5*f) - 6804*A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*ta...
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2087 vs. $2(112) = 224$.

Time = 0.28 (sec) , antiderivative size = 2087, normalized size of antiderivative = 18.15

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*
sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^
7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2...
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.48

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^5}\,dx =$$

$$-\frac{2\left(315\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8 - 630\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 + 315\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 + 2310\,Aa^2\tan(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 +
 105*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 94
5*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*
a^2*tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2
*tan(1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan
(1/2*f*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*
x + 1/2*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.88

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{265\,A\,a^2\cos(2\,e+2\,f\,x)}{2}-\frac{49\,B\,a^2}{8}-\frac{4967\,A\,a^2}{16}-\frac{89\,A\,a^2\cos(3\,e+3\,f\,x)}{4}-\frac{49\,A\,a^2\cos(4\,e+4\,f\,x)}{16}+\frac{35\,B\,a^2\,c}{}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x
)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (4
967*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x)
)/16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B
*a^2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin
(3*e + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*
f*x))/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35
*B*a^2*cos(e + f*x))/8 + (2205*A*a^2*sin(e + f*x))/8 - (945*B*a^2*sin(e +
f*x))/8))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/
2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5
*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*co
s((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

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3.36.1 Optimal result

Integrand size = 36, antiderivative size = 156

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\ &=\frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}\\ &\quad+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}\end{aligned}$$

output
```
1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*
sin(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

3.36.2 Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.83

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\ &=\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+\right.}\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^6,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
 + 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*C
os[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))
/2] + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(
e + f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310
*B*Sin[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f
*x))/2] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c
^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

3.36.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(3A-8B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-8B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\left(\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3150$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{(3A-8B)\left(\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}\right)}{11c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + ((3*A -
8*B)*(Cos[e + f*x]^5/(9*f*(c - c*Sin[e + f*x])^7) + (2*(Cos[e + f*x]^5/(7*
f*(c - c*Sin[e + f*x])^6) + Cos[e + f*x]^5/(35*c*f*(c - c*Sin[e + f*x])^5)
))/(9*c)))/(11*c))
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.36.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{2a^2\left(A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(12A-\frac{B}{3}\right)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\left(-10A+\frac{7B}{3}\right)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right.$
derivativedivides	$\frac{2a^2\left(-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{128}{11\left(\tan\left(\right.\right.}\right.}{f}$
default	$\frac{2a^2\left(-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{128}{11\left(\tan\left(\right.\right.}\right.}{f}$
risch	$-\frac{4Aa^2}{1155}-\frac{16iBa^2e^{5i(fx+e)}}{5}-\frac{44iAa^2e^{5i(fx+e)}}{5}+\frac{32Ba^2}{3465}+\frac{32iBa^2e^{i(fx+e)}}{315}-\frac{4iAa^2e^{i(fx+e)}}{105}+\frac{8iBa^2e^{7i(fx+e)}}{3}-\frac{32Ba^2e^{2i(f}}{63}$
norman	$-\frac{912Aa^2-122Ba^2}{3465cf}-\frac{2Aa^2\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(3Aa^2-Ba^2\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{2\left(45Aa^2-Ba^2\right)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2\left(87Aa^2\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^2*(A*tan(1/2*f*x+1/2*e)^10+(-3*A+B)*tan(1/2*f*x+1/2*e)^9+(12*A-1/3*B)
*tan(1/2*f*x+1/2*e)^8+2*(-10*A+7/3*B)*tan(1/2*f*x+1/2*e)^7+2/5*(81*A-13/3*
B)*tan(1/2*f*x+1/2*e)^6+2/5*(-71*A+16*B)*tan(1/2*f*x+1/2*e)^5+4/7*(41*A-2*
B)*tan(1/2*f*x+1/2*e)^4+2/7*(-34*A+9*B)*tan(1/2*f*x+1/2*e)^3+1/21*(89*A+2/
3*B)*tan(1/2*f*x+1/2*e)^2+1/105*(-47*A+61/3*B)*tan(1/2*f*x+1/2*e)+152/1155
*A-61/3465*B)/f/c^6/(tan(1/2*f*x+1/2*e)-1)^11
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(152) = 304$.

Time $= 0.25$ (sec) , antiderivative size $= 407$, normalized size of antiderivative $= 2.61$

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^6}\,dx =$$
$$-\frac{2\,(3\,A - 8\,B)a^2\cos(fx+e)^6 + 12\,(3\,A - 8\,B)a^2\cos(fx+e)^5 - 25\,(3\,A - 8\,B)a^2\cos(fx+e)^4 - 35\,(6}{3465\left(c^6 f\cos(fx+e)^6 - 5\,c^6 f\cos(\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
-1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e
)^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)
^3 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) +
1260*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*
cos(f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*c
os(f*x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x +
 e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e
)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x
 + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6
*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c
^6*f)*sin(f*x + e))
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(141) = 282$.

Time = 44.16 (sec) , antiderivative size = 4816, normalized size of antiderivative = 30.87

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) - 83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) + 138600*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2
)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x...
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(152) = 304$.

Time = 0.31 (sec) , antiderivative size = 2604, normalized size of antiderivative = 16.69

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/
(cos(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6
*A*a^2*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12
936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(c...
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.

Time $= 0.36$ (sec) , antiderivative size $= 353$, normalized size of antiderivative $= 2.26$

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^6}\,dx =$$
$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10} - 10395\,Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9} + 3465\,Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9} + 41580\,A\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/
2*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2
*e)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*
e)^7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2
*e)^6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*
e)^5 + 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*
e)^4 - 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e
)^3 + 8910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)
^2 + 110*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) +
671*B*a^2*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x
 + 1/2*e) - 1)^11)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.71

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{38163\,A\,a^2}{8}-\frac{1283\,B\,a^2}{8}-\frac{11931\,A\,a^2\cos(2e+2fx)}{4}+\frac{9609\,A\,a^2\cos(3e+3fx)}{16}+\frac{1383\,A\,a^2\cos(4e+4fx)}{8}-\frac{22}{}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos
(2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e
+ 4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x
))/4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 +
 (45*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a
^2*sin(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(
5*e + 5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e +
 3*f*x))/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x)
)/32 - 2091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^
2*sin(e + f*x))/4 + (23331*B*a^2*sin(e + f*x))/16))/(3465*c^6*f*((231*2^(1
/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f
*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2
)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (
9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

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3.37.1 Optimal result

Integrand size = 36, antiderivative size = 197

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx\\ &=\frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}\\ &\quad+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}\end{aligned}$$

output

```
1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c
*sin(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/
15015*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5
```

3.37.2 Mathematica [A] (verified)

Time = 10.42 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.59

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(6006(8A+7B)\cos\left(\frac{1}{2}(e+fx)\right)-1716(11A\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^7,x]
```

output
```
-1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*
(6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/
2] - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*C
os[(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x)
)/2] + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(
3*(e + f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/
2] - 8580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[
(9*(e + f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))
/(c^7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

3.37.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(4A-9B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}dx}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(4A-9B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^8}dx}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3151$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos}{13f(c-c\sin}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos}{13f(c-c\sin}\right.$$

$$\downarrow 3150$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{(4A-9B)\left(\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{3\left(\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+f}{7f(c-c\sin(e-}\right.}{9c}\right.}{11c}\right.}{13c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + ((4*A -
9*B)*(Cos[e + f*x]^5/(11*f*(c - c*Sin[e + f*x])^8) + (3*(Cos[e + f*x]^5/(9
*f*(c - c*Sin[e + f*x])^7) + (2*(Cos[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])^
6) + Cos[e + f*x]^5/(35*c*f*(c - c*Sin[e + f*x])^5)))/(9*c)))/(11*c)))/(13
*c))
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.37.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{4ia^2\left(4iA+2860iA\,e^{4i(fx+e)}+15015B\,e^{9i(fx+e)}+21021iB\,e^{8i(fx+e)}-48048A\,e^{7i(fx+e)}-312iA\,e^{2i(fx+e)}-42042B\,e^{}\right.}{}$
parallelrisch	$-\frac{2a^2\left(A\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A-B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-40A+7B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{256A+256B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8320A+7680B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{12}{3\left(\tan\left(\right.\right.}}{}$
default	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{256A+256B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8320A+7680B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{12}{3\left(\tan\left(\right.\right.}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETUR
NVERBOSE)
```

output
```
-4/15015*I*a^2*(4*I*A+2860*I*A*exp(4*I*(f*x+e))+15015*B*exp(9*I*(f*x+e))+2
1021*I*B*exp(8*I*(f*x+e))-48048*A*exp(7*I*(f*x+e))-312*I*A*exp(2*I*(f*x+e)
)-42042*B*exp(7*I*(f*x+e))-9*I*B+18876*A*exp(5*I*(f*x+e))-26598*I*B*exp(6*
I*(f*x+e))+32604*B*exp(5*I*(f*x+e))+702*I*B*exp(2*I*(f*x+e))+1144*A*exp(3*
I*(f*x+e))+24024*I*A*exp(8*I*(f*x+e))-2574*B*exp(3*I*(f*x+e))+8580*I*B*exp
(4*I*(f*x+e))-52*A*exp(I*(f*x+e))-54912*I*A*exp(6*I*(f*x+e))+117*B*exp(I*(
f*x+e)))/f/c^7/(exp(I*(f*x+e))-I)^13
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(192) = 384$.

Time $= 0.27$ (sec) , antiderivative size $= 475$, normalized size of antiderivative $= 2.41$

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^7}\,dx$$
$$= \frac{2\,(4\,A - 9\,B)a^2\cos\left(fx+e\right)^7 - 12\,(4\,A - 9\,B)a^2\cos\left(fx+e\right)^6 - 49\,(4\,A - 9\,B)a^2\cos\left(fx+e\right)^5 + 70\,(4\,A}{15015\left(c^7 f\cos\left(fx+e\right)^7 +}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="fricas")
```

output
```
1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e
)^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^
4 + 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x +
e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a
^2*cos(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2
*cos(f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^
2*cos(f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f
*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*co
s(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 -
6*c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3
 + 80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e
))
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(178) = 356$.

Time = 71.06 (sec) , antiderivative size = 6669, normalized size of antiderivative = 33.85

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

output
```
Piecewise((-30030*A*a**2*tan(e/2 + f*x/2)**12/(15015*c**7*f*tan(e/2 + f*x/
2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x
/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 +
f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2
+ f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/
2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e
/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/
2 + f*x/2) - 15015*c**7*f) + 120120*A*a**2*tan(e/2 + f*x/2)**11/(15015*c**
7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c*
*7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725
*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 257657
40*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 1932
4305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 42
94290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 19
5195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 540540*A*a**2*tan(e/2 + f*x
/2)**10/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2
)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x
/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 +
f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2
+ f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan...
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3120 vs. $2(192) = 384$.

Time = 0.34 (sec) , antiderivative size = 3120, normalized size of antiderivative = 15.84

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="maxima")
```

output
```
-2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18
7330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c
^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f
*x + e)^13/(cos(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x +
e) + 1) - 28626*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26512
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 276276*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - ...
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(192) = 384$.

Time $= 0.40$ (sec) , antiderivative size $= 421$, normalized size of antiderivative $= 2.14$

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx =$$
$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{12} - 60060\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 15015\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 2702\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="giac")
```

output

```
-2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x +
1/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x
 + 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*
f*x + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1
/2*f*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan
(1/2*f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*
tan(1/2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a
^2*tan(1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A
*a^2*tan(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*
A*a^2*tan(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*
A*a^2*tan(1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*
a^2*tan(1/2*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 -
213*B*a^2)/(c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.54

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx =$$

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{994249\,A\,a^2}{32}-\frac{63639\,B\,a^2}{32}-\frac{1609013\,A\,a^2\cos(2\,e+2\,f\,x)}{64}+\frac{85687\,A\,a^2\cos(3\,e+3\,f\,x)}{16}+\frac{79591\,A\,a^2\cos(4\,e+4\,$$

input

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x
)
```

output

```
-(2*cos(e/2 + (f*x)/2)*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*
a^2*cos(2*e + 2*f*x))/64 + (85687*A*a^2*cos(3*e + 3*f*x))/16 + (79591*A*a^
2*cos(4*e + 4*f*x))/32 - (5261*A*a^2*cos(5*e + 5*f*x))/16 - (1771*A*a^2*co
s(6*e + 6*f*x))/64 + (140553*B*a^2*cos(2*e + 2*f*x))/64 - (4431*B*a^2*cos(
3*e + 3*f*x))/8 - (10161*B*a^2*cos(4*e + 4*f*x))/32 + 36*B*a^2*cos(5*e + 5
*f*x) + (231*B*a^2*cos(6*e + 6*f*x))/64 + (636207*A*a^2*sin(2*e + 2*f*x))/
64 + (309309*A*a^2*sin(3*e + 3*f*x))/32 - (7007*A*a^2*sin(4*e + 4*f*x))/4
- (12389*A*a^2*sin(5*e + 5*f*x))/32 + (1755*A*a^2*sin(6*e + 6*f*x))/64 - (
121407*B*a^2*sin(2*e + 2*f*x))/64 - (39039*B*a^2*sin(3*e + 3*f*x))/32 + (3
003*B*a^2*sin(4*e + 4*f*x))/16 + (1599*B*a^2*sin(5*e + 5*f*x))/32 - (195*B
*a^2*sin(6*e + 6*f*x))/64 - (93221*A*a^2*cos(e + f*x))/8 + (3291*B*a^2*cos
(e + f*x))/8 - (704847*A*a^2*sin(e + f*x))/16 + (125697*B*a^2*sin(e + f*x)
)/16))/(15015*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (
429*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/
4 + (5*f*x)/2))/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (3
9*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 -
 pi/4 + (11*f*x)/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

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3.38.1 Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx \\
&= \frac{11}{256}a^3(10A-3B)c^6x + \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} \\
&\quad + \frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f} \\
&\quad + \frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} \\
&\quad + \frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} - \frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} \\
&\quad + \frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} \\
&\quad + \frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}
\end{aligned}$$

output

```
11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/25
6*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos
(f*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f
-1/10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(
f*x+e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-
c^6*sin(f*x+e))/f
```

3.38.2 Mathematica [A] (verified)

Time = 9.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6(27720(10A-3B)(e+fx)+5040(33A-19B)\cos(e+fx)+33}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^6,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*
x) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)]
 + 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 2
80*(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 252
0*(6*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*
(6*A - 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

3.38.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$
$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\downarrow 3339$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\int \cos^6(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^3}{10f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\int \cos(e+fx)^6(c-c\sin(e+fx))^3dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^3}{10f}\right)$$

$$\downarrow 3157$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\int \cos^6(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)-\frac{B\cos^7($$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\int \cos(e+fx)^6(c-c\sin(e+fx))^2dx+\frac{c\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)-\frac{B\cos^7($$

$$\downarrow 3157$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\int \cos^6(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)+\frac{cc}{}\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\int \cos(e+fx)^6(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)+\frac{cc}{}\right.\right.$$

$$\downarrow 3148$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\int \cos^6(e+fx)dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)+\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx\right.\right.\right.\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int\cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(}{}\right.\right.\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int\sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\right.\right.\right.$$

$\downarrow$ 3115

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+}{6f}\right.\right.\right.\right.\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos}{6f}\right.\right.\right.\right.\right.$$

$\downarrow$ 3115

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin}{}\right.\right.\right.\right.\right.$$

$\downarrow$ 24

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}+\frac{9}{8}c\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+f}{6f}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

output
```
a^3*c^3*(-1/10*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^3)/f + ((10*A - 3*B)
*((c*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^2)/(9*f) + (11*c*((Cos[e + f*x]^7
*(c^2 - c^2*Sin[e + f*x]))/(8*f) + (9*c*((c*Cos[e + f*x]^7)/(7*f) + c*((Co
s[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
 + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6)))/8))/9))/10)
```

3.38. $\int(a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.38.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{3\left(\left(\frac{29A}{9}-\frac{5B}{3}\right)\cos(3fx+3e)+\left(A-\frac{B}{3}\right)\cos(5fx+5e)+\left(\frac{3A}{28}+\frac{5B}{84}\right)\cos(7fx+7e)+\left(-\frac{A}{108}+\frac{B}{36}\right)\cos(9fx+9e)+\left(6A-\frac{25B}{24}\right)$
risch	$\frac{55a^3c^6xA}{128}-\frac{33a^3c^6xB}{256}+\frac{33c^6a^3\cos(fx+e)A}{128f}-\frac{19c^6a^3\cos(fx+e)B}{128f}-\frac{Ba^3c^6\sin(10fx+10e)}{5120f}-\frac{c^6a^3\cos(9fx$
parts	$-\frac{(-6Aa^3c^6-6Ba^3c^6)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}+\frac{(-6Aa^3c^6+8Ba^3c^6)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin($}{f}$
derivativedivides	$3Aa^3c^6\cos(fx+e)-3Ba^3c^6\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aa^3c^6\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}\right.}{9}$
default	$3Aa^3c^6\cos(fx+e)-3Ba^3c^6\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aa^3c^6\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}\right.}{9}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
3/64*((29/9*A-5/3*B)*cos(3*f*x+3*e)+(A-1/3*B)*cos(5*f*x+5*e)+(3/28*A+5/84*
B)*cos(7*f*x+7*e)+(-1/108*A+1/36*B)*cos(9*f*x+9*e)+(6*A-25/24*B)*sin(2*f*x
+2*e)+(1/2*A+7/12*B)*sin(4*f*x+4*e)+(-2/9*A+17/48*B)*sin(6*f*x+6*e)+(-1/16
*A+5/96*B)*sin(8*f*x+8*e)-1/240*B*sin(10*f*x+10*e)+(11/2*A-19/6*B)*cos(f*x
+e)+55/6*f*x*A-11/4*f*x*B+1856/189*A-320/63*B)*c^6*a^3/f
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx =$$

$$-\frac{8960\,(A-3\,B)a^3c^6\cos(fx+e)^9-46080\,(A-B)a^3c^6\cos(fx+e)^7-3465\,(10\,A-3\,B)a^3c^6fx+21\,(}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
-1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*co
s(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x +
 e)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*
cos(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B
)*a^3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(252) = 504$.

Time = 1.51 (sec) , antiderivative size = 1948, normalized size of antiderivative = 7.35

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^6 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(
e + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A
*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e +
 f*x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**
3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x
)**2*cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2
 - 105*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6
/2 - 9*A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e
 + f*x)**8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(
128*f) - 8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*
c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)
**5*cos(e + f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5
*f) + 6*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e
 + f*x)**3*cos(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e
+ f*x)**3/(3*f) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A
*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e +
f*x)**2*cos(e + f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f +
 105*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(
e + f*x)*cos(e + f*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**
3/(4*f) - 128*A*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(...
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(252) = 504$.

Time = 0.23 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^6\,dx =$$

$$-\frac{2048\left(35\cos(fx+e)^9 - 180\cos(fx+e)^7 + 378\cos(fx+e)^5 - 420\cos(fx+e)^3 + 315\cos(fx+e)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)
^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x
+ e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*
x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*
e))*A*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
+ 4*e) - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 61
44*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(
f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*c
os(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3
*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*
sin(2*f*x + 2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8
*f*x + 8*e) - 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20
160*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*
f*x + 2*e))*B*a^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6
 - 1935360*A*a^3*c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx \\
&= -\frac{Ba^3c^6\sin(10fx+10e)}{5120f} + \frac{11}{256}\left(10Aa^3c^6-3Ba^3c^6\right)x \\
&\quad -\frac{(Aa^3c^6-3Ba^3c^6)\cos(9fx+9e)}{2304f} + \frac{(9Aa^3c^6+5Ba^3c^6)\cos(7fx+7e)}{1792f} \\
&\quad +\frac{(3Aa^3c^6-Ba^3c^6)\cos(5fx+5e)}{64f} + \frac{(29Aa^3c^6-15Ba^3c^6)\cos(3fx+3e)}{192f} \\
&\quad +\frac{(33Aa^3c^6-19Ba^3c^6)\cos(fx+e)}{128f} - \frac{(6Aa^3c^6-5Ba^3c^6)\sin(8fx+8e)}{2048f} \\
&\quad -\frac{(32Aa^3c^6-51Ba^3c^6)\sin(6fx+6e)}{3072f} + \frac{(6Aa^3c^6+7Ba^3c^6)\sin(4fx+4e)}{256f} \\
&\quad +\frac{(144Aa^3c^6-25Ba^3c^6)\sin(2fx+2e)}{512f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^
6)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a
^3*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*
cos(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/
f + 1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c
^6 - 5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6
)*sin(6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/
f + 1/512*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

3.38.9 Mupad [B] (verification not implemented)

Time = 16.01 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.06

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

output

```
(tan(e/2 + (f*x)/2)^18*(6*A*a^3*c^6 - 2*B*a^3*c^6) + tan(e/2 + (f*x)/2)^16
*(22*A*a^3*c^6 - 18*B*a^3*c^6) + tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B
*a^3*c^6) + tan(e/2 + (f*x)/2)^14*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + tan(
e/2 + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + tan(e/2 + (f*x)/
2)^10*(116*A*a^3*c^6 - 60*B*a^3*c^6) - tan(e/2 + (f*x)/2)^19*((73*A*a^3*c^
6)/64 + (33*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (
58*B*a^3*c^6)/21) + tan(e/2 + (f*x)/2)^12*((328*A*a^3*c^6)/3 - 72*B*a^3*c^
6) + tan(e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - tan(
e/2 + (f*x)/2)^13*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + tan(e/2 + (f
*x)/2)^6*((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^5*((
449*A*a^3*c^6)/48 - (577*B*a^3*c^6)/160) - tan(e/2 + (f*x)/2)^15*((449*A*a
^3*c^6)/48 - (577*B*a^3*c^6)/160) + tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)
/192 - (705*B*a^3*c^6)/128) - tan(e/2 + (f*x)/2)^17*((2117*A*a^3*c^6)/192
- (705*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*
B*a^3*c^6)/64) - tan(e/2 + (f*x)/2)^11*((699*A*a^3*c^6)/32 - (2749*B*a^3*c
^6)/64) + tan(e/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (5
8*A*a^3*c^6)/63 - (10*B*a^3*c^6)/21)/(f*(10*tan(e/2 + (f*x)/2)^2 + 45*tan(
e/2 + (f*x)/2)^4 + 120*tan(e/2 + (f*x)/2)^6 + 210*tan(e/2 + (f*x)/2)^8 + 2
52*tan(e/2 + (f*x)/2)^10 + 210*tan(e/2 + (f*x)/2)^12 + 120*tan(e/2 + (f*x)
/2)^14 + 45*tan(e/2 + (f*x)/2)^16 + 10*tan(e/2 + (f*x)/2)^18 + tan(e/2 ...
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

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3.39.1 Optimal result

Integrand size = 36, antiderivative size = 222

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{5}{128}a^3(9A-2B)c^5x + \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} \\
&\quad + \frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f} + \frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f} \\
&\quad + \frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} \\
&\quad + \frac{a^3(9A-2B)\cos^7(e+fx)\,(c^5-c^5\sin(e+fx))}{72f}
\end{aligned}$$

output

```
5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*
(9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3
*sin(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c
^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-
c^5*sin(f*x+e))/f
```

3.39.2 Mathematica [A] (verified)

Time = 8.78 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+504(20A-13B)\cos(e+fx)+336(1$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^5,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
 + 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 1
008*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[
9*(e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e
+ f*x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6)
```

3.39.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$
$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\downarrow 3339$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\int \cos^6(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\int \cos(e+fx)^6(c-c\sin(e+fx))^2dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)$$

$$\downarrow 3157$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\int \cos^6(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos^7(e+\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\int \cos(e+fx)^6(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos^7(e+\right.$$

$$\downarrow 3148$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\int \cos^6(e+fx)dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos^7(\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int \cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)}{\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e\right.\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\right.\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right.\right.\right.\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\right.\right.\right.\right.$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}+\frac{9}{8}c\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

output
```
a^3*c^3*(-1/9*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^2)/f + ((9*A - 2*B)*(
(Cos[e + f*x]^7*(c^2 - c^2*Sin[e + f*x]))/(8*f) + (9*c*((c*Cos[e + f*x]^7)
/(7*f) + c*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e + f*x]^3*Sin[
e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6)))/8
))/9)
```

3.39.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.39.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{c^5\left(\left(3A-\frac{11B}{6}\right)\cos(3fx+3e)+\left(A-\frac{B}{2}\right)\cos(5fx+5e)+\frac{\left(A-\frac{B}{8}\right)\cos(7fx+7e)}{7}+(8A-B)\sin(2fx+2e)+\frac{\left(\frac{5A}{2}+B\right)\sin(4fx+4}{2}}{32f}$
risch	$\frac{45a^3c^5xA}{128}-\frac{5a^3c^5xB}{64}+\frac{5c^5a^3\cos(fx+e)A}{32f}-\frac{13c^5a^3\cos(fx+e)B}{128f}+\frac{Ba^3c^5\cos(9fx+9e)}{2304f}-\frac{\sin(8fx+8e)Aa^3c}{1024f}$
parts	$\frac{\left(-2Aa^3c^5-2Ba^3c^5\right)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{\left(-2Aa^3c^5+Ba^3c^5\right)\cos(fx+e)}{f}+\frac{\left(-Aa^3c^5+2Ba^3c^5\right)\left(-\right.}{}$
derivativedivides	$Aa^3c^5(fx+e)+2Aa^3c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{6Aa^3c^5\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}$
default	$Aa^3c^5(fx+e)+2Aa^3c^5\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{6Aa^3c^5\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}$
norman	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
1/32*c^5*((3*A-11/6*B)*cos(3*f*x+3*e)+(A-1/2*B)*cos(5*f*x+5*e)+1/7*(A-1/8*
B)*cos(7*f*x+7*e)+(8*A-B)*sin(2*f*x+2*e)+1/2*(5/2*A+B)*sin(4*f*x+4*e)+1/16
*(-1/2*A+B)*sin(8*f*x+8*e)+1/72*cos(9*f*x+9*e)*B+1/3*B*sin(6*f*x+6*e)+(5*A
-13/4*B)*cos(f*x+e)+45/4*f*x*A-5/2*f*x*B+64/7*A-352/63*B)*a^3/f
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{896\,Ba^3c^5\cos(fx+e)^9+2304\,(A-B)a^3c^5\cos(fx+e)^7+315\,(9\,A-2\,B)a^3c^5fx-21\,(48\,(A-2\,B)a^3}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
 + 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 -
 8*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e
)^3 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(209) = 418$.

Time = 1.14 (sec) , antiderivative size = 1753, normalized size of antiderivative = 7.90

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^5 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e
+ f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a
**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
6/32 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*
sin(e + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*c
os(e + f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a*
*3*c**5*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)*
*6*cos(e + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f
) - 11*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(
e + f*x)**4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)
/f + 385*A*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c*
*5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*
cos(e + f*x)**5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f -
6*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)
*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f
) + A*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)*
*7/(35*f) + 16*A*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f
*x)**3/f + 2*A*a**3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8
/64 + 35*B*a**3*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c...
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(210) = 420$.

Time $= 0.32$ (sec) , antiderivative size $= 617$, normalized size of antiderivative $= 2.78$

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^5\, dx$$

$$= \frac{18432\left(5\cos(fx+e)^7 - 21\cos(fx+e)^5 + 35\cos(fx+e)^3 - 35\cos(fx+e)\right)Aa^3c^5 + 129024\left(3\cos(f$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3
 - 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^
3 + 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*
A*a^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 
8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*si
n(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e
))*A*a^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*
(f*x + e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*c
os(f*x + e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(
5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e)
)*B*a^3*c^5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(12
8*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*
x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 +
 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 604
80*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161
280*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x
 + e) - 322560*B*a^3*c^5*cos(f*x + e))/f
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{Ba^3c^5\cos(9fx+9e)}{2304f} + \frac{Ba^3c^5\sin(6fx+6e)}{96f} \\
&\quad + \frac{5}{128}\left(9Aa^3c^5 - 2Ba^3c^5\right)x + \frac{(8Aa^3c^5 - Ba^3c^5)\cos(7fx+7e)}{1792f} \\
&\quad + \frac{(2Aa^3c^5 - Ba^3c^5)\cos(5fx+5e)}{64f} + \frac{(18Aa^3c^5 - 11Ba^3c^5)\cos(3fx+3e)}{192f} \\
&\quad + \frac{(20Aa^3c^5 - 13Ba^3c^5)\cos(fx+e)}{128f} - \frac{(Aa^3c^5 - 2Ba^3c^5)\sin(8fx+8e)}{1024f} \\
&\quad + \frac{(5Aa^3c^5 + 2Ba^3c^5)\sin(4fx+4e)}{128f} + \frac{(8Aa^3c^5 - Ba^3c^5)\sin(2fx+2e)}{32f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f +
5/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos
(7*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/19
2*(18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 -
 13*B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x
 + 8*e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8
*A*a^3*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

3.39.9 Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.18

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}\left(4Aa^3c^5 - 2Ba^3c^5\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}\left(8Aa^3c^5 - 8Ba^3c^5\right) + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{8Aa^3c^5}{7} - \frac{8}{}\right.}{} \\
&\quad + \frac{5a^3c^5\,\mathrm{atan}\left(\frac{5a^3c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(9A-2B)}{64\left(\frac{45Aa^3c^5}{64} - \frac{5Ba^3c^5}{32}\right)}\right)(9A-2B)}{64f}
\end{aligned}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

output
```
(tan(e/2 + (f*x)/2)^16*(4*A*a^3*c^5 - 2*B*a^3*c^5) + tan(e/2 + (f*x)/2)^14
*(8*A*a^3*c^5 - 8*B*a^3*c^5) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*
B*a^3*c^5)/7) + tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + tan(e/
2 + (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + tan(e/2 + (f*x)/2)^12*(24*A
*a^3*c^5 - (16*B*a^3*c^5)/3) + tan(e/2 + (f*x)/2)^10*(40*A*a^3*c^5 - 40*B*
a^3*c^5) + tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - ta
n(e/2 + (f*x)/2)^17*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + tan(e/2 + (f*
x)/2)^5*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^13*(
(149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) + tan(e/2 + (f*x)/2)^3*((189*A*a^3
*c^5)/32 - (191*B*a^3*c^5)/48) - tan(e/2 + (f*x)/2)^15*((189*A*a^3*c^5)/32
 - (191*B*a^3*c^5)/48) + tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B
*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^11*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5
)/16) + tan(e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a
^3*c^5)/7 - (22*B*a^3*c^5)/63)/(f*(9*tan(e/2 + (f*x)/2)^2 + 36*tan(e/2 + (
f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^8 + 126*tan(e
/2 + (f*x)/2)^10 + 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^14 + 9
*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 + 1)) + (5*a^3*c^5*atan((5*
a^3*c^5*tan(e/2 + (f*x)/2)*(9*A - 2*B))/(64*((45*A*a^3*c^5)/64 - (5*B*a^3*
c^5)/32)))*(9*A - 2*B))/(64*f)
```

3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

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3.40.1 Optimal result

Integrand size = 36, antiderivative size = 181

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{5}{128}a^3(8A-B)c^4x + \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} \\
&\quad + \frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f} + \frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f} \\
&\quad + \frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{a^3B\cos^7(e+fx)\,(c^4-c^4\sin(e+fx))}{8f}
\end{aligned}$$

output

```
5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*cos(f*x+e)^7/f+5/128*a^3*(8*A
-B)*c^4*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*cos(f*x+e)^3*sin(f*x
+e)/f+1/48*a^3*(8*A-B)*c^4*cos(f*x+e)^5*sin(f*x+e)/f-1/8*a^3*B*cos(f*x+e)^
7*(c^4-c^4*sin(f*x+e))/f
```

3.40.2 Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4(840(8A-B)(e+fx)+1680(A-B)\cos(e+fx)+1008(A-B$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^4,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) +
1680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Co
s[5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f
*x)] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*
B*Sin[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

3.40.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$
$$\downarrow 3042$$
$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\downarrow 3339$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\int \cos^6(e+fx)(c-c\sin(e+fx))dx - \frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\int \cos(e+fx)^6(c-c\sin(e+fx))dx - \frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\downarrow 3148$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\int \cos^6(e+fx)dx + \frac{c\cos^7(e+fx)}{7f}\right) - \frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^6 dx + \frac{c\cos^7(e+fx)}{7f}\right) - \frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\int \cos^4(e+fx)dx + \frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right) + \frac{c\cos^7(e+fx)}{7f}\right) - \frac{B\cos^7(e+fx)(c-\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4 dx + \frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right) + \frac{c\cos^7(e+fx)}{7f}\right) - \frac{B\cos^7(e+f\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(e+fx)dx + \frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right) + \frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right) + \frac{c\cos^7}{}\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx + \frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right) + \frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right) +\right.\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2} + \frac{\sin(e+fx)\cos(e+fx)}{2f}\right) + \frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right) + \frac{\sin(e+fx)\cos^5(}{6f}\right.\right.\right.$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)}{2}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a^3*c^3*(-1/8*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x]))/f + ((8*A - B)*((c*C
os[e + f*x]^7)/(7*f) + c*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e
 + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))/4))/6)))/8)
```

3.40.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.40.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{c^4\left(3(A-B)\cos(3fx+3e)+(A-B)\cos(5fx+5e)+\frac{(A-B)\cos(7fx+7e)}{7}+(15A-B)\sin(2fx+2e)+\left(\frac{B}{2}+3A\right)\sin(4fx+4e)+\right.}{64f}$
risch	$\frac{5a^3c^4xA}{16}-\frac{5a^3c^4xB}{128}+\frac{5c^4a^3\cos(fx+e)A}{64f}-\frac{5c^4a^3\cos(fx+e)B}{64f}+\frac{Ba^3c^4\sin(8fx+8e)}{1024f}+\frac{c^4a^3\cos(7fx+7e)A}{448f}$
parts	$\frac{(-3Aa^3c^4-Ba^3c^4)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-3Aa^3c^4+3Ba^3c^4)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(}{5f}$
derivativedivides	$\frac{Aa^3c^4(fx+e)+3Aa^3c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+Ba^3c^4\left(-\frac{\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\right.}{}\right.}{}$
default	$\frac{Aa^3c^4(fx+e)+3Aa^3c^4\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+Ba^3c^4\left(-\frac{\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\right.}{}\right.}{}$
norman	$\frac{c^4a^3(488A-397B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192f}+\left(\frac{5}{16}Aa^3c^4-\frac{5}{128}Ba^3c^4\right)x+\frac{c^4a^3(88A+5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}-\frac{5c^4a^3(136A-353B)\left(\tan^9\left(\frac{fx}{2}+\right.\right.}{192f}$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output

```
1/64*c^4*(3*(A-B)*cos(3*f*x+3*e)+(A-B)*cos(5*f*x+5*e)+1/7*(A-B)*cos(7*f*x+
7*e)+(15*A-B)*sin(2*f*x+2*e)+(1/2*B+3*A)*sin(4*f*x+4*e)+1/3*(A+B)*sin(6*f*
x+6*e)+1/16*B*sin(8*f*x+8*e)+5*(A-B)*cos(f*x+e)+20*f*x*A-5/2*f*x*B+64/7*A-
64/7*B)*a^3/f
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.76

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx$$
$$= \frac{384\,(A-B)a^3c^4\cos(fx+e)^7 + 105\,(8\,A-B)a^3c^4fx + 7\,(48\,Ba^3c^4\cos(fx+e)^7 + 8\,(8\,A-B)a^3c^4\cos($$
$$2688\,f$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7
*(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8
*A - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*
x + e))/f
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. 2(163) = 326.

Time = 0.87 (sec) , antiderivative size = 1579, normalized size of antiderivative = 8.72

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3
*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)*
*2*cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x
*cos(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*
cos(e + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*
x)/f + 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*
sin(e + f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e +
f*x)/f + 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c
**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A
*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos
(e + f*x)**5/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3
*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)
**7/(35*f) + 8*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f
*x)**3/f + A*a**3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/1
28 + 35*B*a**3*c**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*
x*sin(e + f*x)**6/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/6
4 - 45*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*
sin(e + f*x)**4/8 + 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32...
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(170) = 340$.

Time $= 0.24$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 3.15$

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^4\, dx$$
$$= \frac{3072\left(5\cos(fx+e)^7 - 21\cos(fx+e)^5 + 35\cos(fx+e)^3 - 35\cos(fx+e)\right)Aa^3c^4 + 21504\left(3\cos(fx\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output

```
1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*
a^3*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) -
 48*sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A
*a^3*c^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*
x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f
*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f
*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2
*e))*B*a^3*c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
+ 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x +
 2*e))*B*a^3*c^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*
x + e))/f
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^4\,dx \\
&= \frac{Ba^3c^4 \sin(8fx+8e)}{1024f} + \frac{5}{128}\left(8Aa^3c^4 - Ba^3c^4\right)x + \frac{(Aa^3c^4 - Ba^3c^4)\cos(7fx+7e)}{448f} \\
&\quad + \frac{(Aa^3c^4 - Ba^3c^4)\cos(5fx+5e)}{64f} + \frac{3(Aa^3c^4 - Ba^3c^4)\cos(3fx+3e)}{64f} \\
&\quad + \frac{5(Aa^3c^4 - Ba^3c^4)\cos(fx+e)}{64f} + \frac{(Aa^3c^4 + Ba^3c^4)\sin(6fx+6e)}{192f} \\
&\quad + \frac{(6Aa^3c^4 + Ba^3c^4)\sin(4fx+4e)}{128f} + \frac{(15Aa^3c^4 - Ba^3c^4)\sin(2fx+2e)}{64f}
\end{aligned}$$

input

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x +
1/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3
*c^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f
 + 5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*
c^4)*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)
/f + 1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

3.40.9 Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.65

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(6\,A\,a^3\,c^4-6\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(2\,A\,a^3\,c^4-2\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(6\,A\,a^3\,c^4-6\right.}{}$$

$$+\frac{5\,a^3\,c^4\,\mathrm{atan}\left(\frac{\frac{5\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8\,A-B)}{64}}{\left(\frac{5\,A\,a^3\,c^4}{8}-\frac{5\,B\,a^3\,c^4}{64}\right)}\right)(8\,A-B)}{64\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*
(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*
c^4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)
/2)^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3
*c^4 - 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4)
 - tan(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 +
 (f*x)/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)
^13*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113
*A*a^3*c^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*
c^4)/24 + (895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 -
 (1765*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B
*a^3*c^4)/192) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64)
+ (2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e
/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*t
an(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14
+ tan(e/2 + (f*x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)
/2)*(8*A - B))/(64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f
)
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

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3.41.1 Optimal result

Integrand size = 36, antiderivative size = 117

$$\begin{aligned}&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx\\&=\frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f}\\&\quad+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}\end{aligned}$$

output
```
5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*cos(f*x+e)^7/f+5/16*a^3*A*c^3*cos(f*x+e)*si
n(f*x+e)/f+5/24*a^3*A*c^3*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*A*c^3*cos(f*x+
e)^5*sin(f*x+e)/f
```

3.41.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\begin{aligned}&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx\\&=\frac{a^3c^3(-192B\cos^7(e+fx)+7A(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))))}{1344f}\end{aligned}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
 + 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

3.41.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))dx$$

$$\downarrow 3148$$

$$a^3c^3\left(A\int \cos^6(e+fx)dx-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^6 dx-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\downarrow 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\int \cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(A\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4 dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\downarrow 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+\right.$$

$$\downarrow 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\right.$$

$$\downarrow 24$$

$$a^3c^3\left(A\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)\right)-\frac{B\cos}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^3*c^3*(-1/7*(B*Cos[e + f*x]^7)/f + A*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f
) + (5*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[
e + f*x])/(2*f)))/4))/6))
```

3.41.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.41.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{c^3a^3(420fxA+7\sin(6fx+6e)A+63\sin(4fx+4e)A+315A\sin(2fx+2e)-105\cos(fx+e)B-3\cos(7fx+7e)B-21\cos(5fx$
risch	$\frac{5a^3Ac^3x}{16}-\frac{5Ba^3c^3\cos(fx+e)}{64f}-\frac{Ba^3c^3\cos(7fx+7e)}{448f}+\frac{Aa^3c^3\sin(6fx+6e)}{192f}-\frac{Ba^3c^3\cos(5fx+5e)}{64f}+\frac{3Aa^3c$
derivativedivides	$\frac{Aa^3c^3(fx+e)-Aa^3c^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+3Aa^3c^3\left(-\frac{\left(\sin^3(fx+e)+\right.\right.}{}$
default	$\frac{Aa^3c^3(fx+e)-Aa^3c^3\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+3Aa^3c^3\left(-\frac{\left(\sin^3(fx+e)+\right.\right.}{}$
parts	$a^3Ac^3x-\frac{Ba^3c^3\cos(fx+e)}{f}-\frac{3Aa^3c^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{3Aa^3c^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right.\right.}{4}}{f}$
norman	$\frac{-\frac{2Ba^3c^3}{7f}-\frac{6Ba^3c^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{10Ba^3c^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2Ba^3c^3\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{5a^3Ac^3x}{16}+\frac{11Aa^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{8f}}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/1344*c^3*a^3*(420*f*x*A+7*sin(6*f*x+6*e)*A+63*sin(4*f*x+4*e)*A+315*A*sin
(2*f*x+2*e)-105*cos(f*x+e)*B-3*cos(7*f*x+7*e)*B-21*cos(5*f*x+5*e)*B-63*cos
(3*f*x+3*e)*B-192*B)/f
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx =$$
$$-\frac{48\,Ba^3c^3\cos(fx+e)^7-105\,Aa^3c^3fx-7\left(8\,Aa^3c^3\cos(fx+e)^5+10\,Aa^3c^3\cos(fx+e)^3+15\,Aa^3c^3\right.}{336\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*c
os(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*s
in(f*x + e))/f
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(116) = 232$.

Time = 0.54 (sec) , antiderivative size = 682, normalized size of antiderivative = 5.83

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx$$
$$= \begin{cases} -\frac{5Aa^3c^3x\sin^6(e+fx)}{16} - \frac{15Aa^3c^3x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9Aa^3c^3x\sin^4(e+fx)}{8} - \frac{15Aa^3c^3x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9Aa^3c^3x}{} \\ x(A + B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c)^3 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3
*c**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)*
*2*cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x
*cos(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*
cos(e + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e +
 f*x)/(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*
cos(e + f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f)
+ 3*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)
**6*cos(e + f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B
*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*
cos(e + f*x)**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f +
3*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)
**7/(35*f) - 8*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f
*x)**3/f - B*a**3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**3*(-c*sin(e) + c)**3, True))
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(107) = 214$.

Time $= 0.22$ (sec) , antiderivative size $= 264$, normalized size of antiderivative $= 2.26$

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx =$$

$$-\frac{35\left(4\sin(2fx+2e)^3 + 60fx + 60e + 9\sin(4fx+4e) - 48\sin(2fx+2e)\right)Aa^3c^3 - 630(12fx+1}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 4
8*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
c^3 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5
 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5
 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 -
 3*cos(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

3.41.8 Giac [A] (verification not implemented)

Time $= 0.32$ (sec) , antiderivative size $= 155$, normalized size of antiderivative $= 1.32$

$$\begin{aligned}
&\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx \\
&= \frac{5}{16}Aa^3c^3x - \frac{Ba^3c^3\cos(7fx+7e)}{448f} - \frac{Ba^3c^3\cos(5fx+5e)}{64f} \\
&\quad - \frac{3Ba^3c^3\cos(3fx+3e)}{64f} - \frac{5Ba^3c^3\cos(fx+e)}{64f} + \frac{Aa^3c^3\sin(6fx+6e)}{192f} \\
&\quad + \frac{3Aa^3c^3\sin(4fx+4e)}{64f} + \frac{15Aa^3c^3\sin(2fx+2e)}{64f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos
(5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f
*x + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x
+ 4*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

3.41.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{5\,A\,a^3\,c^3\,x}{16}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\left(\frac{a^3\,c^3\,(672\,B-735\,A\,(e+f\,x))}{336}+\frac{35\,A\,a^3\,c^3\,(e+f\,x)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\left(\frac{a^3\,c^3\,(2016\,B-2205\,A\,(e+f\,x))}{336}+\frac{105}{}\right.}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

output
```
(5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e +
f*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3
*(2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/
2 + (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3
*(e + f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e
 + f*x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2
 + (f*x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*
tan(e/2 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*
a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

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3.42.1 Optimal result

Integrand size = 36, antiderivative size = 138

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{1}{16}a^3(6A+B)c^2x - \frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f} + \frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f}$$
$$+ \frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{Bc^2\cos^5(e+fx)\,(a^3+a^3\sin(e+fx))}{6f}$$

output
```
1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B
)*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)
/f-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f
```

3.42.2 Mathematica [A] (verified)

Time = 6.89 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^3c^2(360Ae+60Be+360Afx+60Bfx-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))-12Ac$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
(a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*
x] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e +
 f*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e +
f*x)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

3.42.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \cos^4(e+fx)(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\int \cos^4(e+fx)(\sin(e+fx)a+a)dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\int \cos(e+fx)^4(\sin(e+fx)a+a)dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\downarrow 3148$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\int\cos^4(e+fx)dx-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\int\sin\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\si}{6f}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+f}{}\right.$$

$$\downarrow 3115$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\right.$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/6*(B*Cos[e + f*x]^5*(a + a*Sin[e + f*x]))/f + ((6*A + B)*(-1/5
*(a*Cos[e + f*x]^5)/f + a*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 +
 (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)))/6)
```

3.42.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.42.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{\left(5(A+B)\cos(3fx+3e)+(A+B)\cos(5fx+5e)+5\left(-4A-\frac{B}{4}\right)\sin(2fx+2e)+\frac{5\left(-A+\frac{B}{2}\right)\sin(4fx+4e)}{2}+\frac{5B\sin(6fx+6e)}{12}+\right.}{80f}$
risch	$\frac{3a^3c^2xA}{8}+\frac{a^3c^2xB}{16}-\frac{c^2a^3\cos(fx+e)A}{8f}-\frac{c^2a^3\cos(fx+e)B}{8f}-\frac{Ba^3c^2\sin(6fx+6e)}{192f}-\frac{c^2a^3\cos(5fx+5e)A}{80f}-$
parts	$-\frac{(-2Aa^3c^2-2Ba^3c^2)(2+\sin^2(fx+e))\cos(fx+e)}{3f}+\frac{(-2Aa^3c^2+Ba^3c^2)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(Aa^3$
derivativedivides	$-\frac{Aa^3c^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+Aa^3c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Aa^3}{}$
default	$-\frac{Aa^3c^2\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+Aa^3c^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Aa^3}{}$
norman	$\frac{(\frac{3}{8}Aa^3c^2+\frac{1}{16}Ba^3c^2)x+(\frac{3}{8}Aa^3c^2+\frac{1}{16}Ba^3c^2)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}Aa^3c^2+\frac{3}{8}Ba^3c^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{4}Aa^3c^2-}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
-1/80*(5*(A+B)*cos(3*f*x+3*e)+(A+B)*cos(5*f*x+5*e)+5*(-4*A-1/4*B)*sin(2*f*
x+2*e)+5/2*(-A+1/2*B)*sin(4*f*x+4*e)+5/12*B*sin(6*f*x+6*e)+10*(A+B)*cos(f*
x+e)-30*f*x*A-5*f*x*B+16*A+16*B)*c^2*a^3/f
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx =$$
$$-\frac{48\,(A+B)a^3c^2\cos(fx+e)^5-15\,(6\,A+B)a^3c^2fx+5\,(8\,Ba^3c^2\cos(fx+e)^5-2\,(6\,A+B)a^3c^2\cos(}{240\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(
8*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A +
 B)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(128) = 256$.

Time $= 0.45$ (sec) , antiderivative size $= 910$, normalized size of antiderivative $= 6.59$

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= \begin{cases} \frac{3Aa^3c^2x\sin^4(e+fx)}{8} + \frac{3Aa^3c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^3c^2x\sin^2(e+fx) + \frac{3Aa^3c^2x\cos^4(e+fx)}{8} - Aa^3c^2x\cos^2(e \\ x(A + B\sin(e))(a\sin(e) + a)^3(-c\sin(e) + c)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*c
os(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*
c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e
+ f*x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a
**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2
*cos(e + f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**
2*cos(e + f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*s
in(e + f*x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15
*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e
+ f*x)**2*cos(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c
**2*x*cos(e + f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2
*x*cos(e + f*x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f)
- B*a**3*c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*
*3*cos(e + f*x)**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f
) - 4*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*si
n(e + f*x)**2*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*si
n(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + ...
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(128) = 256$.

Time $= 0.22$ (sec) , antiderivative size $= 360$, normalized size of antiderivative $= 2.61$

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2\, dx =$$

$$-\frac{64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aa^3c^2 + 640\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*
c^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5
 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 -
3*cos(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*si
n(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*
x + 2*e))*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x +
 e))/f
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= -\frac{Ba^3c^2\sin(6fx+6e)}{192f} + \frac{1}{16}\left(6Aa^3c^2 + Ba^3c^2\right)x - \frac{(Aa^3c^2 + Ba^3c^2)\cos(5fx+5e)}{80f}$$
$$- \frac{(Aa^3c^2 + Ba^3c^2)\cos(3fx+3e)}{16f} - \frac{(Aa^3c^2 + Ba^3c^2)\cos(fx+e)}{8f}$$
$$+ \frac{(2Aa^3c^2 - Ba^3c^2)\sin(4fx+4e)}{64f} + \frac{(16Aa^3c^2 + Ba^3c^2)\sin(2fx+2e)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1
/80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c
^2)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64
*(2*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3
*c^2)*sin(2*f*x + 2*e)/f
```

3.42.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.88

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$

$$= \frac{a^3c^2\,\mathrm{atan}\left(\frac{a^3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(6A+B)}{8\left(\frac{3Aa^3c^2}{4}+\frac{Ba^3c^2}{8}\right)}\right)(6A+B)}{8f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(4Aa^3c^2+4Ba^3c^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(2Aa^3c^2+2Ba^3c^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4Aa^3c^2+}{}$$

$$-\frac{a^3c^2(6A+B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{8f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

output
```
(a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 +
 (B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 +
4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2
 + (f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*
c^2 + 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)
/5) - tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 +
(f*x)/2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*
A*a^3*c^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (4
7*B*a^3*c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/
24) - tan(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)
/5 + (2*B*a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4
 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2
)^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f
*x)/2)) - (f*x)/2))/(8*f)
```

3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

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3.43.1 Optimal result

Integrand size = 34, antiderivative size = 140

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx \\
&= \frac{1}{8}a^3(5A+2B)cx - \frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} \\
&\quad + \frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f} - \frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} \\
&\quad - \frac{(5A+2B)c\cos^3(e+fx)\,(a^3+a^3\sin(e+fx))}{20f}
\end{aligned}$$

output

```
1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B
)*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/
20*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

3.43.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{a^3c\cos(e+fx)\left(-30(5A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(-8(10A+7B)+15(3A-2B)\,s}{120f\sqrt{\cos^2(e+f}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
(a^3*c*Cos[e + f*x]*(-30*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(-8*(10*A + 7*B) + 15*(3*A - 2*B)*Sin[e + f*x] +
16*(5*A + 2*B)*Sin[e + f*x]^2 + 30*(A + 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e +
 f*x]^4)))/(120*f*Sqrt[Cos[e + f*x]^2])
```

3.43.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)^2(A+B\sin(e+fx))dx$$
$$\downarrow 3042$$
$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)^2(A+B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$ac\left(\frac{1}{5}(5A+2B)\int \cos^2(e+fx)(\sin(e+fx)a+a)^2dx - \frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^2}{5f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A+2B)\int \cos(e+fx)^2(\sin(e+fx)a+a)^2dx - \frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^2}{5f}\right)$$

$$\downarrow 3157$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\int \cos^2(e+fx)(\sin(e+fx)a+a)dx - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right) - \frac{B\cos^3(e+f}{}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\int \cos(e+fx)^2(\sin(e+fx)a+a)dx - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right) - \frac{B\cos^3(e+f}{}\right.$$

$$\downarrow 3148$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\int \cos^2(e+fx)dx - \frac{a\cos^3(e+fx)}{3f}\right) - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right) - \frac{B\cos^3(e}{}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx - \frac{a\cos^3(e+fx)}{3f}\right) - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right) - \frac{B}{}\right.$$

$$\downarrow 3115$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\left(\frac{\int 1dx}{2} + \frac{\sin(e+fx)\cos(e+fx)}{2f}\right) - \frac{a\cos^3(e+fx)}{3f}\right) - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+\right.}{4f}\right.\right.$$

$$\downarrow 24$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\left(\frac{\sin(e+fx)\cos(e+fx)}{2f} + \frac{x}{2}\right) - \frac{a\cos^3(e+fx)}{3f}\right) - \frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right.}{4f}\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/5*(B*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^2)/f + ((5*A + 2*B)*(-1/4
*(Cos[e + f*x]^3*(a^2 + a^2*Sin[e + f*x]))/f + (5*a*(-1/3*(a*Cos[e + f*x]^
3)/f + a*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4))/5)
```

3.43.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.43.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{\left(\left(-\frac{2A}{3}-\frac{5B}{12}\right)\cos(3fx+3e)+\left(-\frac{A}{8}-\frac{B}{4}\right)\sin(4fx+4e)+\frac{\cos(5fx+5e)B}{20}+A\sin(2fx+2e)+\left(-2A-\frac{3B}{2}\right)\cos(fx+e)+\frac{5fxA}{2}\right.}{4f}$
risch	$\frac{5a^3cxA}{8}+\frac{a^3cxB}{4}-\frac{a^3c\cos(fx+e)A}{2f}-\frac{3a^3c\cos(fx+e)B}{8f}+\frac{Ba^3c\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)Aa^3c}{32f}-\frac{\sin(4}{}$
parts	$\frac{(-Aa^3c-2Ba^3c)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(2Aa^3c+Ba^3c)\cos(fx+e)}{f}+a^3cxA+$
derivativedivides	$-Aa^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Aa^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+\frac{Ba^3c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(}{5}\right.}{}$
default	$-Aa^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Aa^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+\frac{Ba^3c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(}{5}\right.}{}$
norman	$\frac{(\frac{5}{8}Aa^3c+\frac{1}{4}Ba^3c)x+(\frac{5}{8}Aa^3c+\frac{1}{4}Ba^3c)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}Aa^3c+\frac{5}{2}Ba^3c)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{25}{4}Aa^3c+\frac{5}{2}Ba^3c$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
1/4*((-2/3*A-5/12*B)*cos(3*f*x+3*e)+(-1/8*A-1/4*B)*sin(4*f*x+4*e)+1/20*cos
(5*f*x+5*e)*B+A*sin(2*f*x+2*e)+(-2*A-3/2*B)*cos(f*x+e)+5/2*f*x*A+f*x*B-8/3
*A-28/15*B)*c*a^3/f
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \frac{24\,Ba^3c\cos(fx+e)^5 - 80\,(A+B)a^3c\cos(fx+e)^3 + 15\,(5\,A+2\,B)a^3cfx - 15\,\left(2\,(A+2\,B)a^3c\cos(fx\right.}{120\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5
*A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a
^3*c*cos(f*x + e))*sin(f*x + e))/f
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(128) = 256.

Time = 0.30 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.47

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{3Aa^3cx\sin^4(e+fx)}{8} - \frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aa^3cx\cos^4(e+fx)}{8} + Aa^3cx + \frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{2Aa^3}{} \\ x(A + B\sin(e))\,(a\sin(e) + a)^3\,(-c\sin(e) + c) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3
*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e +
 f*x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e
 + f*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**
4/4 - 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e +
f*x)**2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*
a**3*c*sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*
c*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/
f + 8*B*a**3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0))
, (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))\,dx =$$

$$-\frac{320\left(\cos\left(fx+e\right)^3 - 3\cos\left(fx+e\right)\right)Aa^3c + 15\left(12\,fx + 12\,e + \sin\left(4\,fx+4\,e\right) - 8\sin\left(2\,fx+2\,e\right)\right)Aa^3}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c -
 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*
(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f
*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^
3*c*cos(f*x + e))/f
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))\,dx \\
&= \frac{Ba^3 c\cos(5fx+5e)}{80f} + \frac{Aa^3 c\sin(2fx+2e)}{4f} \\
&\quad + \frac{1}{8}(5Aa^3 c + 2Ba^3 c)x - \frac{(8Aa^3 c + 5Ba^3 c)\cos(3fx+3e)}{48f} \\
&\quad - \frac{(4Aa^3 c + 3Ba^3 c)\cos(fx+e)}{8f} - \frac{(Aa^3 c + 2Ba^3 c)\sin(4fx+4e)}{32f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(4*f*x + 4*e)/f
```

3.43.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.79

$$\begin{aligned}
&\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))\,dx \\
&= \frac{a^3 c\,\mathrm{atan}\left(\frac{a^3 c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(5A+2B)}{4\left(\frac{5Aa^3 c}{4}+\frac{Ba^3 c}{2}\right)}\right)(5A+2B)}{4f} \\
&\quad - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8 (4Aa^3 c + 2Ba^3 c) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aa^3 c}{4} - \frac{Ba^3 c}{2}\right) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3 \left(\frac{7Aa^3 c}{2} + 3Ba^3 c\right)}{f\left(} \\
&\quad - \frac{a^3 c(5A+2B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right) - \frac{fx}{2}\right)}{4f}
\end{aligned}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

output

```
(a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*
a^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^
3*c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/
2)^3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3
*B*a^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 +
 (f*x)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3
+ (8*B*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) +
 (4*A*a^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2
+ (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2
+ (f*x)/2)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2))/(4*f)
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

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3.44.1 Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= -\frac{5a^3(3A+4B)x}{2c} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf}$$
$$+ \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c^3\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}$$

output

```
-5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B
)*cos(f*x+e)*sin(f*x+e)/c/f+a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
4+2*a^3*(3*A+4*B)*c^3*cos(f*x+e)^5/f/(c^2-c^2*sin(f*x+e))^2
```

3.44.2 Mathematica [A] (verified)

Time = 11.64 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(16$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3
*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 +
5*e + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B
*Cos[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

3.44.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^3}dx}{c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\int\frac{\cos^4(e+fx)}{c-c\sin(e+fx)}dx}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\int\frac{\cos(e+fx)^4}{c-c\sin(e+fx)}dx}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\downarrow 3115$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) - ((3*A + 4*B
)*((-2*Cos[e + f*x]^5)/(c*f*(c - c*Sin[e + f*x])^2) + (5*(-1/3*Cos[e + f*x
]^3/(c*f) + (x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))/c))/c^2))/c)
```

3.44.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.44.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{65a^3\left(\frac{4\left(4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A+4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA-4fxB+\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A+\frac{68B}{65}\right.\right.}{8cf\cos(fx+e)}$
derivativedivides	$\frac{2a^3\left(-\frac{\left(2B+\frac{A}{2}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-2B-\frac{A}{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
default	$\frac{2a^3\left(-\frac{\left(2B+\frac{A}{2}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-2B-\frac{A}{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{fc}$
risch	$-\frac{15a^3xA}{2c}-\frac{10a^3xB}{c}+\frac{2a^3e^{i(fx+e)}A}{cf}+\frac{31a^3e^{i(fx+e)}B}{8cf}+\frac{2a^3e^{-i(fx+e)}A}{cf}+\frac{31a^3e^{-i(fx+e)}B}{8cf}+\frac{16a^3A}{fc\left(e^{i(fx+e)}\right.}$
norman	$\frac{-\frac{17Aa^3+20Ba^3}{cf}+\frac{5a^3(3A+4B)x}{2c}-\frac{\left(5Aa^3+2Ba^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(17Aa^3+18Ba^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(21Aa^3+34Ba^3\right)\left(\ta\right.}{3cf}}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
65/8*a^3*(4/65*(4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A+4*B)*sin(3*f*x+3*e)-1/1
95*B*cos(4*f*x+4*e)+4/13*(-3*f*x*A-4*f*x*B+24/5*A+94/15*B)*cos(f*x+e)+(A+6
8/65*B)*sin(f*x+e)+16/13*A+19/13*B)/c/f/cos(f*x+e)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\,Ba^3\cos(fx+e)^4-(3\,A+10\,B)a^3\cos(fx+e)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos(fx+e$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
 4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3
*A + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e
)^3 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 1
2*B)*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) -
c*f*sin(f*x + e) + c*f)
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. $2(144) = 288$.

Time = 3.79 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.28

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 +
 f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c
*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*ta
n(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5
 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 +
 f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*
A*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2
+ f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18
*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*
x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(
e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*
c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x
/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*
tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*t...
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(152) = 304$.

Time = 0.35 (sec) , antiderivative size = 1139, normalized size of antiderivative = 7.30

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c)
+ 18*A*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1
) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((
sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*ar...
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{\frac{15\,(3\,Aa^3+4\,Ba^3)(fx+e)}{c}+\frac{96\,(Aa^3+Ba^3)}{c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)}+\frac{2\left(3\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+12\,Ba^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-24\,Aa^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-42\,Ba\right.}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*
f*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*
f*x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x +
1/2*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)
^2 - 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a
^3 - 46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

3.44.9 Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{24\,A\,a^3 - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(7\,A\,a^3+\frac{34\,B\,a^3}{3}\right)+\frac{94\,B\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(9\,A\,a^3+18\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(17}{f\left(-c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-3\,c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\right.}$$
$$-\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3\,A+4\,B)}{15\,A\,a^3+20\,B\,a^3}\right)(3\,A+4\,B)}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

output
```
(24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - t
an(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3
+ 20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)
/2)^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/
(f*(c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f
*x)/2)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2
 + (f*x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f
*x)/2)*(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

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3.45.1 Optimal result

Integrand size = 36, antiderivative size = 163

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{5a^3(2A+5B)x}{2c^2} - \frac{5a^3(2A+5B)\cos(e+fx)}{2c^2 f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} \\
&\quad - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}$$

output
```
5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c
^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*cos(f*x+e)^3/f/(c^2-c^2*sin(f*x+e))
```

3.45.2 Mathematica [A] (verified)

Time = 11.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.72

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(32(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}
\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

3.45.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}dx}{3c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^4}dx}{3c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{3c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{3c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - ((2*A + 5
*B)*((2*Cos[e + f*x]^5)/(c*f*(c - c*Sin[e + f*x])^3) - (5*((3*(x/c - Cos[e
 + f*x]/(c*f)))/(2*c) - Cos[e + f*x]^3/(2*f*(c^2 - c^2*Sin[e + f*x])))/c^
2))/(3*c))
```

3.45.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.45.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc^2}$
default	$\frac{2a^3\left(\frac{\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(-A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A+5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}\right.}{fc^2}$
parallelrisch	$-\frac{\left((30fxA+75fxB-56A-\frac{607}{4}B)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+(-10fxA-25fxB-\frac{19}{3}A-\frac{115}{12}B)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+(-30fxA-75fxB\right.}{2fc^2\left(-3\right.}$
risch	$\frac{5a^3xA}{c^2}+\frac{25a^3xB}{2c^2}+\frac{iBa^3e^{2i(fx+e)}}{8c^2f}-\frac{a^3e^{i(fx+e)}A}{2c^2f}-\frac{5a^3e^{i(fx+e)}B}{2c^2f}-\frac{a^3e^{-i(fx+e)}A}{2c^2f}-\frac{5a^3e^{-i(fx+e)}B}{2c^2f}-$
norman	$\frac{\left(8Aa^3+25Ba^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{46Aa^3+118Ba^3}{3cf}-\frac{5a^3(2A+5B)x}{2c}-\frac{\left(34Aa^3+77Ba^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{\left(38Aa^3+93Ba^3\right)\text{ta}}{cf}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^2*((1/2*B*tan(1/2*f*x+1/2*e)^3+(-A-5*B)*tan(1/2*f*x+1/2*e)^2-1/2
*B*tan(1/2*f*x+1/2*e)-A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A+5*B)*arct
an(tan(1/2*f*x+1/2*e))-(-4*A-12*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(16*A+16*B)/
(tan(1/2*f*x+1/2*e)-1)^3-1/2*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{3\,Ba^3\cos(fx+e)^4 - 6\,(A+4\,B)a^3\cos(fx+e)^3 - 30\,(2\,A+5\,B)a^3fx - 16\,(A+B)a^3 + (15\,(2\,A+5\,B$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5
*B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^
3)*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x
 + e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a
^3*cos(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 7
9*B)*a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*
x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(151) = 302$.

Time = 7.40 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.62

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7
 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2
*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2
 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(
e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)
**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c
**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(
e/2 + f*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*ta
n(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 2
10*A*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*
f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2
+ f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**
2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/
2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c
**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(
e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/
2) - 6*c**2*f) - 150*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5...
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. 2(156) = 312.

Time = 0.34 (sec) , antiderivative size = 1386, normalized size of antiderivative = 8.50

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) -
 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 - c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) -
11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)...
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{15\left(2Aa^3+5Ba^3\right)(fx+e)}{c^2} + \frac{6\left(Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 2Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 10Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 2Aa^3 - 10Ba^3\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2 c^2}}{6f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^
3 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a
^3*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*
e)^2 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A
*a^3 + 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.45.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{5a^3\operatorname{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2A+5B)}{10Aa^3+25Ba^3}\right)(2A+5B)}{c^2 f}$$
$$- \frac{\frac{46Aa^3}{3} - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)(38Aa^3+93Ba^3) + \frac{118Ba^3}{3} + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(8Aa^3+25Ba^3) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{f\left(-c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7 + 3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 - 5c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x
)
```

output
```
(5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(2*A + 5*B))/(10*A*a^3 + 25*B*a^3))*
(2*A + 5*B))/(c^2*f) - ((46*A*a^3)/3 - tan(e/2 + (f*x)/2)*(38*A*a^3 + 93*B
*a^3) + (118*B*a^3)/3 + tan(e/2 + (f*x)/2)^6*(8*A*a^3 + 25*B*a^3) - tan(e/
2 + (f*x)/2)^5*(34*A*a^3 + 77*B*a^3) - tan(e/2 + (f*x)/2)^3*(72*A*a^3 + 16
6*B*a^3) + tan(e/2 + (f*x)/2)^4*((106*A*a^3)/3 + (328*B*a^3)/3) + tan(e/2
+ (f*x)/2)^2*((128*A*a^3)/3 + (359*B*a^3)/3))/(f*(5*c^2*tan(e/2 + (f*x)/2)
^2 - 7*c^2*tan(e/2 + (f*x)/2)^3 + 7*c^2*tan(e/2 + (f*x)/2)^4 - 5*c^2*tan(e
/2 + (f*x)/2)^5 + 3*c^2*tan(e/2 + (f*x)/2)^6 - c^2*tan(e/2 + (f*x)/2)^7 +
c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

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3.46.1 Optimal result

Integrand size = 36, antiderivative size = 153

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx\\ &= -\frac{a^3(A+6B)x}{c^3}+\frac{a^3(A+6B)\cos(e+fx)}{c^3 f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}\\ &\quad -\frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+6B)c^3\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}\end{aligned}$$

output

```
-a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e
))^4+2/3*a^3*(A+6*B)*c^3*cos(f*x+e)^3/f/(c^3-c^3*sin(f*x+e))^2
```

3.46.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. $2(153) = 306$.

Time = 11.41 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.07

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(11A+21B)\right.}\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B
*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e
 + f*x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e
 + f*x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(
e + f*x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*(c - c*Sin[e + f*x])^3)
```

3.46.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}dx}{5c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^5}dx}{5c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}dx}{3c^2}\right)}{5c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^3}dx}{3c^2}\right)}{5c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - ((A + 6*B
)*((2*Cos[e + f*x]^5)/(3*c*f*(c - c*Sin[e + f*x])^4) - (5*((-3*(x/c - Cos[
e + f*x]/(c*f)))/c^2 + (2*Cos[e + f*x]^3)/(c*f*(c - c*Sin[e + f*x])^2)))/(
3*c^2)))/(5*c))
```

3.46.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.46.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{3(\text{ta}}\right.}{f\,c^3}$
default	$\frac{2a^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A+6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{}{3(\text{ta}}\right.}{f\,c^3}$
risch	$-\frac{a^3xA}{c^3}-\frac{6a^3xB}{c^3}+\frac{B\,a^3\mathrm{e}^{i(fx+e)}}{2c^3f}+\frac{B\,a^3\mathrm{e}^{-i(fx+e)}}{2c^3f}+\frac{-\frac{112A\,a^3\mathrm{e}^{2i(fx+e)}}{3}-24iA\,a^3\mathrm{e}^{3i(fx+e)}+\frac{56iA\,a^3\mathrm{e}^{i(fx+e)}}{3}+}{}$
parallelrisch	$-\frac{\left(\left(\left(\frac{233}{2}-60fx\right)B-10fxA+24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-\frac{33}{2}+30fx\right)B+5fxA-\frac{16A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+(fxA+6fxB-\frac{243}{10}B\right.}{f\,c^3\left(-10\,\text{c}\right.}$
norman	$-\frac{5a^3(A+6B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{30a^3(A+6B)x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+\frac{5a^3(A+6B)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{14a^3(A+6B)x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}+$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A+6*B)*arctan(tan(1/2*f*x+1/2*e))
-(2*A+6*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(8*A-8*B)/(tan(1/2*f*x+1/2*e)-1)^2-1
/5*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(40*A+24*B)/(tan(1/2*f*x+1/2*e
)-1)^3-1/4*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(150) = 300$.

Time $= 0.26$ (sec) , antiderivative size $= 337$, normalized size of antiderivative $= 2.20$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{15\,Ba^3\cos(fx+e)^4+60\,(A+6\,B)a^3fx-24\,(A+B)a^3-(15\,(A+6\,B)a^3fx-(46\,A+231\,B)a^3)\cos(}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (1
5*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a
^3*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*
A + 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*
f*x + 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f
*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin
(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)
*sin(f*x + e))
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.

Time = 13.60 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*
c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f
*x*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)
**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5
/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3
*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e
/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/
2) - 15*c**3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 +
f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5
 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c
**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*
A*a**3*f*x*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*
tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2
+ f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)
**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6...
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(150) = 300.

Time = 0.35 (sec) , antiderivative size = 1685, normalized size of antiderivative = 11.01

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 11*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x +
e) + 1) - 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c
^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f
*x + e) + 1) - 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3
- 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + ...
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{30\,Ba^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^3} - \frac{15\,(Aa^3+6\,Ba^3)(fx+e)}{c^3} - \frac{4\left(15\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Ba\right.}{15}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(
f*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x +
 1/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*
e)^3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 50*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a
^3 + 63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.46.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.20

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{\frac{52Aa^3}{15} - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{40Aa^3}{3}+82Ba^3\right) + \frac{94Ba^3}{5} + \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4Aa^3+12Ba^3) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(8A}{f\left(-c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7 + 5c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 - 11c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5 + 1}$$
$$- \frac{2a^3\operatorname{atan}\left(\frac{2a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A+6B)}{2Aa^3+12Ba^3}\right)(A+6B)}{c^3 f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x
)
```

output
```
((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)
/5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A
*a^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e
/2 + (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*
a^3)/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2
 + (f*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5
+ 5*c^3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(
e/2 + (f*x)/2))) - (2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a
^3 + 12*B*a^3))*(A + 6*B))/(c^3*f)
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

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3.47.1 Optimal result

Integrand size = 36, antiderivative size = 151

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx\\ &=\frac{a^3Bx}{c^4}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\\ &\quad+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}\end{aligned}$$

output
```
a^3*B*x/c^4+1/7*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7-2/5*a^3*B*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5+2/3*a^3*B*c^2*cos(f*x+e)^3/f/(c^2-c^2*
sin(f*x+e))^3-2*a^3*B*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 356 vs. $2(151) = 302$.

Time = 11.47 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.36

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx\\ &=\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(120(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12(15A+29\right.}{}\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^4,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e
 + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x
)/2] - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f
*x)/2] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
 f*x)/2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e
 + f*x)/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*(c - c*Sin[e + f*x])^4)
```

3.47.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}dx}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^6}dx}{c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}dx}{c^2}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^4}dx}{c^2}\right)}{c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}}{c^2}\right)}{c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^2}dx}{c^2}}{c^2}\right)}{c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{\int 1dx}{c^2}}{c^2}}{c^2}\right)}{c}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{x}{c^2}}{c^2}}{c^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7) - (B*((2*Co
s[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^5) - ((2*Cos[e + f*x]^3)/(3*c*f*
(c - c*Sin[e + f*x])^3) - (-(x/c^2) + (2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e
 + f*x])))/c^2)/c^2))/c)
```

3.47.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.47.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4\left(\tan\left(\right.\right.\right.}}{f\,c^4}$
default	$\frac{2a^3\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160}{4\left(\tan\left(\right.\right.\right.}}{f\,c^4}$
parallelrisch	$-\frac{2\left(-\frac{B\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{7}{2}fxB+A-B\right)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(-\frac{21fx}{2}+8\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35}{2}fxB+5A-\frac{55}{3}B\right)\left(t\right.\right.}{}$
risch	$\frac{a^3Bx}{c^4}-\frac{2\left(-337B\,a^3-15A\,a^3-2520iB\,a^3e^{5i(fx+e)}+6160iB\,a^3e^{3i(fx+e)}-1624iB\,a^3e^{i(fx+e)}+105A\,a^3e^{6i(fx+e)}+735\right.}{105\left(e^{i(fx+e)}-i\right)^7}$
norman	$-\frac{2336B\,a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{6544B\,a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{a^3xB}{c}-\frac{304B\,a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cf}+\frac{7a^3xB\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}-\frac{\left(870A\,a^3-3142B\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^4*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)-1)-1/2
*(12*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(60*A+20*B)/(tan(1/2*f*x+1/2*e)-1
)^3-1/7*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/4*(160*A+96*B)/(tan(1/2*f*x
+1/2*e)-1)^4-1/6*(192*A+192*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(240*A+208*B)/
(tan(1/2*f*x+1/2*e)-1)^5)
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(149) = 298$.

Time $= 0.27$ (sec) , antiderivative size $= 363$, normalized size of antiderivative $= 2.40$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{840\,Ba^3fx+(105\,Ba^3fx+(15\,A+337\,B)a^3)\cos(fx+e)^4+120\,(A+B)a^3-(315\,Ba^3fx+(45\,A-6}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
 + 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 -
 24*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (
A - 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^
3*f*x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*
B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*s
in(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(
f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^
4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(141) = 282$.

Time = 23.73 (sec) , antiderivative size = 2951, normalized size of antiderivative = 19.54

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*a**3*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7
 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675
*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*
f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 1050*A
*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan
(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 +
 f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2
)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 630*A*a**3*tan(e/2 + f*
x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*
c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*
tan(e/2 + f*x/2) - 105*c**4*f) - 30*A*a**3/(105*c**4*f*tan(e/2 + f*x/2)**7
 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675
*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*
f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 105*B*
a**3*f*x*tan(e/2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/
2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 735*B*a**3*f*x*tan(e
/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f...
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. 2(149) = 298.

Time = 0.39 (sec) , antiderivative size = 2118, normalized size of antiderivative = 14.03

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(c
os(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x
+ e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e
) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x +
e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/
(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^...
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{\frac{105\,(fx+e)Ba^3}{c^4} - \frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 + 840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 1925\,Ba^3\tan\right.}{c^4}}{105}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105
*B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a
^3*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3
*tan(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*ta
n(1/2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*
a^3)/(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.47.9 Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{B\,a^3\,x}{c^4}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(\frac{a^3\,(1680\,B-2205\,B\,(e+f\,x))}{105}+21\,B\,a^3\,(e+f\,x)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^3\,(7840\,B-3675\,B\,(e+f\,x))}{105}+35\right.$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x
)
```

output
```
(B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/1
05 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e
+ f*x)))/105 + 35*B*a^3*(e + f*x)) + (a^3*(30*A - 334*B + 105*B*(e + f*x))
)/105 + tan(e/2 + (f*x)/2)^6*((a^3*(210*A - 210*B + 735*B*(e + f*x)))/105
- 7*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^2*((a^3*(630*A - 5334*B + 2205*B
*(e + f*x)))/105 - 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^4*((a^3*(1050*
A - 3850*B + 3675*B*(e + f*x)))/105 - 35*B*a^3*(e + f*x)) + tan(e/2 + (f*x
)/2)*((a^3*(2128*B - 735*B*(e + f*x)))/105 + 7*B*a^3*(e + f*x)) - B*a^3*(e
 + f*x))/(c^4*f*(tan(e/2 + (f*x)/2) - 1)^7)
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

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3.48.1 Optimal result

Integrand size = 36, antiderivative size = 77

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8} + \frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

output
```
1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*c
os(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

3.48.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(77) = 154.

Time = 12.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.68

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B)\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^5,x]
```

output

```
-1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(31
5*(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(
5*(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9
*B*Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2]
+ 105*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e
 + f*x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*S
in[(9*(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)
```

3.48.3 Rubi [A] (verified)

Time $= 0.51$ (sec) , antiderivative size $= 75$, normalized size of antiderivative $= 0.97$, number of steps used $= 6$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used $= \{3042, 3446, 3042, 3338, 3042, 3150\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A-8B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A-8B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

$$\downarrow 3150$$

$$a^3c^3\left(\frac{(A-8B)\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + ((A - 8*B
)*Cos[e + f*x]^7)/(63*c*f*(c - c*Sin[e + f*x])^7))
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time $= 1.16$ (sec) , antiderivative size $= 173$, normalized size of antiderivative $= 2.25$

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-A+B)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(23A+5B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+5(-A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A+3B}{fc^5\left(\tan\left(\frac{fx}{2}+\right.\right.}$
derivativedivides	$\frac{2a^3\left(-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{992A+8}{6\left(\tan\left(\frac{fx}{2}+\right.\right.}{fc^5}$
default	$\frac{2a^3\left(-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{14A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{992A+8}{6\left(\tan\left(\frac{fx}{2}+\right.\right.}{fc^5}$
risch	$\frac{16Ba^3}{63}-\frac{2Aa^3}{63}-10iAa^3e^{5i(fx+e)}+\frac{2iBa^3e^{i(fx+e)}}{7}-2iBa^3e^{7i(fx+e)}+2iAa^3e^{7i(fx+e)}-\frac{2iAa^3e^{i(fx+e)}}{7}+10iBa^3e^{5i(fx-}$
norman	$-\frac{16Aa^3-2Ba^3}{63cf}-\frac{2Aa^3\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{2\left(Aa^3-Ba^3\right)\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{\left(2Aa^3-2Ba^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7cf}+\frac{6\left(3Aa^3-3Ba^3\right)\left(t}{cf}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^8+(-A+B)*tan(1/2*f*x+1/2*e)^7+1/3*(23*A+5*B)*tan(
1/2*f*x+1/2*e)^6+5*(-A+B)*tan(1/2*f*x+1/2*e)^5+(11*A+3*B)*tan(1/2*f*x+1/2*
e)^4+3*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/7*(25*A+3*B)*tan(1/2*f*x+1/2*e)^2+1/7
*(-A+B)*tan(1/2*f*x+1/2*e)+8/63*A-1/63*B)*a^3/f/c^5/(tan(1/2*f*x+1/2*e)-1)
^9
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(75) = 150$.

Time $= 0.26$ (sec) , antiderivative size $= 331$, normalized size of antiderivative $= 4.30$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{(A-8B)a^3\cos(fx+e)^5-(4A+31B)a^3\cos(fx+e)^4+(19A+37B)a^3\cos(fx+e)^3+4(13A+}{63\left(c^5f\cos(fx+e)^5+5c^5f\cos(fx+e)^4-8c^5f\cos\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (1
9*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(
A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 +
 (5*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*
(A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x +
e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*
f)*sin(f*x + e))
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(68) = 136$.

Time $= 40.13$ (sec) , antiderivative size $= 3262$, normalized size of antiderivative $= 42.36$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-126*A*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9
- 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*
c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f
*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e
/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*A*a**3*tan
(e/2 + f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x
/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6
 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 529
2*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*
f*tan(e/2 + f*x/2) - 63*c**5*f) - 966*A*a**3*tan(e/2 + f*x/2)**6/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e
/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f
*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)*
*3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c*
*5*f) + 630*A*a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 56
7*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5
*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan
(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 +
 f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 1386*A*a**3*tan(e/
2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x...
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2701 vs. $2(75) = 150$.

Time $= 0.30$ (sec) , antiderivative size $= 2701$, normalized size of antiderivative $= 35.08$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output

```
-2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*
sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^
7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3...
```

3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(75) = 150$.

Time $= 0.50$ (sec) , antiderivative size $= 285$, normalized size of antiderivative $= 3.70$

$$\int \frac{(a + a\sin(e+fx))^3(A + B\sin(e+fx))}{(c - c\sin(e+fx))^5}\,dx =$$
$$-\frac{2\left(63\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8 - 63\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7 + 63\,Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7 + 483\,Aa^3\tan\left(\frac{1}{2}fx\right.\right.}{}$$

input

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
 63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^
3*tan(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*ta
n(1/2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x
 + 1/2*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e)
+ 8*A*a^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.49

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1013Aa^3}{16}+\frac{149Ba^3}{16}-\frac{113Aa^3\cos(2e+2fx)}{4}+\frac{37Aa^3\cos(3e+3fx)}{8}+\frac{7Aa^3\cos(4e+4fx)}{16}-\frac{41Ba^3\cos(}{4}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2
*e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x)
)/16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*
B*a^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(
3*e + 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*
x))/8 - (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (25
7*A*a^3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*
x))/2 + (63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4
+ (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2
)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*
f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

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3.49.1 Optimal result

Integrand size = 36, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{a^3(2A-9B)c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

output
```
1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^
2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(
c-c*sin(f*x+e))^7
```

3.49.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.

Time = 12.46 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.65

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+2\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^6,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
 + 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Co
s[(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/
2] + 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*
(e + f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A
*Sin[(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*
x))/2] - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*S
in[(9*(e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

3.49.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(2A-9B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(2A-9B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(2A-9B)\left(\frac{\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(2A-9B)\left(\frac{\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\downarrow 3150$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{(2A-9B)\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + ((2*A -
9*B)*(Cos[e + f*x]^7/(9*f*(c - c*Sin[e + f*x])^8) + Cos[e + f*x]^7/(63*c*f
*(c - c*Sin[e + f*x])^7)))/(11*c))
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.49.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(B-2A)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35A}{3}+B\right)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\left(-\frac{23A}{3}+3B\right)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2($
risch	$-\frac{2ia^3\left(2iA+2970iA\,\mathrm{e}^{4i(fx+e)}+693B\,\mathrm{e}^{9i(fx+e)}+693iB\,\mathrm{e}^{8i(fx+e)}-2310A\,\mathrm{e}^{7i(fx+e)}-110iA\,\mathrm{e}^{2i(fx+e)}-4158B\,\mathrm{e}^{7i(fx+e}$
derivativedivides	$\frac{2a^3\left(-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2960A+1968B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1460A+}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}\right.}{f}$
default	$\frac{2a^3\left(-\frac{3008A+2880B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2960A+1968B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{1460A+}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}\right.}{f}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^10+(B-2*A)*tan(1/2*f*x+1/2*e)^9+(35/3*A+B)*tan(1/
2*f*x+1/2*e)^8+2*(-23/3*A+3*B)*tan(1/2*f*x+1/2*e)^7+2*(46/3*A+B)*tan(1/2*f
*x+1/2*e)^6+2*(-11*A+4*B)*tan(1/2*f*x+1/2*e)^5+12/7*(13*A+B)*tan(1/2*f*x+1
/2*e)^4+2/7*(-25*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/7*(269/9*A+2*B)*tan(1/2*f*
x+1/2*e)^2+1/7*(-16/9*A+B)*tan(1/2*f*x+1/2*e)+79/693*A-1/77*B)*a^3/f/c^6/(
tan(1/2*f*x+1/2*e)-1)^11
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(115) = 230$.

Time $= 0.27$ (sec) , antiderivative size $= 405$, normalized size of antiderivative $= 3.43$

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\, dx$$
$$= \frac{(2\,A - 9\,B)a^3\cos(fx + e)^6 + 6\,(2\,A - 9\,B)a^3\cos(fx + e)^5 - (25\,A + 234\,B)a^3\cos(fx + e)^4 + 7\,(23\,A +}{693\,\left(c^6 f\cos(fx + e)^6 - 5\,c^6 f\cos(fx + e\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
 (25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 +
28*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(
A + B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x +
 e)^4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x +
e)^2 + 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*
f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c
^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32
*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x
 + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(105) = 210$.

Time = 64.76 (sec) , antiderivative size = 4816, normalized size of antiderivative = 40.81

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-1386*A*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)*
*11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9
 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 -
320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 22
8690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 3811
5*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f)
+ 2772*A*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*
c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c*
*6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6
*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f
*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*ta
n(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*A*a
**3*tan(e/2 + f*x/2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan
(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e
/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2
 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
 f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*
x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 21252*A*a**3*tan(e/
2 + f*x/2)**7/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x
/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*...
```

3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3390 vs. $2(115) = 230$.

Time = 0.31 (sec) , antiderivative size = 3390, normalized size of antiderivative = 28.73

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/
(cos(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9
*A*a^3*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12
936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(c...
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(115) = 230$.

Time $= 0.76$ (sec) , antiderivative size $= 353$, normalized size of antiderivative $= 2.99$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx =$$
$$-\frac{2\left(693\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{10} - 1386\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{9} + 693\,Ba^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{9} + 8085\,Aa^3\tan\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e
)^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8
 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
 4158*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 +
1386*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5
544*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 11
88*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178
*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*
a^3*tan(1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan
(1/2*f*x + 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^
11)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.46

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx =$$

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(565\,A\,a^3\cos(2e+2fx)-\frac{837\,B\,a^3}{16}-922\,A\,a^3-\frac{3527\,A\,a^3\cos(3e+3fx)}{32}-29\,A\,a^3\cos(4e\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x
)
```

output
```
-(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*
A*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81
*A*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*
cos(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e +
5*f*x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x
))/32 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 +
 (693*B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^
3*sin(4*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f
*x) + (13629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^
6*f*((231*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2
 - pi/4 + (3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/3
2 + (55*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e
)/2 + pi/4 + (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/
32))
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

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3.50.1 Optimal result

Integrand size = 36, antiderivative size = 156

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx\\ &=\frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}\\ &\quad+\frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}\end{aligned}$$

output
```
1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)
*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^
7/f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e
))^7
```

3.50.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 339 vs. $2(156) = 312$.

Time = 13.83 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx =$$
$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^7,x]
```

output
```
-1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*
(6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2]
 - 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[
(7*(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2
] + 130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(
e + f*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2]
- 6864*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[
(7*(e + f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2]
 + 3*A*Sin[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

3.50.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(3A-10B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-10B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\left(\frac{\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\downarrow 3150$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{(3A-10B)\left(\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}\right)}{13c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + ((3*A -
 10*B)*(Cos[e + f*x]^7/(11*f*(c - c*Sin[e + f*x])^9) + (2*(Cos[e + f*x]^7/
(9*f*(c - c*Sin[e + f*x])^8) + Cos[e + f*x]^7/(63*c*f*(c - c*Sin[e + f*x])
^7)))/(11*c)))/(13*c))
```

3.50.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.50.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{2\left(A\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(17A+\frac{B}{3}\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-33A+\frac{23B}{3}\right)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+$
derivativedivides	$2a^3\left(-\frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{8832A+8576B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{}{\tan}$
default	$2a^3\left(-\frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{8832A+8576B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{}{\tan}$
risch	$-\frac{4\left(-10Ba^3+3Aa^3-30030iBa^3e^{7i(fx+e)}+39iAa^3e^{i(fx+e)}+12012iBa^3e^{9i(fx+e)}-130iBa^3e^{i(fx+e)}-234Aa^3e^{2i(f}\right.$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^12+(-3*A+B)*tan(1/2*f*x+1/2*e)^11+(17*A+1/3*B)*ta
n(1/2*f*x+1/2*e)^10+(-33*A+23/3*B)*tan(1/2*f*x+1/2*e)^9+(72*A-B)*tan(1/2*f
*x+1/2*e)^8+(-82*A+50/3*B)*tan(1/2*f*x+1/2*e)^7+(666/7*A-38/21*B)*tan(1/2*
f*x+1/2*e)^6+(-426/7*A+90/7*B)*tan(1/2*f*x+1/2*e)^5+(857/21*A-2/63*B)*tan(
1/2*f*x+1/2*e)^4+(-263/21*A+215/63*B)*tan(1/2*f*x+1/2*e)^3+(37/231*B+389/7
7*A)*tan(1/2*f*x+1/2*e)^2+(-79/231*A+97/693*B)*tan(1/2*f*x+1/2*e)+310/3003
*A-97/9009*B)*a^3/f/c^7/(tan(1/2*f*x+1/2*e)-1)^13
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(152) = 304$.

Time $= 0.27$ (sec) , antiderivative size $= 475$, normalized size of antiderivative $= 3.04$

$$\int \frac{(a + a\sin(e+fx))^3 (A + B\sin(e+fx))}{(c - c\sin(e+fx))^7}\,dx =$$

$$-\frac{2\,(3\,A - 10\,B)a^3\cos(fx+e)^7 - 12\,(3\,A - 10\,B)a^3\cos(fx+e)^6 - 49\,(3\,A - 10\,B)a^3\cos(fx+e)^5 + 7}{9009\,\left(c^7 f\cos(fx+\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="fricas")
```

output
```
-1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
 e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x
+ e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f
*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 1
0*B)*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 1
0*B)*a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A +
21*B)*a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^
3)*sin(f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f
*cos(f*x + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*
c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x +
 e)^6 - 6*c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*
x + e)^3 + 80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin
(f*x + e))
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(143) = 286$.

Time = 102.64 (sec) , antiderivative size = 6669, normalized size of antiderivative = 42.75

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

output
```
Piecewise((-18018*A*a**3*tan(e/2 + f*x/2)**12/(9009*c**7*f*tan(e/2 + f*x/2
)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2
)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x
/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f
*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 +
 f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 +
 f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f
*x/2) - 9009*c**7*f) + 54054*A*a**3*tan(e/2 + f*x/2)**11/(9009*c**7*f*tan(
e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(
e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*ta
n(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*
tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*
f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*
f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*
tan(e/2 + f*x/2) - 9009*c**7*f) - 306306*A*a**3*tan(e/2 + f*x/2)**10/(9009
*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702
*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 64414
35*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 1545
9444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11
594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 ...
```

3.50.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4078 vs. $2(152) = 304$.

Time = 0.39 (sec) , antiderivative size = 4078, normalized size of antiderivative = 26.14

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="maxima")
```

output
```
-2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18
7330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c
^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f
*x + e)^13/(cos(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x +
e) + 1) - 28626*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26512
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 276276*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - ...
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(152) = 304$.

Time $= 0.43$ (sec) , antiderivative size $= 421$, normalized size of antiderivative $= 2.70$

$$\int \frac{(a + a\sin(e+fx))^3(A + B\sin(e+fx))}{(c - c\sin(e+fx))^7}\,dx =$$
$$-\frac{2\left(9009\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12} - 27027\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11} + 9009\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11} + 153153\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="giac")
```

output

```
-2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/
2*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x +
1/2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x
+ 1/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x
 + 1/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x
 + 1/2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f
*x + 1/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*
f*x + 1/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/
2*f*x + 1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2
*f*x + 1/2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2
*f*x + 1/2*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f
*x + 1/2*e) + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7
*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.21

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$

$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{2363\,B\,a^3}{32}-\frac{279183\,A\,a^3}{16}+\frac{220269\,A\,a^3\cos(2e+2fx)}{16}-\frac{46095\,A\,a^3\cos(3e+3fx)}{16}-\frac{20829\,A\,a^3\cos(4e+4fx)}{16}-\right.}{}$$

input

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x
)
```

output

```
(2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3
*cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*c
os(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*
e + 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3
*f*x))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x)
)/16 - (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 -
 (81081*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (
3237*A*a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*
B*a^3*sin(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a
^3*sin(4*e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(
6*e + 6*f*x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) +
(173745*A*a^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16))/(9009*c^7*
f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/
2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64
 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e
)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2
))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

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3.51.1 Optimal result

Integrand size = 36, antiderivative size = 197

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9}$$
$$+ \frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8} + \frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}$$

output
```
1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)
*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^
7/f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e
))^8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7
```

3.51.2 Mathematica [A] (verified)

Time = 14.87 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.85

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6435(72A+47B)\cos\left(\frac{1}{2}(e+fx)\right)-10010(26\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^8,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(6435*(72*
A + 47*B)*Cos[(e + f*x)/2] - 10010*(26*A + 23*B)*Cos[(3*(e + f*x))/2] - 72
072*A*Cos[(5*(e + f*x))/2] - 117117*B*Cos[(5*(e + f*x))/2] + 5460*A*Cos[(7
*(e + f*x))/2] + 30030*B*Cos[(7*(e + f*x))/2] - 420*A*Cos[(11*(e + f*x))/2
] + 1155*B*Cos[(11*(e + f*x))/2] + 4*A*Cos[(15*(e + f*x))/2] - 11*B*Cos[(1
5*(e + f*x))/2] + 437580*A*Sin[(e + f*x)/2] + 373230*B*Sin[(e + f*x)/2] +
240240*A*Sin[(3*(e + f*x))/2] + 285285*B*Sin[(3*(e + f*x))/2] - 60060*A*Si
n[(5*(e + f*x))/2] - 150150*B*Sin[(5*(e + f*x))/2] - 45045*B*Sin[(7*(e + f
*x))/2] - 1820*A*Sin[(9*(e + f*x))/2] + 5005*B*Sin[(9*(e + f*x))/2] + 60*A
*Sin[(13*(e + f*x))/2] - 165*B*Sin[(13*(e + f*x))/2]))/(1441440*c^8*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^8)
```

3.51.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(4A-11B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}dx}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(4A-11B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{10}}dx}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\downarrow 3151$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\left(\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)}{15f(c-c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)}{15f(c-c}\right.$$

$\downarrow$ 3150

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{(4A-11B)\left(\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3\left(\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7}{9f(c-c}\right.}{11c}\right.}{13c}\right.}{15c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + ((4*A -
 11*B)*(Cos[e + f*x]^7/(13*f*(c - c*Sin[e + f*x])^10) + (3*(Cos[e + f*x]^7
/(11*f*(c - c*Sin[e + f*x])^9) + (2*(Cos[e + f*x]^7/(9*f*(c - c*Sin[e + f*
x])^8) + Cos[e + f*x]^7/(63*c*f*(c - c*Sin[e + f*x])^7)))/(11*c)))/(13*c))
)/(15*c))
```

3.51. $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.51.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4ia^3\left(-4iA-5460iA\,e^{4i(fx+e)}+45045B\,e^{11i(fx+e)}-302445iB\,e^{8i(fx+e)}-240240A\,e^{9i(fx+e)}+420iA\,e^{2i(fx+e)}-285285\right.}{}$
parallelrisch	$-\frac{2\left(A\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4A+B)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(71A-B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+10(-6A+B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(74}{}\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{24320}{13\left(\tan\left(\right.\right.}\right.}{}$
default	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{24320}{13\left(\tan\left(\right.\right.}\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x,method=_RETUR
NVERBOSE)
```

output
```
4/45045*I*a^3*(-4*I*A-5460*I*A*exp(4*I*(f*x+e))+45045*B*exp(11*I*(f*x+e))-
302445*I*B*exp(8*I*(f*x+e))-240240*A*exp(9*I*(f*x+e))+420*I*A*exp(2*I*(f*x
+e))-285285*B*exp(9*I*(f*x+e))+11*I*B+437580*A*exp(7*I*(f*x+e))+72072*I*A*
exp(10*I*(f*x+e))+373230*B*exp(7*I*(f*x+e))+117117*I*B*exp(10*I*(f*x+e))-6
0060*A*exp(5*I*(f*x+e))+230230*I*B*exp(6*I*(f*x+e))-150150*B*exp(5*I*(f*x+
e))-1155*I*B*exp(2*I*(f*x+e))-1820*A*exp(3*I*(f*x+e))-463320*I*A*exp(8*I*(
f*x+e))+5005*B*exp(3*I*(f*x+e))-30030*I*B*exp(4*I*(f*x+e))+60*A*exp(I*(f*x
+e))+260260*I*A*exp(6*I*(f*x+e))-165*B*exp(I*(f*x+e)))/f/c^8/(exp(I*(f*x+e
))-I)^15
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(192) = 384.

Time = 0.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{2\,(4\,A-11\,B)a^3\cos(fx+e)^8+16\,(4\,A-11\,B)a^3\cos(fx+e)^7-49\,(4\,A-11\,B)a^3\cos(fx+e)^6-168}{45045\,(c}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="fricas")
```

output
```
1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
 e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x
+ e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f
*x + e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f
*x + e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A
 - 11*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*
A - 11*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3
*A + 8*B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A +
 B)*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32
*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4
 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x
+ e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8
*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 19
2*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8821 vs. $2(178) = 356$.

Time $= 162.97$ (sec) , antiderivative size $= 8821$, normalized size of antiderivative $= 44.78$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
```

output
```
Piecewise((-90090*A*a**3*tan(e/2 + f*x/2)**14/(45045*c**8*f*tan(e/2 + f*x/
2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x
/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 +
 f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan
(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f
*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c*
*8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*
c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c
**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 360360*A*a**3*tan(e/2 + f*x/2)**1
3/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14
+ 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**
12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x
/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2
+ f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(
e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*t
an(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*
tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 213
2130*A*a**3*tan(e/2 + f*x/2)**12/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 6756
75*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 204
95475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**1...
```

3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4765 vs. $2(192) = 384$.

Time $= 0.41$ (sec) , antiderivative size $= 4765$, normalized size of antiderivative $= 24.19$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="maxima")
```

output
```
2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 8
91345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 1
95195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f
*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 10
5*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
+ 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(co
s(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(1
7715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + ...
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(192) = 384$.

Time $= 0.45$ (sec) , antiderivative size $= 489$, normalized size of antiderivative $= 2.48$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\,dx =$$
$$-\frac{2\left(45045\,Aa^3\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{14} - 180180\,Aa^3\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{13} + 45045\,Ba^3\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{13} + 106\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="giac")
```

output
```
-2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
 1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f
*x + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1
/2*f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*
tan(1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*
A*a^3*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 1412
4825*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 -
13178880*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)
^7 + 11026015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/
2*e)^6 - 6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x +
 1/2*e)^5 + 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*
x + 1/2*e)^4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*
f*x + 1/2*e)^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*
f*x + 1/2*e)^2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x
 + 1/2*e) + 4243*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{544369\,A\,a^3}{4}-\frac{21791\,B\,a^3}{4}-\frac{257861\,A\,a^3\cos(2\,e+2\,f\,x)}{2}+\frac{3497111\,A\,a^3\cos(3\,e+3\,f\,x)}{128}+\frac{72047\,A\,a^3\cos(4\,e+4\,f\,x}{4}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x
)
```

output

```
(2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*
cos(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*
cos(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*co
s(6*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(
2*e + 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*
e + 4*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*
f*x))/32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f
*x))/64 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*
f*x))/8 - (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f
*x))/64 + (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*
x))/64 - (854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x
))/8 + (46475*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/
64 - (385*B*a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 +
 (126929*B*a^3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (39
70395*B*a^3*sin(e + f*x))/128))/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4
 + (f*x)/2))/128 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3
003*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)
/2 - pi/4 + (7*f*x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2)
)/128 - (105*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*
cos((13*e)/2 + pi/4 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 +...
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

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3.52.1 Optimal result

Integrand size = 36, antiderivative size = 190

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx \\
&= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} \\
&\quad - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} \\
&\quad - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))}
\end{aligned}$$

output

```
-35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*cos(f*x+e)^3/a/f-35/8*(4*A-5*B
)*c^4*cos(f*x+e)*sin(f*x+e)/a/f-a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+
e))^5-2*a^2*(4*A-5*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^3-7/4*(4*A-5*B)*
c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
```

3.52.2 Mathematica [A] (verified)

Time = 12.48 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(3072(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-420(4A-5B)(\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B
)*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
) + 24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)]
 + 3*B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

3.52.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3446$$
$$a^4c^4\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}\,dx$$
$$\downarrow 3042$$

$$a^4c^4\int\frac{\cos(e+fx)^8(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\downarrow 3338$$

$$a^4c^4\left(-\frac{(4A-5B)\int\frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(4A-5B)\int\frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3159$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3158$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3161$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3115$$

$$a^4c^4\left(-\frac{\left((4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos}{f(a\sin(e+}\right.$$

$$\downarrow 24$$

$$a^4c^4\left(-\frac{\left((4A-5B)\left(\frac{7\left(\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{4a}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(}{f(a\sin(e+f}\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```

output
```
a^4*c^4*(-(((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5)) - ((4*A -
5*B)*((2*Cos[e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^3) + (7*(Cos[e + f*x]^5
/(4*f*(a^2 + a^2*Sin[e + f*x])) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos
[e + f*x]*Sin[e + f*x])/(2*f))/a))/(4*a)))/a^2))/a)
```

3.52.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.52.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{2c^4\left(-\frac{16A-16B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A-15B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5A}{2}-\frac{55B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(35A-55B)\left(t\right.}{\left(\right.}$
default	$\frac{2c^4\left(-\frac{16A-16B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(11A-15B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5A}{2}-\frac{55B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(35A-55B)\left(t\right.}{\left(\right.}$
parallelrisch	$-\frac{7c^4\left(\left(30fxA-\frac{75}{2}fxB+\frac{1189}{14}A-\frac{1433}{14}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(30fxA-\frac{75}{2}fxB+\frac{139}{14}A-\frac{215}{14}B\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+9\left(A-\frac{3B}{2}\right)\cos\right.}{}$
risch	$-\frac{35c^4xA}{2a}+\frac{175c^4xB}{8a}-\frac{47c^4e^{i(fx+e)}A}{8af}+\frac{75c^4e^{i(fx+e)}B}{8af}-\frac{47c^4e^{-i(fx+e)}A}{8af}+\frac{75c^4e^{-i(fx+e)}B}{8af}-\frac{32c^4}{fa\left(e^{i(fx-}\right.}$
norman	$\frac{-\frac{166Ac^4-206Bc^4}{3af}-\frac{35(4A-5B)c^4x}{8a}-\frac{\left(108Ac^4-167Bc^4\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4af}-\frac{\left(148Ac^4-175Bc^4\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4af}-\frac{\left(204Ac^4-3\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f*c^4/a*(-(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)-((5/2*A-47/8*B)*tan(1/2*f*x
+1/2*e)^7+(11*A-15*B)*tan(1/2*f*x+1/2*e)^6+(5/2*A-55/8*B)*tan(1/2*f*x+1/2*
e)^5+(35*A-55*B)*tan(1/2*f*x+1/2*e)^4+(-5/2*A+55/8*B)*tan(1/2*f*x+1/2*e)^3
+(107/3*A-175/3*B)*tan(1/2*f*x+1/2*e)^2+(-5/2*A+47/8*B)*tan(1/2*f*x+1/2*e)
+35/3*A-55/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^4-35/8*(4*A-5*B)*arctan(tan(1/2*f
*x+1/2*e)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{6\,Bc^4\cos(fx+e)^5 - 8\,(A-5\,B)c^4\cos(fx+e)^4 + (52\,A-113\,B)c^4\cos(fx+e)^3 + 105\,(4\,A-5\,B)c^4$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 1
13*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*co
s(f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*
B)*c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*
x + e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 -
3*(76*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*co
s(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6690 vs. 2(173) = 346.

Time = 6.94 (sec) , antiderivative size = 6690, normalized size of antiderivative = 35.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

output

```
Piecewise((-420*A*c**4*f*x*tan(e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9
 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/
2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4
 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2
 + f*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 +
 f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*
a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 +
f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a
*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*
f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/
2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f
*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2
)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2
)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan
(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**
5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(
e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(
e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2
 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3...
```

3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1796 vs. $2(182) = 364$.

Time $= 0.33$ (sec) , antiderivative size $= 1796$, normalized size of antiderivative $= 9.45$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output

```
1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 + 4*a*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) +
39*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 24*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x
+ e)/(cos(f*x + e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 3*a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 3
9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e)...
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{105\,(4\,Ac^4-5\,Bc^4)(fx+e)}{a}+\frac{768\,(Ac^4-Bc^4)}{a(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}+\frac{2\left(60\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7-141\,Bc^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+264\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^6-36\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1
/2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan
(1/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2
*f*x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x
+ 1/2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1
/2*e)^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)
^3 + 856*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2
- 60*A*c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c
^4 - 440*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

3.52.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{55Ac^4}{3}-\frac{299Bc^4}{12}\right)+\frac{166Ac^4}{3}-\frac{206Bc^4}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(27Ac^4-\frac{167Bc^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+}$$
$$-\frac{35c^4\,\mathrm{atan}\left(\frac{35c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A-5B)}{140Ac^4-175Bc^4}\right)(4A-5B)}{4af}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*((55*A*c^4)/3 - (299*B*c^4)/12) + (166*A*c^4)/3 - (2
06*B*c^4)/3 + tan(e/2 + (f*x)/2)^7*(27*A*c^4 - (167*B*c^4)/4) + tan(e/2 +
(f*x)/2)^8*(37*A*c^4 - (175*B*c^4)/4) + tan(e/2 + (f*x)/2)^5*(75*A*c^4 - (
495*B*c^4)/4) + tan(e/2 + (f*x)/2)^6*(155*A*c^4 - (687*B*c^4)/4) + tan(e/2
 + (f*x)/2)^4*(257*A*c^4 - (1153*B*c^4)/4) + tan(e/2 + (f*x)/2)^3*((199*A*
c^4)/3 - (1235*B*c^4)/12) + tan(e/2 + (f*x)/2)^2*((583*A*c^4)/3 - (2795*B*
c^4)/12))/(f*(a + a*tan(e/2 + (f*x)/2) + 4*a*tan(e/2 + (f*x)/2)^2 + 4*a*ta
n(e/2 + (f*x)/2)^3 + 6*a*tan(e/2 + (f*x)/2)^4 + 6*a*tan(e/2 + (f*x)/2)^5 +
 4*a*tan(e/2 + (f*x)/2)^6 + 4*a*tan(e/2 + (f*x)/2)^7 + a*tan(e/2 + (f*x)/2
)^8 + a*tan(e/2 + (f*x)/2)^9)) - (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(
4*A - 5*B))/(140*A*c^4 - 175*B*c^4))*(4*A - 5*B))/(4*a*f)
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

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3.53.1 Optimal result

Integrand size = 36, antiderivative size = 157

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= -\frac{5(3A-4B)c^3x}{2a} - \frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af}$$
$$- \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a^3(3A-4B)c^3\cos^5(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^2}$$

output
```
-5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^
3*cos(f*x+e)*sin(f*x+e)/a/f-a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^
4-2*a^3*(3*A-4*B)*c^3*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^2
```

3.53.2 Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)+(48$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
]),x]
```

output
```
(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
 f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*
(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 +
 5*e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B
*Cos[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

3.53.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3338$$

$$a^3c^3\left(-\frac{(3A-4B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}\,dx}{a} - \frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(3A-4B)\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3115$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

output
```
a^3*c^3*(-(((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4)) - ((3*A -
4*B)*((2*Cos[e + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3
/(3*a*f) + (x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))/a))/a^2))/a)
```

3.53.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.53.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{65\left(\frac{4\left(-4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A-4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA+4fxB-\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A-\frac{68B}{65}\right)$ $8af\cos(fx+e)$
derivativedivides	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-5}{fa}$
default	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-5}{fa}$
risch	$-\frac{15c^3xA}{2a}+\frac{10c^3xB}{a}-\frac{2c^3e^{i(fx+e)}A}{af}+\frac{31c^3e^{i(fx+e)}B}{8af}-\frac{2c^3e^{-i(fx+e)}A}{af}+\frac{31c^3e^{-i(fx+e)}B}{8af}-\frac{16c^3A}{fa\left(e^{i(fx+e)}-$
norman	$\frac{-\frac{72Ac^3-94Bc^3}{3af}-\frac{5(3A-4B)c^3x}{2a}-\frac{\left(9Ac^3-18Bc^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(17Ac^3-20Bc^3\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(21Ac^3-34Bc^3\right)\tan}{3af}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
65/8*(4/65*(-4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A-4*B)*sin(3*f*x+3*e)-1/195*
B*cos(4*f*x+4*e)+4/13*(-3*f*x*A+4*f*x*B-24/5*A+94/15*B)*cos(f*x+e)+(A-68/6
5*B)*sin(f*x+e)-16/13*A+19/13*B)*c^3/a/f/cos(f*x+e)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{2Bc^3\cos(fx+e)^4+(3A-10B)c^3\cos(fx+e)^3+15(3A-4B)c^3fx+24(A-2B)c^3\cos(fx+e)}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
 4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3
*A - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e
)^3 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 1
2*B)*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) +
a*f*sin(f*x + e) + a*f)
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. $2(139) = 278$.

Time = 3.83 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.10

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^3}{a + a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
 f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a
*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 +
 f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*
A*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
+ f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*
x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(
e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*
a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*
tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*t...
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(151) = 302$.

Time = 0.33 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.13

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^3}{a + a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) -
 3*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/
(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a +
 a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) +...
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{15\,(3\,Ac^3-4\,Bc^3)(fx+e)}{a}+\frac{96\,(Ac^3-Bc^3)}{a(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}+\frac{2\left(3\,Ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-12\,Bc^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+24\,Ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-42\,Bc^3\right.}{}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*
f*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*
f*x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x +
1/2*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)
^2 - 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c
^3 - 46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

3.53.9 Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.03

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(7Ac^3-\frac{34Bc^3}{3}\right)+24Ac^3-\frac{94Bc^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(9Ac^3-18Bc^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(1}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+3a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\right.}$$
$$-\frac{5c^3\,\mathrm{atan}\left(\frac{5c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3A-4B)}{15Ac^3-20Bc^3}\right)(3A-4B)}{af}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*(7*A*c^3 - (34*B*c^3)/3) + 24*A*c^3 - (94*B*c^3)/3 +
 tan(e/2 + (f*x)/2)^5*(9*A*c^3 - 18*B*c^3) + tan(e/2 + (f*x)/2)^6*(17*A*c^
3 - 20*B*c^3) + tan(e/2 + (f*x)/2)^3*(16*A*c^3 - 32*B*c^3) + tan(e/2 + (f*
x)/2)^4*(56*A*c^3 - 62*B*c^3) + tan(e/2 + (f*x)/2)^2*(63*A*c^3 - 76*B*c^3)
)/(f*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 +
(f*x)/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e
/2 + (f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7)) - (5*c^3*atan((5*c^3*tan(e/2 +
(f*x)/2)*(3*A - 4*B))/(15*A*c^3 - 20*B*c^3))*(3*A - 4*B))/(a*f)
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

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3.54.1 Optimal result

Integrand size = 36, antiderivative size = 118

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx\\ &=-\frac{3(2A-3B)c^2x}{2a}-\frac{3(2A-3B)c^2\cos(e+fx)}{2af}\\ &\quad-\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}\end{aligned}$$

output
```
-3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*cos(f*x+e)/a/f-a^2*(A-B)*c^2*cos(
f*x+e)^5/f/(a+a*sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*
x+e))
```

3.54.2 Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.59

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+4(\right.}{4af}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
]),x]
```

output
```
-1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos
[(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[
2*(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e
+ 9*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

3.54.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^3}\,dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^3}\,dx$$

$$\downarrow 3338$$

$$a^2c^2\left(-\frac{(2A-3B)\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}\,dx}{a} - \frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(-\frac{(2A-3B)\int \frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}\,dx}{a} - \frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3161$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

output
```
a^2*c^2*(-(((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3)) - ((2*A -
3*B)*((3*(x/a + Cos[e + f*x]/(a*f)))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^
2*Sin[e + f*x]))))/a)
```

3.54.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.54.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{4c^2\left(\frac{(-A+3B)\cos(2fx+2e)}{8}-\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA+\frac{9}{2}fxB-5A+7B\right)\cos(fx+e)}{4}+\left(A-\frac{33B}{32}\right)\sin(fx+e)-\frac{9A}{8}+\frac{11B}{8}\right)}{af\cos(fx+e)}$
derivativedivides	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fa}$
default	$\frac{2c^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-3B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fa}$
risch	$-\frac{3c^2xA}{a}+\frac{9c^2xB}{2a}-\frac{c^2e^{i(fx+e)}A}{2af}+\frac{3c^2e^{i(fx+e)}B}{2af}-\frac{c^2e^{-i(fx+e)}A}{2af}+\frac{3c^2e^{-i(fx+e)}B}{2af}-\frac{8c^2A}{fa\left(e^{i(fx+e)}+i\right)}+$
norman	$\frac{\left(6Ac^2-4Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(8Ac^2-9Bc^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(20Ac^2-15Bc^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(22Ac^2-20Bc^2\right)\left(\tan^5\left(\frac{fx}{2}\right.\right.}{af}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
4*c^2*(1/8*(-A+3*B)*cos(2*f*x+2*e)-1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A+9/2
*f*x*B-5*A+7*B)*cos(f*x+e)+(A-33/32*B)*sin(f*x+e)-9/8*A+11/8*B)/a/f/cos(f*
x+e)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.52

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{Bc^2\cos(fx+e)^3-3(2A-3B)c^2fx-2(A-3B)c^2\cos(fx+e)^2-8(A-B)c^2-(3(2A-3B)c^2fx}{2(af\,co}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*
x + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos
(f*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^
2*cos(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(
f*x + e) + a*f)
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(99) = 198$.

Time $= 2.03$ (sec) , antiderivative size $= 2365$, normalized size of antiderivative $= 20.04$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)*
*4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
- 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c*
*2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 16*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*t
an(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
f*x/2) + 2*a*f) - 36*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)...
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(112) = 224$.

Time = 0.31 (sec) , antiderivative size = 608, normalized size of antiderivative = 5.15

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^2}{a + a\sin(e+fx)}\,dx$$

$$= \frac{Bc^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 4}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1} + \frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right) - 2Ac^2\left(\frac{}{a+\frac{a\sin}{\cos}}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
(B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
  + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x
  + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a) + 4*B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x +
  e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x +
  e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
 + 1))) - 2*A*c^2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^2}{a + a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{3\left(2Ac^2 - 3Bc^2\right)(fx+e)}{a} + \frac{16\left(Ac^2 - Bc^2\right)}{a\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)+1\right)} - \frac{2\left(Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 2Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 6Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Bc^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2 a}}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) -
 2*A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

3.54.9 Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2Ac^2-5Bc^2)+10Ac^2-14Bc^2+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(2Ac^2-7Bc^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(8A$$
$$f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+2a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right.$$
$$-\frac{3c^2\,\mathrm{atan}\left(\frac{3c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2A-3B)}{6Ac^2-9Bc^2}\right)(2A-3B)}{af}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*(2*A*c^2 - 5*B*c^2) + 10*A*c^2 - 14*B*c^2 + tan(e/2
+ (f*x)/2)^3*(2*A*c^2 - 7*B*c^2) + tan(e/2 + (f*x)/2)^4*(8*A*c^2 - 9*B*c^2
) + tan(e/2 + (f*x)/2)^2*(18*A*c^2 - 21*B*c^2))/(f*(a + a*tan(e/2 + (f*x)/
2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*
x)/2)^4 + a*tan(e/2 + (f*x)/2)^5)) - (3*c^2*atan((3*c^2*tan(e/2 + (f*x)/2)
*(2*A - 3*B))/(6*A*c^2 - 9*B*c^2))*(2*A - 3*B))/(a*f)
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

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3.55.1 Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af} - \frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

output
```
-(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

3.55.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.

Time = 5.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{\left(-((A-2B)x)+\frac{B\cos(e)\cos(fx)}{f}-\frac{B\sin(e)\sin(fx)}{f}+\frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(c-c\sin(e+f}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
,x]
```

output
```
((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A -
 B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)
```

3.55.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3446, 3042, 3336, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^2}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^2}\,dx$$

$$\downarrow 3336$$

$$ac\left(-\frac{\int (a(A-2B)+aB\sin(e+fx))dx}{a^3} - \frac{2(A-B)\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}\right)$$

$$\downarrow 2009$$

$$ac\left(-\frac{ax(A-2B) - \frac{aB\cos(e+fx)}{f}}{a^3} - \frac{2(A-B)\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
a*c*(-((a*(A - 2*B)*x - (a*B*Cos[e + f*x])/f)/a^3) - (2*(A - B)*Cos[e + f*
x])/(f*(a^2 + a^2*Sin[e + f*x])))
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.55.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{2c\left(-\frac{-2B+2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$
default	$\frac{2c\left(-\frac{-2B+2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa}$
parallelrisch	$\frac{2c\left(\frac{B\cos(2fx+2e)}{4}+\left(-\frac{1}{2}fxA+fxB-A+\frac{3}{2}B\right)\cos(fx+e)+(A-B)\sin(fx+e)-A+\frac{5B}{4}\right)}{af\cos(fx+e)}$
risch	$-\frac{cxA}{a}+\frac{2cxB}{a}+\frac{Bce^{i(fx+e)}}{2af}+\frac{Bce^{-i(fx+e)}}{2af}-\frac{4cA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{4cB}{fa\left(e^{i(fx+e)}+i\right)}$
norman	$\frac{\frac{2Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{4Ac-6Bc}{af}-\frac{(A-2B)cx}{a}+\frac{2Bc\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(4Ac-5Bc)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(4Ac-4Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*c/a*(-(-2*B+2*A)/(tan(1/2*f*x+1/2*e)+1)+B/(1+tan(1/2*f*x+1/2*e)^2)-(A-
2*B)*arctan(tan(1/2*f*x+1/2*e)))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time $= 0.26$ (sec) , antiderivative size $= 117$, normalized size of antiderivative $= 2.05$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{a + a\sin(e+fx)}\,dx = -\frac{(A-2B)cfx - Bc\cos(fx+e)^2 + 2(A-B)c + ((A-2B)cfx + (2A-3B)c)\cos(fx+e) + ((A-}{af\cos(fx+e) + af\sin(fx+e) + af}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x +
(2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A -
 B)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(49) = 98$.

Time $= 1.06$ (sec) , antiderivative size $= 828$, normalized size of antiderivative $= 14.53$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{a + a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan
(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)*
*2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/
2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2
)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e
/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x
/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*
tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + 2*B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
 f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*
f) + 2*B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 +
 f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) + 6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c
)/(a*sin(e) + a), True))
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(57) = 114$.

Time $= 0.40$ (sec) , antiderivative size $= 256$, normalized size of antiderivative $= 4.49$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{a + a\sin(e+fx)}\,dx$$
$$= \frac{2\left(Bc\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1} + \frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right) - Ac\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(
a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time $= 0.46$ (sec) , antiderivative size $= 115$, normalized size of antiderivative $= 2.02$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{a + a\sin(e+fx)}\,dx$$
$$= -\frac{\frac{(Ac-2Bc)(fx+e)}{a} + \frac{2\left(2Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 2Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 2Ac - 3Bc\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 1\right)a}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(
1/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f
*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

3.55.9 Mupad [B] (verification not implemented)

Time $= 13.59$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 1.93$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{a + a\sin(e+fx)}\,dx$$
$$= -\frac{(4Ac - 4Bc)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 - 2Bc\tan\left(\frac{e}{2}+\frac{fx}{2}\right) + 4Ac - 6Bc}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3 + a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 + a\tan\left(\frac{e}{2}+\frac{fx}{2}\right) + a\right)} - \frac{Acfx - 2Bcfx}{af}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
- (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 +
(f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/
2 + (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

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3.56.1 Optimal result

Integrand size = 36, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

output
```
B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f
```

3.56.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]))
,x]
```

output
```
(B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

3.56.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3148$$

$$\frac{A\int \sec^2(e+fx)dx + \frac{B\sec(e+fx)}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx + \frac{B\sec(e+fx)}{f}}{ac}$$

$$\downarrow 4254$$

$$\frac{\frac{B\sec(e+fx)}{f} - \frac{A\int 1d(-\tan(e+fx))}{f}}{ac}$$

$$\downarrow 24$$

$$\frac{\frac{A\tan(e+fx)}{f} + \frac{B\sec(e+fx)}{f}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

output
```
((B*Sec[e + f*x])/f + (A*Tan[e + f*x])/f)/(a*c)
```

3.56.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
parallelrisch	$\frac{-2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2B}{fac\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	42
risch	$\frac{2iA+2B\,e^{i(fx+e)}}{\left(e^{i(fx+e)}-i\right)\left(e^{i(fx+e)}+i\right)acf}$	56
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{acf}$	57
default	$\frac{-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{acf}$	57
norman	$\frac{-\frac{2B}{acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	123

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
(-2*A*tan(1/2*f*x+1/2*e)-2*B)/f/a/c/(tan(1/2*f*x+1/2*e)^2-1)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{A\sin(fx+e)+B}{acf\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
(A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.

Time $= 0.72$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 2.37$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c - c\sin(e+fx))}\,dx = \begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} - \frac{2B}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B
/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin
(e) + a)*(-c*sin(e) + c)), True))
```

3.56.7 Maxima [A] (verification not implemented)

Time $= 0.20$ (sec) , antiderivative size $= 35$, normalized size of antiderivative $= 1.00$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c - c\sin(e+fx))}\,dx = \frac{\frac{A\tan(fx+e)}{ac} + \frac{B}{ac\cos(fx+e)}}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
(A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = -\frac{2\left(A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)acf}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = -\frac{2\left(B+A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

output
```
-(2*(B + A*tan(e/2 + (f*x)/2)))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

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3.57.1 Optimal result

Integrand size = 36, antiderivative size = 63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{(2A-B)\tan(e+fx)}{3ac^2f}$$

output
```
1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2
/f
```

3.57.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)(6B-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)-4B\sin(e+fx)}{12ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2),x]
```

output
```
(Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
 + 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e +
 f*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

3.57.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{\frac{(2A-B)\int \sec^2(e+fx)dx}{3c} + \frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(2A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx}{3c} + \frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

$$\downarrow 4254$$

$$\frac{\frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))} - \frac{(2A-B)\int 1d(-\tan(e+fx))}{3cf}}{ac}$$

$$\downarrow 24$$

$$\frac{\frac{(2A-B)\tan(e+fx)}{3cf} + \frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

output
```
(((A + B)*Sec[e + f*x])/(3*f*(c - c*Sin[e + f*x])) + ((2*A - B)*Tan[e + f*
x])/(3*c*f))/(a*c)
```

3.57.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.57.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{2i\left(4iA\,e^{i(fx+e)}-2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}+2A-B\right)}{3\left(e^{i(fx+e)}-i\right)^3\left(e^{i(fx+e)}+i\right)a\,c^2 f}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2 f}$
default	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{a\,c^2 f}$
parallelrisch	$\frac{-6A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-6B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2A-2B}{3fa\,c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
norman	$\frac{\frac{2A-4B}{6acf}-\frac{4(2A-B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}+\frac{(2A-4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{(14A-16B)\left(\tan^2\right.}{6acf}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*I*(4*I*A*exp(I*(f*x+e))-2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))+2*A
-B)/(exp(I*(f*x+e))-I)^3/(exp(I*(f*x+e))+I)/a/c^2/f
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= -\frac{(2A-B)\cos(fx+e)^2+(2A-B)\sin(fx+e)-A+2B}{3\left(ac^2 f\cos(fx+e)\sin(fx+e)-ac^2 f\cos(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*
f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(51) = 102$.

Time = 2.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.17

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f} + \frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*
c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6
*A*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/
2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 +
f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 +
6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/
2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*
B*tan(e/2 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*ta
n(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 +
f*x/2) - 3*a*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin
(e) + c)**2), True))
```

3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(60) = 120$.

Time = 0.23 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.22

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} - \frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) +
1)^4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= -\frac{\frac{3\,(A-B)}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+B}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
-1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/
2*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)
/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.57.9 Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2\left(\frac{3\,B}{2} + A\cos(e+f\,x) + B\cos(e+f\,x) + 2\,A\sin(e+f\,x) - B\sin(e+f\,x) + A\cos(2\,e+2\,f\,x) - \frac{B}{}\right.}{3\,a\,c^2\,f\,(2\cos(e+f\,x) - \sin(2\,e+2\,f\,x))}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

output
```
(2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e
 + f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x
))/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f
*x)))
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

3.58.1 Optimal result 598
3.58.2 Mathematica [A] (verified) 598
3.58.3 Rubi [A] (verified) 599
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3.58.5 Fricas [A] (verification not implemented) 602
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3.58.8 Giac [A] (verification not implemented) 604
3.58.9 Mupad [B] (verification not implemented) 605

3.58.1 Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2(3A-2B)\tan(e+fx)}{15ac^3f}$$

output
```
1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/
f/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f
```

3.58.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)(80B+5(-9A+B)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+9A\cos(3(e+fx))-B\cdots}{240}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3),x]
```

output
```
-1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Co
s[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e +
 f*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] -
 24*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(a*c^3*f*(-1 + Sin[e + f*
x])^3*(1 + Sin[e + f*x]))
```

3.58.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^2}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{\frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5c} + \frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(3A-2B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))}\,dx}{5c} + \frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\downarrow 3151$$

$$\frac{\frac{(3A-2B)\left(\frac{2\int \sec^2(e+fx)dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(3A-2B)\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\downarrow 4254$$

$$\frac{\frac{(3A-2B)\left(\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}-\frac{2\int 1d(-\tan(e+fx))}{3cf}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\downarrow 24$$

$$\frac{\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}+\frac{(3A-2B)\left(\frac{2\tan(e+fx)}{3cf}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

output
```
(((A + B)*Sec[e + f*x])/(5*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*(Sec[e
 + f*x]/(3*f*(c - c*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*c*f)))/(5*c))/(a*
c)
```

3.58.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.58.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4\left(-3iA+12A\,e^{i(fx+e)}+10B\,e^{3i(fx+e)}+2iB-8B\,e^{i(fx+e)}-10iB\,e^{2i(fx+e)}+15iA\,e^{2i(fx+e)}\right)}{15\left(e^{i(fx+e)}-i\right)^5\left(e^{i(fx+e)}+i\right)fc^3a}$
parallelrisch	$\frac{-30A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(60A-30B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A+40B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-40B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A+8$ $15fc^3a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5$
derivativedivides	$\frac{-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{ac^3f}$
default	$\frac{-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{ac^3f}$
norman	$-\frac{12A+2B}{15afc}+\frac{2(6A-7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3afc}-\frac{2A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2(-8B+7A)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}+\frac{2(2A-B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2(9A-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(-3*I*A+12*A*exp(I*(f*x+e))+10*B*exp(3*I*(f*x+e))+2*I*B-8*B*exp(I*(f
*x+e))-10*I*B*exp(2*I*(f*x+e))+15*I*A*exp(2*I*(f*x+e)))/(exp(I*(f*x+e))-I)
^5/(exp(I*(f*x+e))+I)/f/c^3/a
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{4\,(3\,A-2\,B)\cos\left(fx+e\right)^2 - \left(2\,(3\,A-2\,B)\cos\left(fx+e\right)^2 - 9\,A+6\,B\right)\sin\left(fx+e\right) - 6\,A+9\,B}{15\left(ac^3 f\cos\left(fx+e\right)^3 + 2\,ac^3 f\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,ac^3 f\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A
+ 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f
*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. 2(85) = 170.

Time = 4.55 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output

```
Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60
*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**
3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60
*A*tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(
e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 +
f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f
*x/2)**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**
5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60
*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*t
an(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2
 + f*x/2) - 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c*
*3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*t
an(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*ta
n(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 +
 f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2
)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)
**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 7
5*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c*
*3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c...
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(98) = 196$.

Time $= 0.23$ (sec) , antiderivative size $= 423$, normalized size of antiderivative $= 4.15$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c - c\sin(e+fx))^3}\,dx =$$

$$-\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right.}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a}{(c}}}{15f}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1)
 - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.64

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{\frac{15\,(A-B)}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx+\right.}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{60\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x
+ 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 +
30*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*
f*x + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) +
63*A - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.58.9 Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{2\left(\frac{5\,B\,\sin(e+f\,x)}{2} - \frac{15\,A\,\cos(e+f\,x)}{4} - \frac{5\,B\,\cos(e+f\,x)}{8} - \frac{15\,A\,\sin(e+f\,x)}{4} - \frac{5\,B}{2} - 3\,A\,\cos(2\,e+2\,f\,x) + \frac{3\,A\,\cos(3\,e+3\,f\,x)}{4}\right.}{15\,a\,c^3\,f\,\left(\frac{\cos(3\,e+3\,f\,x)}{4} - \frac{5}{}\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

output
```
(2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 -
(15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*
f*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*
f*x) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*
f*x))/2))/(15*a*c^3*f*(cos(3*e + 3*f*x)/4 - (5*cos(e + f*x))/4 + sin(2*e +
 2*f*x)))
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

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3.59.1 Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)} + \frac{2(4A-3B)\tan(e+fx)}{35ac^4f}$$

output

```
1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/
f/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e)
)+2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f
```

3.59.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(560B+(-406A+182B)\cos(e$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x
)] + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)
] + 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*
A*Sin[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288
*B*Sin[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(224
0*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

3.59.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\downarrow 3042$$

$$\frac{(4A-3B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^2}dx}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 3151$$
$$\frac{(4A-3B)\left(\frac{3\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}dx}{5c} + \frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 3042$$
$$\frac{(4A-3B)\left(\frac{3\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))}dx}{5c} + \frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 3151$$
$$\frac{(4A-3B)\left(\frac{3\left(\frac{2\int \sec^2(e+fx)dx}{3c} + \frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c} + \frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 3042$$
$$\frac{(4A-3B)\left(\frac{3\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx}{3c} + \frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c} + \frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 4254$$
$$\frac{(4A-3B)\left(\frac{3\left(\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))} - \frac{2\int 1d(-\tan(e+fx))}{3cf}\right)}{5c} + \frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c} + \frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}$$
$$ac$$
$$\downarrow 24$$
$$\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3} + \frac{(4A-3B)\left(\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2} + \frac{3\left(\frac{2\tan(e+fx)}{3cf} + \frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}\right)}{7c}$$
$$ac$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

3.59. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

output
```
(((A + B)*Sec[e + f*x])/(7*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*(Sec[e
 + f*x]/(5*f*(c - c*Sin[e + f*x])^2) + (3*(Sec[e + f*x]/(3*f*(c - c*Sin[e
+ f*x])) + (2*Tan[e + f*x])/(3*c*f)))/(5*c)))/(7*c))/(a*c)
```

3.59.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.59.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
risch	$\frac{4i\left(56iA\,e^{3i(fx+e)}-42iB\,e^{3i(fx+e)}+35B\,e^{4i(fx+e)}-24iA\,e^{i(fx+e)}+56A\,e^{2i(fx+e)}+18iB\,e^{i(fx+e)}-42B\,e^{2i(fx+e)}-4A+\right.}{35\left(e^{i(fx+e)}-i\right)^7\left(e^{i(fx+e)}+i\right)a\,c^4f}$
parallelrisch	$\frac{-70A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-70B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-350A+140B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-210B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{35f\,c^4a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\right.}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{a\,c^4f}$
default	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-}{a\,c^4f}$
norman	$\frac{(6A-2B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{26A-2B}{35afc}-\frac{12(4A-3B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}-\frac{2A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{2(4A-18B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5afc}-\frac{4(3A-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}\right.\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNV
ERBOSE)
```

output
```
4/35*I*(56*I*A*exp(3*I*(f*x+e))-42*I*B*exp(3*I*(f*x+e))+35*B*exp(4*I*(f*x+
e))-24*I*A*exp(I*(f*x+e))+56*A*exp(2*I*(f*x+e))+18*I*B*exp(I*(f*x+e))-42*B
*exp(2*I*(f*x+e))-4*A+3*B)/(exp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)/a/c^4/f
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$$
$$=\frac{2\left(4\,A-3\,B\right)\cos\left(fx+e\right)^4-9\left(4\,A-3\,B\right)\cos\left(fx+e\right)^2+\left(6\left(4\,A-3\,B\right)\cos\left(fx+e\right)^2-20\,A+15\,B\right)\sin}{35\left(3\,ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)-\left(ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)\right)\sin\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*
A - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^
4*f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*
a*c^4*f*cos(f*x + e))*sin(f*x + e))
```

3.59.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. $2(122) = 244$.

Time = 9.50 (sec) , antiderivative size = 2468, normalized size of antiderivative = 17.38

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-70*A*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 21
0*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*
c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4
*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 21
0*A*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*ta
n(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 +
f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*A*tan(e/2 +
 f*x/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2
)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5
 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*tan(e/2 + f*x/2)**4/(3
5*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*
c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4
*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*t
an(e/2 + f*x/2) - 35*a*c**4*f) + 14*A*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan
(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2
 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f
*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2
) - 35*a*c**4*f) - 154*A*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x...
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(137) = 274$.

Time = 0.24 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.36

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{2\left(\frac{A\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{175\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{35\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-13\right)}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}} - \frac{}{ac^{}}\right)}{35\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

output
```
-2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/(a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{\frac{35\,(A-B)}{ac^4(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}+\frac{525\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^6+35\,B\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^6-1960\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+280\,B\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+4025\,A\tan(\frac{1}{2}\,f}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="giac")
```

output
```
-1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
 + 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5
+ 280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan
(1/2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x +
 1/2*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 -
 1176*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/
(a*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.59.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.68

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$$

$$=\frac{2\left(\frac{35\,B}{4}+\frac{91\,A\cos(e+f\,x)}{4}-\frac{7\,B\cos(e+f\,x)}{4}+14\,A\sin(e+f\,x)-\frac{21\,B\sin(e+f\,x)}{2}+14\,A\cos(2\,e+2\,f\,x)-\frac{39\,A\cos}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

output
```
(2*((35*B)/4 + (91*A*cos(e + f*x))/4 - (7*B*cos(e + f*x))/4 + 14*A*sin(e +
 f*x) - (21*B*sin(e + f*x))/2 + 14*A*cos(2*e + 2*f*x) - (39*A*cos(3*e + 3*
f*x))/4 - A*cos(4*e + 4*f*x) - (21*B*cos(2*e + 2*f*x))/2 + (3*B*cos(3*e +
3*f*x))/4 + (3*B*cos(4*e + 4*f*x))/4 - (91*A*sin(2*e + 2*f*x))/4 - 6*A*sin
(3*e + 3*f*x) + (13*A*sin(4*e + 4*f*x))/8 + (7*B*sin(2*e + 2*f*x))/4 + (9*
B*sin(3*e + 3*f*x))/2 - (B*sin(4*e + 4*f*x))/8))/(35*a*c^4*f*((7*cos(e + f
*x))/2 - (3*cos(3*e + 3*f*x))/2 - (7*sin(2*e + 2*f*x))/2 + sin(4*e + 4*f*x
)/4))
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

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3.60.1 Optimal result

Integrand size = 36, antiderivative size = 240

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} \\
&\quad + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} \\
&\quad - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} \\
&\quad + \frac{6a^4(4A-7B)c^5\cos^7(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

output

```
105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A
-7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a
+a*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6
*a^4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^
5*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))
```

3.60.2 Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(2048(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-1024(A-B)\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B
)*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) -
1024*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 + 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2
4*(95*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*
(A - 7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7
*A - 24*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

3.60.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.87, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow 3446$$

$$a^5c^5\int\frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\downarrow 3042$$

$$a^5c^5\int\frac{\cos(e+fx)^{10}(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\downarrow 3338$$

$$a^5c^5\left(-\frac{(4A-7B)\int\frac{\cos^{10}(e+fx)}{(\sin(e+fx)a+a)^6}dx}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\int\frac{\cos(e+fx)^{10}}{(\sin(e+fx)a+a)^6}dx}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3159$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\int\frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\int\frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^4}dx}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3159$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3158$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)}{3f(a\sin($$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)}{3f(a\sin($$

$$\downarrow 3161$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-$$

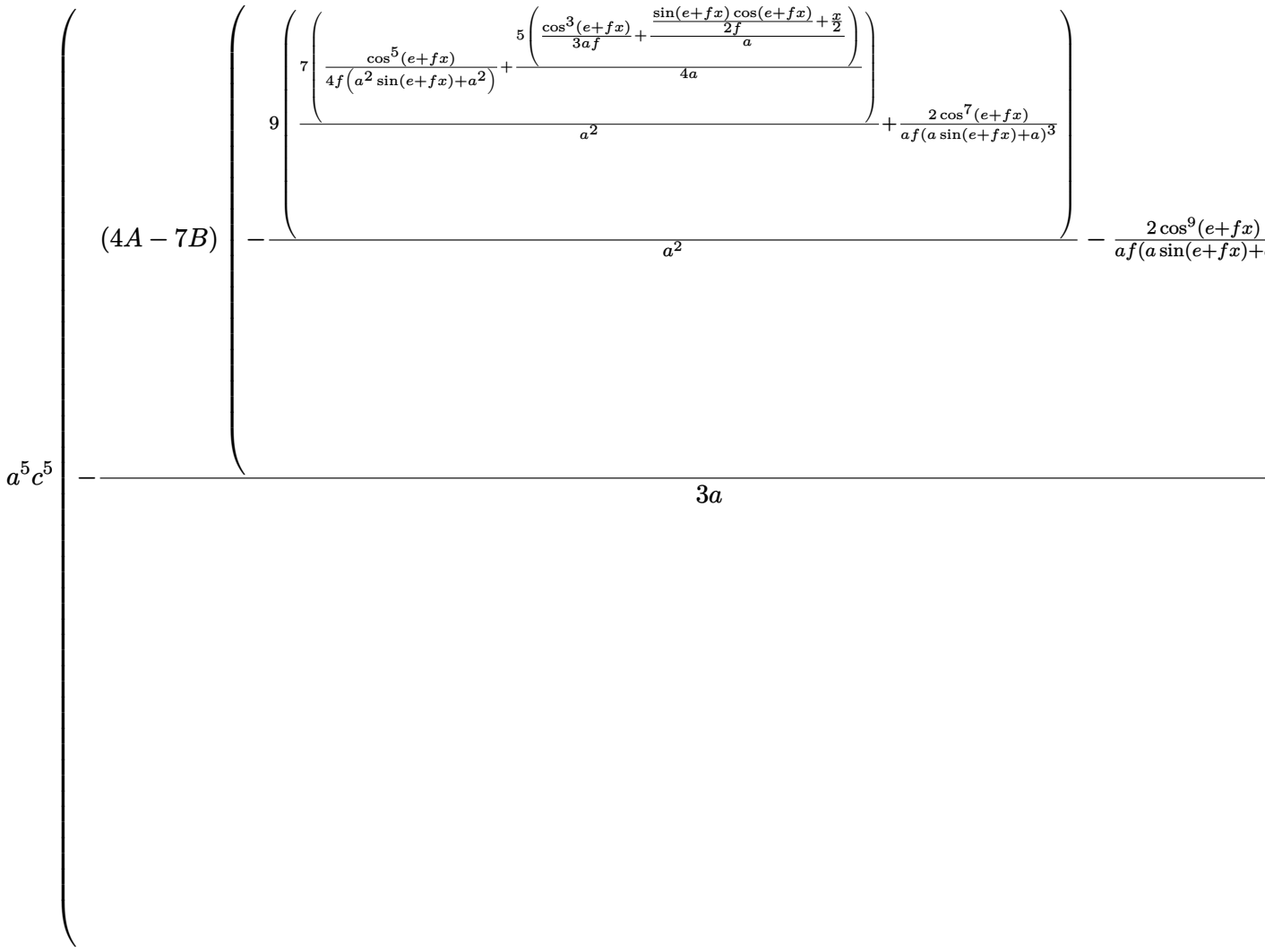
$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}\right.$$

$$\downarrow 3115$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+\right.}{af(a\sin(e+f}\right.}{3a}\right.$$

$$\downarrow 24$$



input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^5*c^5*(-1/3*((A - B)*Cos[e + f*x]^11)/(f*(a + a*Sin[e + f*x])^7) - ((4*A
 - 7*B)*((-2*Cos[e + f*x]^9)/(a*f*(a + a*Sin[e + f*x])^5) - (9*((2*Cos[e +
 f*x]^7)/(a*f*(a + a*Sin[e + f*x])^3) + (7*(Cos[e + f*x]^5/(4*f*(a^2 + a^2
*Sin[e + f*x])) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos[e + f*x]*Sin[e
+ f*x])/(2*f))/a))/(4*a)))/a^2))/a^2))/(3*a))
```

3.60.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.60.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.05

method	result
derivativedivides	$2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(23A-49B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{7A}{2}-\frac{103B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(71A-161B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\cdots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right.$
default	$2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(23A-49B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{7A}{2}-\frac{103B}{8}\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(71A-161B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\cdots}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right.$
parallelrisch	$-\frac{75c^5\left(\frac{\left(-84fxA+147fxB-\frac{2543}{15}A+\frac{4514}{15}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{5}+\frac{\left(-49fxB+28fxA+\frac{712}{45}B-\frac{481}{45}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{5}+\frac{\left(-84fxA+147f\right.}{\cdots}$
risch	$\frac{105c^5xA}{2a^2}-\frac{735c^5xB}{8a^2}+\frac{7ic^5e^{2i(fx+e)}A}{8a^2f}-\frac{3ic^5e^{2i(fx+e)}B}{a^2f}+\frac{95c^5e^{i(fx+e)}A}{8a^2f}-\frac{217c^5e^{i(fx+e)}B}{8a^2f}+\frac{95c^5e^{-i(fx}{8a^2f}$
norman	Expression too large to display

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^5/a^2*(((7/2*A-95/8*B)*tan(1/2*f*x+1/2*e)^7+(23*A-49*B)*tan(1/2*f*x+
1/2*e)^6+(7/2*A-103/8*B)*tan(1/2*f*x+1/2*e)^5+(71*A-161*B)*tan(1/2*f*x+1/2
*e)^4+(-7/2*A+103/8*B)*tan(1/2*f*x+1/2*e)^3+(215/3*A-497/3*B)*tan(1/2*f*x+
1/2*e)^2+(-7/2*A+95/8*B)*tan(1/2*f*x+1/2*e)+71/3*A-161/3*B)/(1+tan(1/2*f*x
+1/2*e)^2)^4+105/8*(4*A-7*B)*arctan(tan(1/2*f*x+1/2*e))-1/2*(-64*A+64*B)/(
tan(1/2*f*x+1/2*e)+1)^2-(-48*A+80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(64*A-64*B
)/(tan(1/2*f*x+1/2*e)+1)^3)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.54

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{6\,Bc^5\cos(fx+e)^6+4\,(2\,A-11\,B)c^5\cos(fx+e)^5+(76\,A-241\,B)c^5\cos(fx+e)^4-2\,(212\,A-431$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A
- 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(
4*A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A
- 3485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 214
1*B)*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(
f*x + e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x +
 3*(164*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)
*c^5*f*x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos
(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f
)*sin(f*x + e))
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. $2(224) = 448$.

Time = 21.52 (sec) , antiderivative size = 10608, normalized size of antiderivative = 44.20

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^5}{(a + a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((1260*A*c**5*f*x*tan(e/2 + f*x/2)**11/(24*a**2*f*tan(e/2 + f*x/2
)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 +
312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2
*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e
/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*tan(e/2
 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)
**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 4
32*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/
2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2
) + 24*a**2*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 +
f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)*
*9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528
*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*
tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2
+ f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c**5*f*x*t
an(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f
*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**
8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 5...
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2982 vs. $2(228) = 456$.

Time = 0.36 (sec) , antiderivative size = 2982, normalized size of antiderivative = 12.42

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^5}{(a + a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)
/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 22*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + a^2*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1
) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^...
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{315\,(4\,Ac^5-7\,Bc^5)(fx+e)}{a^2} + \frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-1\right.}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output

```
1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1
/2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e)
- 36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x +
 1/2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f
*x + 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x
+ 1/2*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2
*e)^5 + 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e
)^4 - 84*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 +
 1720*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5
- 1288*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

3.60.9 Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(391\,Ac^5-\frac{2729\,Bc^5}{4}\right)+\frac{494\,Ac^5}{3}-\frac{866\,Bc^5}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(103\,Ac^5-\frac{735\,Bc^5}{4}\right)+\tan\left(\frac{e}{2}+\frac{f}{2}\right.}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+3\,a^2\right.}$$

$$+\frac{105\,c^5\,\mathrm{atan}\left(\frac{105\,c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A-7B)}{420\,Ac^5-735\,Bc^5}\right)(4A-7B)}{4\,a^2\,f}$$

input

```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^2,x
)
```

output

```
(tan(e/2 + (f*x)/2)*(391*A*c^5 - (2729*B*c^5)/4) + (494*A*c^5)/3 - (866*B*
c^5)/3 + tan(e/2 + (f*x)/2)^10*(103*A*c^5 - (735*B*c^5)/4) + tan(e/2 + (f*
x)/2)^9*(323*A*c^5 - (2213*B*c^5)/4) + tan(e/2 + (f*x)/2)^7*(1332*A*c^5 -
2253*B*c^5) + tan(e/2 + (f*x)/2)^4*(1632*A*c^5 - 2943*B*c^5) + tan(e/2 + (
f*x)/2)^8*((2002*A*c^5)/3 - (3637*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*((4420*
A*c^5)/3 - (7621*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((2489*A*c^5)/3 - (17609
*B*c^5)/12) + tan(e/2 + (f*x)/2)^6*((4594*A*c^5)/3 - (16805*B*c^5)/6) + ta
n(e/2 + (f*x)/2)^5*((6274*A*c^5)/3 - (21299*B*c^5)/6))/(f*(7*a^2*tan(e/2 +
 (f*x)/2)^2 + 13*a^2*tan(e/2 + (f*x)/2)^3 + 18*a^2*tan(e/2 + (f*x)/2)^4 +
22*a^2*tan(e/2 + (f*x)/2)^5 + 22*a^2*tan(e/2 + (f*x)/2)^6 + 18*a^2*tan(e/2
 + (f*x)/2)^7 + 13*a^2*tan(e/2 + (f*x)/2)^8 + 7*a^2*tan(e/2 + (f*x)/2)^9 +
 3*a^2*tan(e/2 + (f*x)/2)^10 + a^2*tan(e/2 + (f*x)/2)^11 + a^2 + 3*a^2*tan
(e/2 + (f*x)/2))) + (105*c^5*atan((105*c^5*tan(e/2 + (f*x)/2)*(4*A - 7*B))
/(420*A*c^5 - 735*B*c^5))*(4*A - 7*B))/(4*a^2*f)
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

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3.61.1 Optimal result

Integrand size = 36, antiderivative size = 180

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{35(A-2B)c^4x}{2a^2} + \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}$$
$$- \frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}$$

output
```
35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^
4*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*
x+e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4
*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

3.61.2 Mathematica [A] (verified)

Time = 11.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-64(A-B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)
*Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128
*(5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 21
0*(A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A -
71*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e +
f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

3.61.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^4 (A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^4 (A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$a^4c^4 \int \frac{\cos^8(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^6}\,dx$$

$$\downarrow 3042$$

$$a^4c^4 \int \frac{\cos(e+fx)^8(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^6}\,dx$$

$$\downarrow 3338$$

$$a^4c^4 \left(-\frac{(A-2B)\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^5}\,dx}{a} - \frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6} \right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(A-2B)\int\frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^5}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3159$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3159$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3161$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a}\right.$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+\right.$$

$$\downarrow 3115$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)-\frac{(A-B)\cos$$

$$\downarrow 24$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9($$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^4*c^4*(-1/3*((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^6) - ((A -
2*B)*((-2*Cos[e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^4) - (7*((2*Cos[e + f*
x]^5)/(a*f*(a + a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (
Cos[e + f*x]*Sin[e + f*x])/(2*f))/a))/a^2))/a^2))/a)
```

3.61.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.61.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-17B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(12A-36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+3}{fa^2}$
default	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6A-17B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(12A-36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+3}{fa^2}$
parallelrisch	$-\frac{21c^4\left(\left(-20fxA+40fxB-\frac{277}{7}A+\frac{1712}{21}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(-40fxB+20fxA+\frac{290}{21}B-\frac{191}{21}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+(-20fxA+4\right.}{}$
risch	$\frac{35c^4xA}{2a^2}-\frac{35c^4xB}{a^2}+\frac{ic^4e^{2i(fx+e)}A}{8a^2f}-\frac{3ic^4e^{2i(fx+e)}B}{4a^2f}+\frac{3c^4e^{i(fx+e)}A}{a^2f}-\frac{71c^4e^{i(fx+e)}B}{8a^2f}+\frac{3c^4e^{-i(fx+e)}A}{a^2f}-$
norman	$\frac{\left(111Ac^4-212Bc^4\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(33Ac^4-70Bc^4\right)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(979Ac^4-2004Bc^4\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{\left(1893Ac^4-\right.}{}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output

```
2/f*c^4/a^2*(((1/2*A-3*B)*tan(1/2*f*x+1/2*e)^5+(6*A-17*B)*tan(1/2*f*x+1/2*
e)^4+(12*A-36*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+3*B)*tan(1/2*f*x+1/2*e)+6*A-
53/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+35/2*(A-2*B)*arctan(tan(1/2*f*x+1/2*e))
-1/2*(-32*A+32*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-16*A+32*B)/(tan(1/2*f*x+1/2*e
)+1)-1/3*(32*A-32*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{2\,Bc^4\cos(fx+e)^5 - (3\,A-16\,B)c^4\cos(fx+e)^4 + 2\,(15\,A-38\,B)c^4\cos(fx+e)^3 - 210\,(A-2\,B)c^4f}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A -
38*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(
A - 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^
4*f*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*
A - 14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4
*cos(f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218
*B)*c^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. 2(175) = 350.

Time = 12.86 (sec) , antiderivative size = 7337, normalized size of antiderivative = 40.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

output

```
Piecewise((105*A*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**
9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**
2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/
2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*
x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36
*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*ta
n(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*
x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a
**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9
+ 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2
+ f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x
/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan
(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x
/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
*2*f) + 1260*A*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9
+ 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a*...
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(172) = 344$.

Time $= 0.35$ (sec) , antiderivative size $= 2094$, normalized size of antiderivative $= 11.63$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output

```
1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) +
 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^
2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x
+ e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x...
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(172) = 344$.

Time $= 0.33$ (sec) , antiderivative size $= 349$, normalized size of antiderivative $= 1.94$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^2}\,dx$$

$$= \frac{105\,(Ac^4 - 2\,Bc^4)(fx+e)}{a^2} + \frac{2\left(99\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8 - 210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8 + 333\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 - 636\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 + 533\,Ac\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e
)^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7
- 636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 11
60*B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980
*B*c^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*
c^4*tan(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^
4*tan(1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*t
an(1/2*f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2
*f*x + 1/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

3.61.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(131\,A\,c^4-260\,B\,c^4\right)+\frac{164\,A\,c^4}{3}-110\,B\,c^4+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(33\,A\,c^4-70\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right.}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+3\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+6\right.}$$

$$+\frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A-2\,B)}{35\,A\,c^4-70\,B\,c^4}\right)(A-2\,B)}{a^2\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 +
tan(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c
^4 - 212*B*c^4) + tan(e/2 + (f*x)/2)^2*(217*A*c^4 - 448*B*c^4) + tan(e/2 +
 (f*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 66
0*B*c^4) + tan(e/2 + (f*x)/2)^6*((533*A*c^4)/3 - (1160*B*c^4)/3) + tan(e/2
 + (f*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3))/(f*(6*a^2*tan(e/2 + (f*x)/2)^2
 + 10*a^2*tan(e/2 + (f*x)/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(
e/2 + (f*x)/2)^5 + 10*a^2*tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^
7 + 3*a^2*tan(e/2 + (f*x)/2)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*ta
n(e/2 + (f*x)/2))) + (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(3
5*A*c^4 - 70*B*c^4))*(A - 2*B))/(a^2*f)
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

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3.62.1 Optimal result

Integrand size = 36, antiderivative size = 162

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{5(2A-5B)c^3x}{2a^2} + \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} \\
&\quad + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{5(2A-5B)c^3\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

output
```
5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c
^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a
+a*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

3.62.2 Mathematica [A] (verified)

Time = 11.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^3\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-32(A-B)\left(\cos\left(\right.\right.\right.}{}
\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*
Sin[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(
7*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*
(2*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*
B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

3.62.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(-\frac{(2A-5B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^4}dx}{3a} - \frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(2A-5B)\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^4}dx}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\int\frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(\frac{(2A-5B)\left(-\frac{5\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(\frac{(2A-5B)\left(-\frac{5\left(\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^3*c^3*(-1/3*((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^5) - ((2*A
- 5*B)*((-2*Cos[e + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^3) - (5*((3*(x/a + C
os[e + f*x]/(a*f)))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^2*Sin[e + f*x])))
)/a^2))/(3*a))
```

3.62.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.62.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{fa^2}$
default	$\frac{2c^3\left(\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+(A-5B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-5B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{5(2A-5B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{-16A+16}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{fa^2}$
parallelrisch	$-\frac{c^3\left(\left(-30fxA+75fxB-56A+\frac{607}{4}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(10fxA-25fxB-\frac{19}{3}A+\frac{115}{12}B\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+(-30fxA+75fx\right.}{2fa^2\left(3\right.}$
risch	$\frac{5c^3xA}{a^2}-\frac{25c^3xB}{2a^2}-\frac{iBc^3e^{2i(fx+e)}}{8a^2f}+\frac{c^3e^{i(fx+e)}A}{2a^2f}-\frac{5c^3e^{i(fx+e)}B}{2a^2f}+\frac{c^3e^{-i(fx+e)}A}{2a^2f}-\frac{5c^3e^{-i(fx+e)}B}{2a^2f}+\frac{iB}{}$
norman	$\frac{\left(8Ac^3-25Bc^3\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(34Ac^3-77Bc^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(38Ac^3-93Bc^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(148Ac^3-352Bc^3\right)\left(\tan\right.}{af}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/f*c^3/a^2*((-1/2*B*tan(1/2*f*x+1/2*e)^3+(A-5*B)*tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)+A-5*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A-5*B)*arctan(tan(1/2*f*x+1/2*e))-1/2*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-4*A+12*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.80

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{3\,Bc^3\cos(fx+e)^4+6\,(A-4\,B)c^3\cos(fx+e)^3-30\,(2\,A-5\,B)c^3fx+16\,(A-B)c^3+(15\,(2\,A-5\,B)}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5
*B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^
3)*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x
 + e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c
^3*cos(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 7
9*B)*c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*
x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(148) = 296$.

Time = 7.42 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.80

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7
 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2
*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(
e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a
**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*ta
n(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 2
10*A*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*
f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
+ f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/
2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5...
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. 2(152) = 304.

Time = 0.33 (sec) , antiderivative size = 1378, normalized size of antiderivative = 8.51

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1)
+ 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) +
 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e...
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{15\,(2\,Ac^3-5\,Bc^3)(fx+e)}{a^2} - \frac{6\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^
3 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c
^3*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*
e)^2 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A
*c^3 - 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.62.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(38Ac^3-93Bc^3\right)+\frac{46Ac^3}{3}-\frac{118Bc^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(8Ac^3-25Bc^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(34}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+3a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+5a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+7\right.}$$
$$+\frac{5c^3\operatorname{atan}\left(\frac{5c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2A-5B)}{10Ac^3-25Bc^3}\right)(2A-5B)}{a^2f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 +
 tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + tan(e/2 + (f*x)/2)^5*(34*A*c^
3 - 77*B*c^3) + tan(e/2 + (f*x)/2)^3*(72*A*c^3 - 166*B*c^3) + tan(e/2 + (f
*x)/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + tan(e/2 + (f*x)/2)^2*((128*A*c^
3)/3 - (359*B*c^3)/3))/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f
*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2
*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (
f*x)/2))) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 -
 25*B*c^3))*(2*A - 5*B))/(a^2*f)
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

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3.63.1 Optimal result

Integrand size = 36, antiderivative size = 108

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx\\ &=\frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}\\ &\quad-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

output
```
(A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e
)^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(108) = 216.

Time = 11.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.17

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx\\ &=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-4(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+\right.\right.}{}\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A
- B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

3.63.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3338$$

$$a^2c^2\left(-\frac{(A-4B)\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^3}\,dx}{3a} - \frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(-\frac{(A-4B)\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^3}dx}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 3161$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^2*c^2*(-1/3*((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^4) - ((A -
4*B)*((-3*(x/a + Cos[e + f*x]/(a*f)))/a^2 - (2*Cos[e + f*x]^3)/(a*f*(a + a
*Sin[e + f*x])^2)))/(3*a))
```

3.63.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.63.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
default	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
risch	$\frac{c^2xA}{a^2}-\frac{4c^2xB}{a^2}-\frac{Bc^2e^{i(fx+e)}}{2a^2f}-\frac{Bc^2e^{-i(fx+e)}}{2a^2f}+\frac{8iAc^2e^{i(fx+e)}+8Ac^2e^{2i(fx+e)}-24iBc^2e^{i(fx+e)}-16Bc^2e^{2i}}{f\,a^2\left(e^{i(fx+e)}+i\right)^3}$
parallelrisch	$\frac{3c^2\left(\left(fxA-4fxB+\frac{4}{3}A-8B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(-\frac{11}{6}+4fx\right)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+\left(fxA-4fxB+\frac{4}{3}A-\frac{14}{3}B\right)\sin\left(\frac{fx}{2}\right.\right.}{f\,a^2\left(3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\right.}$
norman	$\frac{c^2(A-4B)x}{a}+\frac{\left(8Ac^2-30Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c^2(A-4B)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{8Ac^2-38Bc^2}{3af}-\frac{8Bc^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{2\left(4Ac^2-61\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^2/a^2*(-1/2*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A-8*B)/(tan(1
/2*f*x+1/2*e)+1)^3-4*B/(tan(1/2*f*x+1/2*e)+1)-B/(1+tan(1/2*f*x+1/2*e)^2)+(
A-4*B)*arctan(tan(1/2*f*x+1/2*e)))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(104) = 208.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{3\,Bc^2\cos(fx+e)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2)\cos(fx+}{3\left(a^2f\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A
 - 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x
+ 2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*
x + e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(
f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2
*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(102) = 204$.

Time = 3.98 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.91

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^2}{(a + a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5
+ 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*
x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f
*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3
/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*
tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f
*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 +
f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9
*A*c**2*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(
e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) + 24*A*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 ...
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(104) = 208.

Time = 0.32 (sec) , antiderivative size = 833, normalized size of antiderivative = 7.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
+ 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2
) - A*c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)...
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(Ac^2-4\,Bc^2)(fx+e)}{a^2} - \frac{6\,Bc^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2} - \frac{8\left(3\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ac^2+4\,Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2
+ 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*
e) + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1
/2*e) + 1)^3))/f
```

3.63.9 Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(8Ac^2-30Bc^2\right)+\frac{8Ac^2}{3}-\frac{38Bc^2}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(8Ac^2-26Bc^2\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{8Ac^2}{3}\right.}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+3a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3a^2\right.}$$
$$+\frac{2c^2\operatorname{atan}\left(\frac{2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A-4B)}{2Ac^2-8Bc^2}\right)(A-4B)}{a^2 f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(8*A*c^2 - 30*B*c^2) + (8*A*c^2)/3 - (38*B*c^2)/3 + ta
n(e/2 + (f*x)/2)^3*(8*A*c^2 - 26*B*c^2) + tan(e/2 + (f*x)/2)^2*((8*A*c^2)/
3 - (74*B*c^2)/3) - 8*B*c^2*tan(e/2 + (f*x)/2)^4)/(f*(4*a^2*tan(e/2 + (f*x
)/2)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan
(e/2 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2))) + (2*c^2*atan((2*c^2*
tan(e/2 + (f*x)/2)*(A - 4*B))/(2*A*c^2 - 8*B*c^2))*(A - 4*B))/(a^2*f)
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

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3.64.1 Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{Bcx}{a^2}+\frac{(A-7B)c\cos(e+fx)}{3a^2f(1+\sin(e+fx))} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

output
```
-B*c*x/a^2+1/3*(A-7*B)*c*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-2/3*(A-B)*c*cos(f
*x+e)/f/(a+a*sin(f*x+e))^2
```

3.64.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(72) = 144.

Time = 6.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.17

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e+\frac{3}{}\right.\right.}{6a^2f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*
f*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Si
n[(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^
2*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

3.64.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3336, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^3}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^3}\,dx$$

$$\downarrow 3336$$

$$ac\left(-\frac{\int \frac{a(A-4B)+3aB\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{3a^3} - \frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\downarrow 3042$$

$$ac\left(-\frac{\int \frac{a(A-4B)+3aB\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{3a^3} - \frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\downarrow 3214$$

$$ac\left(-\frac{a(A-7B)\int \frac{1}{\sin(e+fx)a+a}\,dx + 3Bx}{3a^3} - \frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\downarrow 3042$$

$$ac\left(-\frac{a(A-7B)\int \frac{1}{\sin(e+fx)a+a}dx+3Bx}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\downarrow 3127$$

$$ac\left(-\frac{3Bx-\frac{a(A-7B)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
a*c*((-2*(A - B)*Cos[e + f*x])/(3*a*f*(a + a*Sin[e + f*x])^2) - (3*B*x - (
a*(A - 7*B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))/(3*a^3))
```

3.64.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.64.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{Bcx}{a^2}+\frac{2Ace^{2i(fx+e)}-8iBce^{i(fx+e)}-6Bce^{2i(fx+e)}-\frac{2Ac}{3}+\frac{14Bc}{3}}{fa^2\left(e^{i(fx+e)}+i\right)^3}$
derivativedivides	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa^2}$
default	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fa^2}$
parallelrisch	$-\frac{2\left(\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(\frac{3}{2}fxB+A+B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+B\left(\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}+\frac{5B}{3}\right)c}{fa^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$-\frac{2Ac+10Bc}{3af}-\frac{Bcx}{a}-\frac{16Bc\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{8Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{(14Ac+22Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(10Ac+2}{}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output

```
-B*c*x/a^2+2/3*(3*A*c*exp(2*I*(f*x+e))-12*I*B*c*exp(I*(f*x+e))-9*B*c*exp(2
*I*(f*x+e))-A*c+7*B*c)/f/a^2/(exp(I*(f*x+e))+I)^3
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(68) = 136$.

Time $= 0.26$ (sec) , antiderivative size $= 166$, normalized size of antiderivative $= 2.31$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{(a + a\sin(e+fx))^2}\,dx$$
$$= \frac{6\,Bcfx - (3\,Bcfx + (A-7\,B)c)\cos(fx+e)^2 + 2\,(A-B)c + (3\,Bcfx + (A+5\,B)c)\cos(fx+e) + (6\,}{3\left(a^2 f\cos(fx+e)^2 - a^2 f\cos(fx+e) - 2\,a^2 f - (a^2 f\cos(fx\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c +
(3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c
*f*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^
2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(70) = 140$.

Time $= 2.09$ (sec) , antiderivative size $= 702$, normalized size of antiderivative $= 9.75$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{(a + a\sin(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{6Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2 f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2 f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2 f} - \frac{2Ac}{3a^2 f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2 f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2 f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2} \\ \frac{x(A+B\sin(e))(-c\sin(e)+c)}{(a\sin(e)+a)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(
3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
 3*a**2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 +
 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*
B*c*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*
tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*
x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
24*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
 f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(
e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
 + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)*
*2, True))
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(68) = 136$.

Time $= 0.30$ (sec) , antiderivative size $= 452$, normalized size of antiderivative $= 6.28$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))}{(a + a\sin(e+fx))^2}\,dx =$$

$$-\frac{2\left(Bc\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\right.}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(}}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
) + B*c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{\frac{3(fx+e)Bc}{a^2}+\frac{2\left(3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac+5Bc\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
-1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/
2*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/
2*f*x + 1/2*e) + 1)^3))/f
```

3.64.9 Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{Bcx}{a^2} - \frac{\left(\frac{c(6A+6B+9B(e+fx))}{3}-3Bc(e+fx)\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+\left(\frac{c(24B+9B(e+fx))}{3}-3Bc(e+fx)\right)\tan\left(\frac{e}{2}+\right.}{a^2 f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
- (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 -
 3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B
*c*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x))/(a^2*f
*(tan(e/2 + (f*x)/2) + 1)^3)
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

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3.65.1 Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))} + \frac{(2A+B)\tan(e+fx)}{3a^2cf}$$

output
```
-1/3*(A-B)*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+1/3*(2*A+B)*tan(f*x+e)/a^2/
c/f
```

3.65.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{\cos(e+fx)(-6B-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-4B\sin(e+f$$
$$12a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)),x]
```

output
```
(Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)
] - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e
+ f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sin(e+fx)a+a}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(\sin(e+fx)a+a)}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{\frac{(2A+B)\int \sec^2(e+fx)dx}{3a} - \frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(2A+B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx}{3a} - \frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\downarrow 4254$$

$$\frac{-\frac{(2A+B)\int 1d(-\tan(e+fx))}{3af} - \frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\downarrow 24$$

$$\frac{\frac{(2A+B)\tan(e+fx)}{3af} - \frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x])/(f*(a + a*Sin[e + f*x])) + ((2*A + B)*Tan[e +
 f*x])/(3*a*f))/(a*c)
```

3.65.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.65.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

method	result
risch	$\frac{2i\left(4iA\,e^{i(fx+e)}+2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2A-B\right)}{3\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)a^2cf}$
parallelrisch	$\frac{-6A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-6A-6B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2A-2B}{3f\,a^2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
derivativedivides	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2cf}$
default	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2cf}$
norman	$\frac{-\frac{2A+4B}{6acf}-\frac{4(2A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}+\frac{A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{(2A+4B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2acf}-\frac{(8A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}-\frac{(14A+16B)\left(\tan\right.}{6acf}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(4*I*A*exp(I*(f*x+e))+2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))-2*A-B)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)/a^2/c/f
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= -\frac{(2\,A+B)\cos(fx+e)^2-(2\,A+B)\sin(fx+e)-A-2\,B}{3\,(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e))}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(51) = 102$.

Time = 2.22 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.32

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a*
*2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6
*A*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/
2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 +
f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 -
6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/
2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*
B*tan(e/2 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*ta
n(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 +
f*x/2) - 3*a**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*
sin(e) + c)), True))
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(58) = 116$.

Time = 0.22 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.27

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

output
```
2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= -\frac{\frac{3\,(A+B)}{a^2c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)} + \frac{9\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-3\,B\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2+12\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+7\,A-B}{a^2c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)^3}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

output
```
-1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.65.9 Mupad [B] (verification not implemented)

Time = 12.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{2\left(\frac{3B}{2} - A\cos(e+fx) + B\cos(e+fx) + 2A\sin(e+fx) + B\sin(e+fx) - A\cos(2e+2fx) - \frac{B}{}\right.}{3a^2cf\,(2\cos(e+fx)+\sin(2e+2fx))}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

output
```
(2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e
 + f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x
))/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f
*x)))
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

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3.66.1 Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}$$

output
```
1/3*B*sec(f*x+e)^3/a^2/c^2/f+A*tan(f*x+e)/a^2/c^2/f+1/3*A*tan(f*x+e)^3/a^2
/c^2/f
```

3.66.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{a^2c^2f}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^2),x]
```

output
```
(B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(
a^2*c^2*f)
```

3.66.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3148$$

$$\frac{A\int \sec^4(e+fx)dx + \frac{B\sec^3(e+fx)}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^4 dx + \frac{B\sec^3(e+fx)}{3f}}{a^2c^2}$$

$$\downarrow 4254$$

$$\frac{\frac{B\sec^3(e+fx)}{3f} - \frac{A\int(\tan^2(e+fx)+1)d(-\tan(e+fx))}{f}}{a^2c^2}$$

$$\downarrow 2009$$

$$\frac{\frac{B\sec^3(e+fx)}{3f} - \frac{A(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx))}{f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x
]
```

output
```
((B*Sec[e + f*x]^3)/(3*f) - (A*(-Tan[e + f*x] - Tan[e + f*x]^3/3))/f)/(a^2
*c^2)
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.66.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

method	result
risch	$\frac{4iA\,e^{2i(fx+e)}+\frac{8B\,e^{3i(fx+e)}}{3}+\frac{4iA}{3}}{\left(e^{i(fx+e)}-i\right)^3\left(e^{i(fx+e)}+i\right)^3a^2c^2f}$
parallelrisch	$\frac{6A\sin(fx+e)+2A\sin(3fx+3e)+3\cos(fx+e)B+\cos(3fx+3e)B+4B}{3a^2c^2f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2c^2f}$
default	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2c^2f}$
norman	$\frac{-\frac{2B}{3acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}-\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{2B\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{acf}-\frac{2B\left(\tan^2\left(\frac{fx}{2}+\right.\right.}{3acf}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
4/3*(3*I*A*exp(2*I*(f*x+e))+2*B*exp(3*I*(f*x+e))+I*A)/(exp(I*(f*x+e))-I)^3
/(exp(I*(f*x+e))+I)^3/a^2/c^2/f
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\left(2\,A\cos\left(fx+e\right)^2+A\right)\sin\left(fx+e\right)+B}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(56) = 112$.

Time = 2.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 7.56

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{6A\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f} + \frac{4A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\ta} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9
*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a
**2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 -
 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3
*a**2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 -
9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*
a**2*c**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6
- 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 -
3*a**2*c**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*ta
n(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne
(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\left(\tan(fx+e)^3+3\tan(fx+e)\right)A}{a^2c^2} + \frac{B}{a^2c^2\cos(fx+e)^3}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x +
e)^3))/f
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^2c^2f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/
2*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2
- 1)^3*a^2*c^2*f)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(3A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+3B\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+3A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+B\right)}{3a^2c^2f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)^3}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x
)
```

output
```
-(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 +
 (f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^
2 - 1)^3)
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

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3.67.1 Optimal result

Integrand size = 36, antiderivative size = 93

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{(4A-B)\tan(e+fx)}{5a^2c^3f} + \frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}$$

output
```
1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a
^2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f
```

3.67.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(93) = 186.

Time = 2.00 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.55

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-240B+54(A+B)\cos(e+f$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^3),x]
```

output

```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*
A*Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*
Cos[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e
+ f*x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e +
 f*x)] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1
+ Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)
```

3.67.3 Rubi [A] (verified)

Time $= 0.54$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 0.83$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used $= \{3042, 3446, 3042, 3338, 3042, 4254, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{a^2c^2}$$

$$\downarrow 3338$$

$$\frac{\frac{(4A-B)\int \sec^4(e+fx)dx}{5c} + \frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{(4A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4 dx}{5c} + \frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}}{a^2c^2}$$

$$\downarrow 4254$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{(4A-B)\int(\tan^2(e+fx)+1)d(-\tan(e+fx))}{5cf}}{a^2c^2}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{(4A-B)(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx))}{5cf}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])) - ((4*A - B)*(-Tan[e
+ f*x] - Tan[e + f*x]^3/3))/(5*c*f))/(a^2*c^2)
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.67.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{4i\left(24iA\,e^{3i(fx+e)}-6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}+8A\,e^{2i(fx+e)}-2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}+4A-B\right.}{15\left(e^{i(fx+e)}+i\right)^3\left(e^{i(fx+e)}-i\right)^5 f\,c^3a^2}$
parallelrisch	$\frac{-30A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(30A-30B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10A+20B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-50A-10B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(}{15f\,c^3a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\tan\right.}$
derivativedivides	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{5A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}}{a^2c^3f}$
default	$\frac{-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{5A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}}{a^2c^3f}$
norman	$\frac{6A-4B}{10afc}-\frac{4(4A-B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15afc}-\frac{A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{(14A-16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10afc}+\frac{(6A-4B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2afc}+\frac{(2A-8B)(}{}$ $\frac{}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\right.\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
-4/15*I*(24*I*A*exp(3*I*(f*x+e))-6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+
e))+8*I*A*exp(I*(f*x+e))+8*A*exp(2*I*(f*x+e))-2*I*B*exp(I*(f*x+e))-2*B*exp
(2*I*(f*x+e))+4*A-B)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)^5/f/c^3/a^2
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(4A-B\right)\cos\left(fx+e\right)^4 - \left(4A-B\right)\cos\left(fx+e\right)^2 + \left(2\left(4A-B\right)\cos\left(fx+e\right)^2 + 4A - B\right)\sin\left(fx+\right.}{15\left(a^2c^3f\cos\left(fx+e\right)^3\sin\left(fx+e\right) - a^2c^3f\cos\left(fx+e\right)^3\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B
)*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e
)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

3.67.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.

Time = 9.03 (sec) , antiderivative size = 2674, normalized size of antiderivative = 28.75

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

output

```
Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 +
 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 +
 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15
*a**2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)*
*8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)*
*6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)*
*3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2)
- 15*a**2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x
/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x
/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x
/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x
/2) - 15*a**2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 +
 f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 +
 f*x/2) - 15*a**2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e
/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e
/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e
/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*ta...
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(88) = 176$.

Time $= 0.24$ (sec) , antiderivative size $= 651$, normalized size of antiderivative $= 7.00$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7} + 3\right)}{a^2c^3 - \frac{2a^2c^3\sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^2c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{6a^2c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{6a^2c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{2a^2c^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{2a^2c^3\sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{a^2c^3\sin(fx+e)^8}{(\cos(fx+e)+1)^8}} - \frac{}{a^2}}{15\,f}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^
2*c^3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
6*a^2*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*
c^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e)
 + 1) - 9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3
- 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time $= 0.34$ (sec) , antiderivative size $= 221$, normalized size of antiderivative $= 2.38$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{5\left(15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-9B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-12B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13A-7B\right)}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{165A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45B\tan}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A
*tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(
tan(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/
2*f*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e
)^3 + 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*t
an(1/2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(
tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.67.9 Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.97

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\frac{8A}{15} - \frac{2B}{15} - \frac{16A\sin(e+fx)}{15} + \frac{4B\sin(e+fx)}{15}\right)\cos(e+fx)^2 + \frac{2A}{15} - \frac{8B}{15} - \frac{8A\sin(e+fx)}{15} + \frac{2B\sin(e+fx)}{15}}{a^2c^3f\left(2\cos(e+fx)^3\sin(e+fx) - 2\cos(e+fx)^3\right)}$$
$$- \frac{\frac{2A}{5} + \frac{2B}{5} - \frac{2A\sin(e+fx)}{5} - \frac{2B\sin(e+fx)}{5}}{a^2c^3f\left(2\sin(e+fx) - 2\right)} - \frac{\cos(e+fx)\left(\frac{16A}{15} - \frac{4B}{15}\right)}{a^2c^3f\left(2\sin(e+fx) - 2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x
)
```

output
```
((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos
(e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f
*x))/15))/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) -
 ((2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^
3*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3
*f*(2*sin(e + f*x) - 2))
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

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3.68.1 Optimal result

Integrand size = 36, antiderivative size = 135

$$\begin{aligned}&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx\\&=\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}\\&\quad+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}\end{aligned}$$

output
```
1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x
+e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105
*(5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f
```

3.68.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

Time = 2.40 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2688B+42(25A+4B)\cos$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^4),x]
```

output
```
-1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[
2*(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos
[4*(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[
5*(e + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e
 + f*x)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e
 + f*x)] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e
 + f*x)] - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + S
in[e + f*x])^2)
```

3.68.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$\downarrow 3338$$

$$\frac{\frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7c} + \frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{(5A-2B)\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{7c} + \frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}$$
$$a^2c^2$$

$$\downarrow 3151$$

$$\frac{(5A-2B)\left(\frac{4\int \sec^4(e+fx)dx}{5c} + \frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c} + \frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}$$
$$a^2c^2$$

$$\downarrow 3042$$

$$\frac{(5A-2B)\left(\frac{4\int \csc\left(e+fx+\frac{\pi}{2}\right)^4 dx}{5c} + \frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c} + \frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}$$
$$a^2c^2$$

$$\downarrow 4254$$

$$\frac{(5A-2B)\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))} - \frac{4\int \left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5cf}\right)}{7c} + \frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}$$
$$a^2c^2$$

$$\downarrow 2009$$

$$\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2} + \frac{(5A-2B)\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))} - \frac{4\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5cf}\right)}{7c}$$
$$a^2c^2$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^2) + ((5*A - 2*B)*(Sec
[e + f*x]^3/(5*f*(c - c*Sin[e + f*x])) - (4*(-Tan[e + f*x] - Tan[e + f*x]^
3/3))/(5*c*f)))/(7*c))/(a^2*c^2)
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.68.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{16\left(-6iB\,e^{2i(fx+e)}+2iB+70iA\,e^{4i(fx+e)}-28iB\,e^{4i(fx+e)}+20A\,e^{i(fx+e)}+40A\,e^{3i(fx+e)}+42B\,e^{5i(fx+e)}+15iA\,e^{2i(f}{105\left(e^{i(fx+e)}-i\right)^7\left(e^{i(fx+e)}+i\right)^3a^2c^4f}$
parallelrisch	$\frac{-210A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(420A-210B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-280A+280B)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-560A-280B)\left(\tan^6\left(\frac{f}{2}\right.\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a^2c^4f}$
default	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a^2c^4f}$
norman	$-\frac{20A+6B}{35afc}-\frac{4(10A+11B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15afc}-\frac{2A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{2(30A-47B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35afc}-\frac{2(7A-4B)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3afc}+\frac{2}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
-16/105*(-6*I*B*exp(2*I*(f*x+e))+2*I*B+70*I*A*exp(4*I*(f*x+e))-28*I*B*exp(
4*I*(f*x+e))+20*A*exp(I*(f*x+e))+40*A*exp(3*I*(f*x+e))+42*B*exp(5*I*(f*x+e
))+15*I*A*exp(2*I*(f*x+e))-8*B*exp(I*(f*x+e))-16*B*exp(3*I*(f*x+e))-5*I*A)
/(exp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)^3/a^2/c^4/f
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{16\,(5\,A-2\,B)\cos(fx+e)^4-8\,(5\,A-2\,B)\cos(fx+e)^2-\left(8\,(5\,A-2\,B)\cos(fx+e)^4-12\,(5\,A-2\right.}{105\left(a^2c^4f\cos(fx+e)^5+2\,a^2c^4f\cos(fx+e)^3\sin(fx+e)-2\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*
(5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*
sin(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*
x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4228 vs. $2(122) = 244$.

Time = 17.58 (sec) , antiderivative size = 4228, normalized size of antiderivative = 31.32

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*tan(e/2 + f*x/2)**9/(105*a**2*c**4*f*tan(e/2 + f*x/2)**1
0 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)
**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 +
f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2
+ f*x/2) - 105*a**2*c**4*f) + 420*A*tan(e/2 + f*x/2)**8/(105*a**2*c**4*f*t
an(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*
f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c*
*4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2
*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a*
*2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*A*tan(e/2 + f*x/2)**7/
(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**
9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)
**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*
x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 +
f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 560*A*ta
n(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*
tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*
f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c
**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a...
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(129) = 258.

Time = 0.25 (sec) , antiderivative size = 835, normalized size of antiderivative = 6.19

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
-2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
 105*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x
 + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8
*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c
^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 21*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x
 + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8
*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8...
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(129) = 258.

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.05

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{\frac{35\left(9A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8A-5B\right)}{a^2c^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{1365A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+210B\tan}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A
*tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/
2*f*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2
*e)^5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14
350*A*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(
1/2*f*x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1
/2*e) + 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x +
 1/2*e) - 1)^7))/f
```

3.68.9 Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.46

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{\left(\frac{32A}{21}-\frac{64B}{105}-\frac{16A\sin(e+fx)}{21}+\frac{32B\sin(e+fx)}{105}\right)\cos(e+fx)^4+\left(\frac{8A}{7}+\frac{12B}{35}-\frac{8A\sin(e+fx)}{7}-\frac{12B\sin(e+fx)}{35}\right.}{a^2c^4f\,\left(4\cos(e+\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x
)
```

output
```
-((10*B)/21 - (4*A)/21 + (10*A*sin(e + f*x))/21 - (4*B*sin(e + f*x))/21 +
cos(e + f*x)^3*((8*A)/7 + (12*B)/35 - (8*A*sin(e + f*x))/7 - (12*B*sin(e +
 f*x))/35 + ((4*sin(e + f*x) - 4)*((4*A)/7 + (6*B)/35))/2) - cos(e + f*x)^
2*((16*A)/21 - (32*B)/105 - (8*A*sin(e + f*x))/7 + (16*B*sin(e + f*x))/35)
 + cos(e + f*x)^4*((32*A)/21 - (64*B)/105 - (16*A*sin(e + f*x))/21 + (32*B
*sin(e + f*x))/105))/(a^2*c^4*f*(4*cos(e + f*x)^3*sin(e + f*x) - 4*cos(e +
 f*x)^3 + 2*cos(e + f*x)^5))
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

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3.69.1 Optimal result

Integrand size = 36, antiderivative size = 175

$$\begin{aligned}&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx\\ &=\frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}\\ &\quad+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}\end{aligned}$$

output

```
1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e
)^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*
sin(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^
2/c^5/f
```

3.69.2 Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.88

$$\begin{aligned}&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx\\ &=\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)(-10752B+180(31A-5B)\,\mathrm{co}}\end{aligned}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^5),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f
*x)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e +
f*x)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e
+ f*x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e
+ f*x] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e +
f*x)] - 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e
 + f*x)] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5
*(e + f*x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*
c^5*f*(-1 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

3.69.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5}dx$$
$$\downarrow 3446$$
$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx}{a^2c^2}$$
$$\downarrow 3042$$
$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))^3}dx}{a^2c^2}$$
$$\downarrow 3338$$

$$\frac{\frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{(2A-B)\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))^2}dx}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 3151$$

$$\frac{\frac{(2A-B)\left(\frac{5\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}dx}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{(2A-B)\left(\frac{5\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 3151$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{4\int \sec^4(e+fx)dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{4\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 4254$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\int\left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5cf}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\downarrow 2009$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}+\frac{(2A-B)\left(\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}+\frac{5\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5cf}\right)}{7c}\right)}{3c}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*(Sec[e
 + f*x]^3/(7*f*(c - c*Sin[e + f*x])^2) + (5*(Sec[e + f*x]^3/(5*f*(c - c*Si
n[e + f*x])) - (4*(-Tan[e + f*x] - Tan[e + f*x]^3/3))/(5*c*f)))/(7*c)))/(3
*c))/(a^2*c^2)
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
 + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.69.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

method	result
risch	$\frac{16i\left(72iA\,e^{5i(fx+e)}-36iB\,e^{5i(fx+e)}+42B\,e^{6i(fx+e)}+4iA\,e^{3i(fx+e)}+54A\,e^{4i(fx+e)}-2iB\,e^{3i(fx+e)}-27B\,e^{4i(fx+e)}-12i\right.}{63\left(e^{i(fx+e)}-i\right)^9\left(e^{i(fx+e)}+i\right)^3 f c^5 a^2}$
parallelrisch	$\frac{-126A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(378A-126B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-546A+252B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-126A-378B)\left(\tan^{8}\right.}{}$
derivativedivides	$-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$
default	$-\frac{-\frac{A}{16}+\frac{B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{7A}{64}-\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$
norman	$\frac{(4A-8B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{(6A-2B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{38A+2B}{63afc}+\frac{8(2A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7acf}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{4(92A-6}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
16/63*I*(72*I*A*exp(5*I*(f*x+e))-36*I*B*exp(5*I*(f*x+e))+42*B*exp(6*I*(f*x
+e))+4*I*A*exp(3*I*(f*x+e))+54*A*exp(4*I*(f*x+e))-2*I*B*exp(3*I*(f*x+e))-2
7*B*exp(4*I*(f*x+e))-12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))+6*I*B*exp
(I*(f*x+e))-12*B*exp(2*I*(f*x+e))-2*A+B)/(exp(I*(f*x+e))-I)^9/(exp(I*(f*x+
e))+I)^3/f/c^5/a^2
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{8\,(2\,A-B)\cos(fx+e)^6 - 36\,(2\,A-B)\cos(fx+e)^4 + 15\,(2\,A-B)\cos(fx+e)^2 + (24\,(2\,A-B)\cos($$
$$63\left(3\,a^2c^5f\cos(fx+e)^5 - 4\,a^2c^5f\cos(fx+e)^3 - (a^2c^5f\cos(fx+$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
 B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x +
 e)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5
 - 4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*co
s(f*x + e)^3)*sin(f*x + e))
```

3.69.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5868 vs. 2(160) = 320.

Time = 33.91 (sec) , antiderivative size = 5868, normalized size of antiderivative = 33.53

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

output

```
Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**1
2 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2
)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f
*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2
 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(
e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**
5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2
*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*
a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4
 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)*
*2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*A*tan(e/2 +
f*x/2)**9/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 +
 f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e
/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5
*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5
*f) - 126*A*tan(e/2 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - ...
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(168) = 336$.

Time $= 0.30$ (sec) , antiderivative size $= 998$, normalized size of antiderivative $= 5.70$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output

```
-2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e
) + 1)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/
(cos(f*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e
) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x
 + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/
(cos(f*x + e) + 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 189*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9 - 63*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6
*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(c...
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx =$$

$$-\frac{\frac{21\left(21A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-15B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+36A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-24B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+19A-13B\right)}{a^2c^5\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{3591A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8+315}{}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 3
6*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c
^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*
tan(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*
x + 1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e
)^6 - 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 10
7730*A*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*t
an(1/2*f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f
*x + 1/2*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*
e) + 2196*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x +
 1/2*e) - 1)^9))/f
```

3.69.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\left(7A - 14B - 14A\sin(e+fx) + 7B\sin(e+fx) + 30A\cos(e+fx)^2 - 76A\cos(e+fx)^3 - 72A\c$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x
)
```

output
```
(2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^
2 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A
*cos(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e +
f*x)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin
(e + f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e +
 f*x) - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x
) + 4*B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B
*cos(e + f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x
) - 2*cos(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

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3.70.1 Optimal result

Integrand size = 36, antiderivative size = 243

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} \\
&\quad - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} \\
&\quad - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} \\
&\quad - \frac{6a^5(3A-8B)c^5\cos^7(e+fx)}{5f\left(a^2+a^2\sin(e+fx)\right)^4} - \frac{42a^5(3A-8B)c^5\cos^5(e+fx)}{5f\left(a^4+a^4\sin(e+fx)\right)^2}
\end{aligned}$$

output

```
-21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B
)*c^5*cos(f*x+e)*sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*s
in(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6-6/5*
a^5*(3*A-8*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^4-42/5*a^5*(3*A-8*B)
*c^5*cos(f*x+e)^5/f/(a^4+a^4*sin(f*x+e))^2
```

3.70.2 Mathematica [A] (verified)

Time = 12.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-384(A-B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)
*Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 12
8*(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 64*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 11
9*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A -
 8*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*
x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

3.70.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3446$$

$$a^5c^5\int\frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^8}dx$$

$$\downarrow 3042$$

$$a^5c^5\int\frac{\cos(e+fx)^{10}(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^8}dx$$

$$\downarrow 3338$$

$$a^5c^5\left(-\frac{(3A-8B)\int\frac{\cos^{10}(e+fx)}{(\sin(e+fx)a+a)^7}dx}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(3A-8B)\int\frac{\cos(e+fx)^{10}}{(\sin(e+fx)a+a)^7}dx}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3159$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\int\frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^5}dx}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\int\frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^5}dx}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3159$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\downarrow 3159$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B}{5f(a\sin}\right)$$

$$\downarrow 3042$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B}{5f(a\sin}\right.$$

$$\downarrow 3161$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}\right.$$

$$\downarrow 3042$$

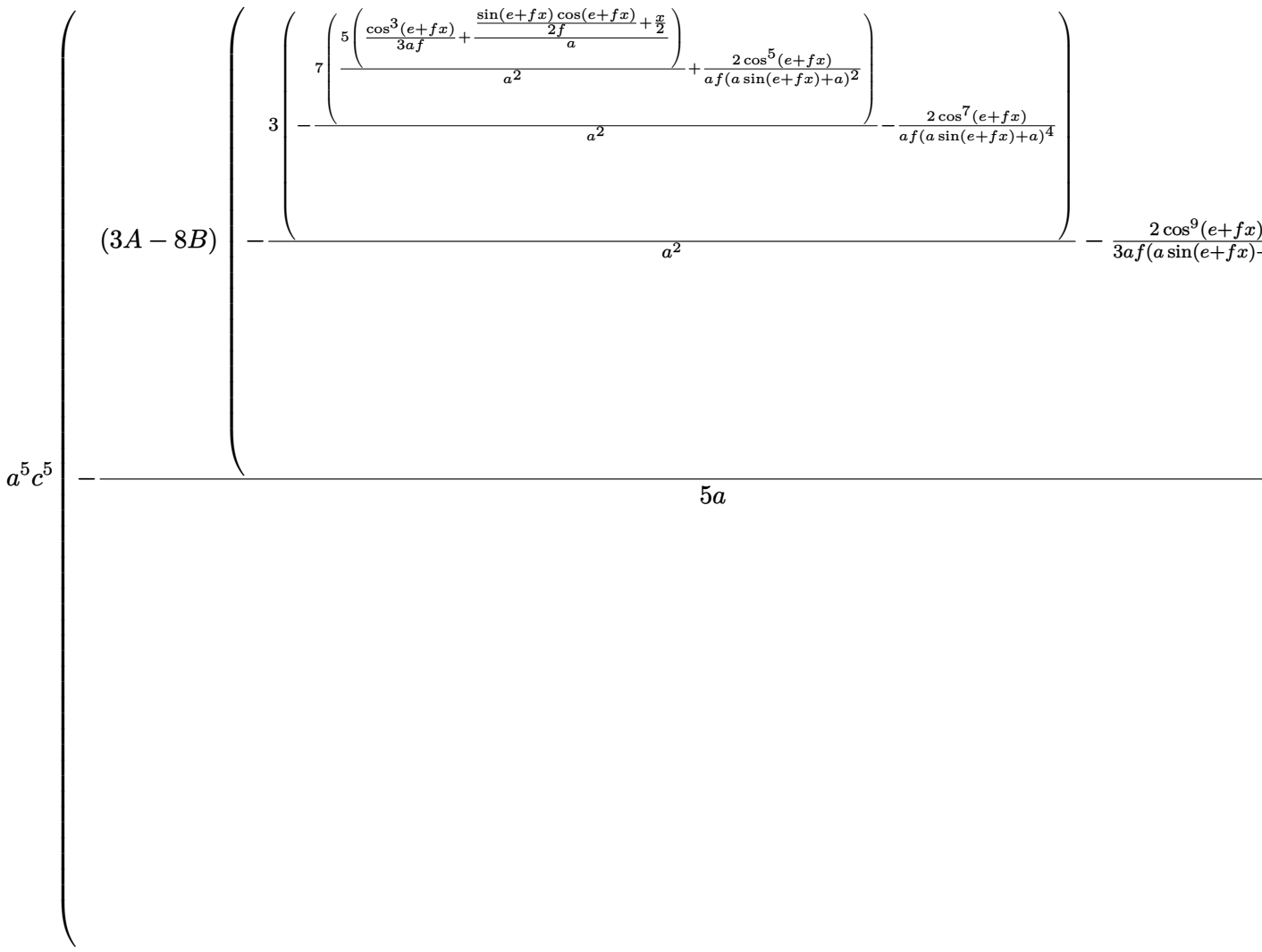
$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}\right.$$

$$\downarrow 3115$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+}{3af(a\sin(e+}}{5a}\right.$$

$\downarrow$ 24

3.70. $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$



input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^5*c^5*(-1/5*((A - B)*Cos[e + f*x]^11)/(f*(a + a*Sin[e + f*x])^8) - ((3*A
 - 8*B)*((-2*Cos[e + f*x]^9)/(3*a*f*(a + a*Sin[e + f*x])^6) - (3*((-2*Cos[
e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^4) - (7*((2*Cos[e + f*x]^5)/(a*f*(a
 + a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos[e + f*x]*S
in[e + f*x])/(2*f))/a))/a^2))/a^2))/a^2))/(5*a))
```

3.70.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.70.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{}$
default	$\frac{2c^5\left(-\frac{\left(\frac{A}{2}-4B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(8A-31B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(16A-64B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{A}{2}+4B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8A-\frac{95B}{3}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}\right.}{}$
parallelrisch	$-\frac{63c^5\left(\left(\frac{19297}{378}B-\frac{1223}{63}A-10fxA+\frac{80}{3}fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{40}{3}fxB+5fxA-\frac{2543}{378}B+\frac{341}{126}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{39439}{3780}\right.\right.}{}$
risch	$-\frac{63c^5xA}{2a^3}+\frac{84c^5xB}{a^3}-\frac{Bc^5e^{3i(fx+e)}}{24a^3f}-\frac{ic^5e^{2i(fx+e)}A}{8a^3f}+\frac{ic^5e^{2i(fx+e)}B}{a^3f}-\frac{4c^5e^{i(fx+e)}A}{a^3f}+\frac{127c^5e^{i(fx+e)}B}{8a^3f}$
norman	Expression too large to display

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output

```
2/f*c^5/a^3*(-((1/2*A-4*B)*tan(1/2*f*x+1/2*e)^5+(8*A-31*B)*tan(1/2*f*x+1/2
*e)^4+(16*A-64*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+4*B)*tan(1/2*f*x+1/2*e)+8*A
-95/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3-21/2*(3*A-8*B)*arctan(tan(1/2*f*x+1/2*
e))-1/4*(-256*A+256*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(32*A-96*B)/(tan(1/2*f
*x+1/2*e)+1)^2-(32*A-80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(96*A-32*B)/(tan(1/2
*f*x+1/2*e)+1)^3-1/5*(128*A-128*B)/(tan(1/2*f*x+1/2*e)+1)^5)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{10\,Bc^5\cos(fx+e)^6+15\,(A-6\,B)c^5\cos(fx+e)^5+10\,(21\,A-74\,B)c^5\cos(fx+e)^4-1260\,(3\,A-8$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*
A - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5
+ (315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*
(3*A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A
 - 8*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x
+ e)^5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x
+ e)^3 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5
*f*x - 2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*
x + 2*(307*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(
f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. $2(228) = 456$.

Time = 37.88 (sec) , antiderivative size = 10608, normalized size of antiderivative = 43.65

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

output

```
Piecewise((-945*A*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2
)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*
tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2
+ f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*t
an(e/2 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)
**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 +
1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a*
*3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan
(e/2 + f*x/2) + 30*a**3*f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3
*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan
(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 +
f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)
**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 3
90*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) -
 23625*A*c**5*f*x*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 15
0*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*
f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*t...
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(231) = 462$.

Time $= 0.38$ (sec) , antiderivative size $= 3282$, normalized size of antiderivative $= 13.51$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output

```
1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
544)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*
a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 + a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e
) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) ...
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{315\,(3\,Ac^5-8\,Bc^5)(fx+e)}{a^3}+\frac{10\left(3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+48\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-186\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+96\,A\right.}{\left(\tan\left(\frac{1}{2}\,fx-\right.\right.}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1
/2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^
4 - 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 3
84*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*
tan(1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)
^3*a^3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2
*e)^4 + 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^
3 + 255*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 +
165*A*c^5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5
 - 94*B*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.70.9 Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.06

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(431\,Ac^5-\frac{3454\,Bc^5}{3}\right)+\frac{496\,Ac^5}{5}-\frac{3958\,Bc^5}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(65\,Ac^5-168\,Bc^5)+\tan\left(\frac{e}{2}+\right.}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+\right.}$$

$$-\frac{21\,c^5\,\mathrm{atan}\left(\frac{21\,c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3\,A-8\,B)}{63\,Ac^5-168\,Bc^5}\right)(3\,A-8\,B)}{a^3\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^3,x
)
```

output

```
- (tan(e/2 + (f*x)/2)*(431*A*c^5 - (3454*B*c^5)/3) + (496*A*c^5)/5 - (3958
*B*c^5)/15 + tan(e/2 + (f*x)/2)^10*(65*A*c^5 - 168*B*c^5) + tan(e/2 + (f*x
)/2)^9*(309*A*c^5 - 838*B*c^5) + tan(e/2 + (f*x)/2)^8*(826*A*c^5 - (6418*B
*c^5)/3) + tan(e/2 + (f*x)/2)^7*(1418*A*c^5 - (11636*B*c^5)/3) + tan(e/2 +
 (f*x)/2)^3*(1654*A*c^5 - (13372*B*c^5)/3) + tan(e/2 + (f*x)/2)^5*(2332*A*
c^5 - (19072*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((4903*A*c^5)/5 - (38884*B*c
^5)/15) + tan(e/2 + (f*x)/2)^6*((11156*A*c^5)/5 - (86708*B*c^5)/15) + tan(
e/2 + (f*x)/2)^4*((11758*A*c^5)/5 - (92224*B*c^5)/15))/(f*(13*a^3*tan(e/2
+ (f*x)/2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*a^3*tan(e/2 + (f*x)/2)^4 +
 46*a^3*tan(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (f*x)/2)^6 + 38*a^3*tan(e/
2 + (f*x)/2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*a^3*tan(e/2 + (f*x)/2)^9
 + 5*a^3*tan(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x)/2)^11 + a^3 + 5*a^3*t
an(e/2 + (f*x)/2))) - (21*c^5*atan((21*c^5*tan(e/2 + (f*x)/2)*(3*A - 8*B))
/(63*A*c^5 - 168*B*c^5))*(3*A - 8*B))/(a^3*f)
```

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

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3.71.1 Optimal result

Integrand size = 36, antiderivative size = 201

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{7(2A-7B)c^4x}{2a^3} - \frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} \\
&\quad - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} \\
&\quad - \frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3} - \frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\,(a^3+a^3\sin(e+fx))}
\end{aligned}$$

output

```
-7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*
c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*cos(f*x+e)^7/
f/(a+a*sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3
-7/6*(2*A-7*B)*c^4*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

3.71.2 Mathematica [A] (verified)

Time = 11.63 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-192(A-B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)
*Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 12
8*(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
64*(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B
)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e +
 f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^3)
```

3.71.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3446$$

$$a^4c^4\int\frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\downarrow 3042$$

$$a^4c^4\int\frac{\cos(e+fx)^8(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\downarrow 3338$$

$$a^4c^4\left(-\frac{(2A-7B)\int\frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^6}dx}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(2A-7B)\int\frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^6}dx}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3159$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^4}dx}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^4}dx}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3159$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\int\frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\downarrow 3158$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-}{5f(a\,s}\right)$$

$$\downarrow 3042$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-}{5f(a\,s}\right)$$

$$\downarrow 3161$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-}{5f(a}\right)$$

$$\downarrow 24$$

$$a^4c^4\left(-\frac{\left((2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)\right)}{5a}-\frac{(A-B$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^4*c^4*(-1/5*((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^7) - ((2*A
 - 7*B)*((-2*Cos[e + f*x]^7)/(3*a*f*(a + a*Sin[e + f*x])^5) - (7*((-2*Cos[e
  + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^3) - (5*((3*(x/a + Cos[e + f*x]/(a*f)
))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^2*Sin[e + f*x]))))/a^2))/(3*a^2)))
/(5*a))
```

3.71.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.71.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\left(\tan^3\left(\frac{f}{2}\right.\right.}{2}\right.}{fa^3}$
default	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\left(\tan^3\left(\frac{f}{2}\right.\right.}{2}\right.}{fa^3}$
parallelrisch	$-\frac{7c^4\left(\left(\frac{471}{7}B-\frac{281}{14}A-10fxA+35fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{35}{2}fxB+5fxA-\frac{761}{84}B+\frac{131}{42}A\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{1937}{140}B+\frac{29}{70}\right.\right.}{}$
risch	$-\frac{7c^4xA}{a^3}+\frac{49c^4xB}{2a^3}+\frac{iBc^4e^{2i(fx+e)}}{8a^3f}-\frac{c^4e^{i(fx+e)}A}{2a^3f}+\frac{7c^4e^{i(fx+e)}B}{2a^3f}-\frac{c^4e^{-i(fx+e)}A}{2a^3f}+\frac{7c^4e^{-i(fx+e)}B}{2a^3f}-$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^4/a^3*(-1/4*(-128*A+128*B)/(tan(1/2*f*x+1/2*e)+1)^4-(8*A-24*B)/(tan(
1/2*f*x+1/2*e)+1)-1/5*(64*A-64*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(64*A-32*B)
/(tan(1/2*f*x+1/2*e)+1)^3+16*B/(tan(1/2*f*x+1/2*e)+1)^2-(-1/2*B*tan(1/2*f*
x+1/2*e)^3+(A-7*B)*tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)+A-7*B)/(1
+tan(1/2*f*x+1/2*e)^2)^2-7/2*(2*A-7*B)*arctan(tan(1/2*f*x+1/2*e)))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(189) = 378.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.95

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{15\,Bc^4\cos(fx+e)^5-30\,(A-6\,B)c^4\cos(fx+e)^4+420\,(2\,A-7\,B)c^4fx+96\,(A-B)c^4-(105\,(2\,A-$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
 - 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 181
9*B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^
4)*cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(
f*x + e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 -
 420*(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(
262*A - 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A -
 241*B)*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2
*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.71.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. $2(185) = 370$.

Time = 23.49 (sec) , antiderivative size = 7337, normalized size of antiderivative = 36.50

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

output

```
Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)
**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 60
0*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f
*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2
 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*
tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**
6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*
a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**
3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*
x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3
+ 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f
) - 4200*A*c**4*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 1
50*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*
f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/
2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2
 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/...
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2394 vs. $2(189) = 378$.

Time $= 0.35$ (sec) , antiderivative size $= 2394$, normalized size of antiderivative $= 11.91$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output

```
1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(co
s(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*
A*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a
^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e)...
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{105\,(2\,Ac^4-7\,Bc^4)(fx+e)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\,B\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2
*e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 -
 B*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2
 + 1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x
+ 1/2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2
*e)^3 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^
2 + 80*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*
c^4 - 59*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.71.9 Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{286\,A\,c^4}{3}-\frac{1007\,B\,c^4}{3}\right)+\frac{334\,A\,c^4}{15}-\frac{1154\,B\,c^4}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(16\,A\,c^4-49\,B\,c^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right.}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+5\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+12\,a\right.}$$
$$-\frac{7\,c^4\,\mathrm{atan}\left(\frac{7\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-7\,B)}{14\,A\,c^4-49\,B\,c^4}\right)(2\,A-7\,B)}{a^3\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x
)
```

output
```
- (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 -
(1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (
f*x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (
1741*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + t
an(e/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^
2*((1006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5
 - (5633*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*
x)/2)^3 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a
^3*tan(e/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f
*x)/2)^8 + a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (
7*c^4*atan((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(
2*A - 7*B))/(a^3*f)
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

3.72.1 Optimal result 730
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3.72.1 Optimal result

Integrand size = 36, antiderivative size = 153

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{(A-6B)c^3x}{a^3} - \frac{(A-6B)c^3\cos(e+fx)}{a^3 f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} \\
&\quad + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{2a^3(A-6B)c^3\cos^3(e+fx)}{3f\left(a^3+a^3\sin(e+fx)\right)^2}
\end{aligned}$$

output

```
-(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*cos(f*x+
e)^7/f/(a+a*sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e
))^4-2/3*a^3*(A-6*B)*c^3*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))^2
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. $2(153) = 306$.

Time = 11.42 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.01

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx \\
&= \frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\tfrac{1}{2}(e+fx)\right)-24(A-B)\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(\right.\right.}{}
\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(
A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^6*(1 + Sin[e + f*x])^3)
```

3.72.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^3(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^6}\,dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^6}\,dx$$

$$\downarrow 3338$$

$$a^3c^3\left(-\frac{(A-6B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^5}\,dx}{5a} - \frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(A-6B)\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^5}dx}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\int\frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^3}dx}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^3}dx}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 3161$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$$\downarrow 24$$

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^3*c^3*(-1/5*((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^6) - ((A -
6*B)*((-2*Cos[e + f*x]^5)/(3*a*f*(a + a*Sin[e + f*x])^4) - (5*((-3*(x/a +
Cos[e + f*x]/(a*f)))/a^2 - (2*Cos[e + f*x]^3)/(a*f*(a + a*Sin[e + f*x])^2)
))/(3*a^2)))/(5*a))
```

3.72.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.72.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{}{5\left(\tan\right.}\right.}{f\,a^3}$
default	$\frac{2c^3\left(\frac{B}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}-(A-6B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{}{5\left(\tan\right.}\right.}{f\,a^3}$
risch	$-\frac{c^3xA}{a^3}+\frac{6c^3xB}{a^3}+\frac{B\,c^3\mathrm{e}^{i(fx+e)}}{2a^3f}+\frac{B\,c^3\mathrm{e}^{-i(fx+e)}}{2a^3f}+\frac{112A\,c^3\mathrm{e}^{2i(fx+e)}}{3}-24iA\,c^3\mathrm{e}^{3i(fx+e)}+\frac{56iA\,c^3\mathrm{e}^{i(fx+e)}}{3}-12A$
parallelrisch	$-\frac{c^3\left(\left(\left(\frac{233}{2}+60fx\right)B-10fxA-24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-30fx-\frac{33}{2}\right)B+5fxA+\frac{16A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{243}{10}B+\frac{24}{5}A+f\right.\right.}{f\,a^3\left(-10\,\mathrm{co}\right.}$
norman	$-\frac{5c^3(A-6B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}-\frac{14c^3(A-6B)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{30c^3(A-6B)x\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{5c^3(A-6B)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{c}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^3/a^3*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A-6*B)*arctan(tan(1/2*f*x+1/2*e))
-1/4*(-64*A+64*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-8*A-8*B)/(tan(1/2*f*x+1/2
*e)+1)^2-(2*A-6*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(32*A-32*B)/(tan(1/2*f*x+1/2
*e)+1)^5-1/3*(40*A-24*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(147) = 294$.

Time $= 0.27$ (sec) , antiderivative size $= 338$, normalized size of antiderivative $= 2.21$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx$$
$$= \frac{15\,Bc^3\cos(fx + e)^4 + 60\,(A - 6\,B)c^3 fx + 24\,(A - B)c^3 - (15\,(A - 6\,B)c^3 fx + (46\,A - 231\,B)c^3)\cos(f$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (1
5*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c
^3*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*
A - 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*
f*x - 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f
*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin
(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)
*sin(f*x + e))
```

3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.

Time = 13.77 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f
*x*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)
**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5
/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e
/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*
A*c**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6...
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. 2(147) = 294.

Time = 0.35 (sec) , antiderivative size = 1679, normalized size of antiderivative = 10.97

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1
5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos
(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e
) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^
3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*
x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 +
 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 1...
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3} - \frac{15\,(Ac^3-6\,Bc^3)(fx+e)}{a^3} - \frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^3\right.}{15\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(
f*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x +
 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*
e)^3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 50*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c
^3 - 63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.72.9 Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.18

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{40Ac^3}{3}-82Bc^3\right)+\frac{52Ac^3}{15}-\frac{94Bc^3}{5}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4Ac^3-12Bc^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(8}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+11a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+1}$$
$$-\frac{2c^3\operatorname{atan}\left(\frac{2c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A-6B)}{2Ac^3-12Bc^3}\right)(A-6B)}{a^3 f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x
)
```

output
```
- (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^
3)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8
*A*c^3 - 58*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan
(e/2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*
A*c^3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e
/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^
5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*ta
n(e/2 + (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A
*c^3 - 12*B*c^3))*(A - 6*B))/(a^3*f)
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

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3.73.1 Optimal result

Integrand size = 36, antiderivative size = 110

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{Bc^2x}{a^3} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2Bc^2\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5-2/3*B*c^2*
cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*B*c^2*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e)
)
```

3.73.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(110) = 220$.

Time = 11.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-12(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(
A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4*(1 + Sin[e + f*x])^3)
```

3.73.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^2(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{B\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a} - \frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{B\int \frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^4}dx}{a} - \frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{B\left(-\frac{\int\frac{\cos^2(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{B\left(-\frac{\int\frac{\cos(e+fx)^2}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{B\left(-\frac{-\frac{\int 1dx}{a^2}-\frac{2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\downarrow 24$$

$$a^2c^2\left(\frac{B\left(-\frac{-\frac{2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}-\frac{x}{a^2}}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^2*c^2*(-1/5*((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^5) + (B*((-
2*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^3) - (-(x/a^2) - (2*Cos[e +
f*x])/(f*(a^2 + a^2*Sin[e + f*x])))/a^2))/a)
```

3.73.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.73.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{fa^3}$
default	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{fa^3}$
parallelrisch	$-\frac{2c^2\left(-\frac{B\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)fx}{2}+\left(-\frac{5}{2}fxB+A-B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-5fx-4)B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-5fxB+2A-\frac{34}{3}B\right)\right.}{fa^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
risch	$\frac{Bc^2x}{a^3}-\frac{2\left(-30Ac^2e^{2i(fx+e)}+15Ac^2e^{4i(fx+e)}+250Bc^2e^{2i(fx+e)}-180iBc^2e^{3i(fx+e)}+140iBc^2e^{i(fx+e)}-75Bc^2e^{4i(}\right.}{15fa^3\left(e^{i(fx+e)}+i\right)^5}$
norman	$\frac{Bc^2x}{a}+\frac{8Bc^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{48Bc^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}+\frac{Bc^2x\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{6Ac^2-46Bc^2}{15fa}+\frac{40Bc^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fa}+\frac{64Bc^2}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^2/a^3*(B*arctan(tan(1/2*f*x+1/2*e))-1/4*(-32*A+32*B)/(tan(1/2*f*x+1/
2*e)+1)^4-(A-B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(16*A-16*B)/(tan(1/2*f*x+1/2*e)
+1)^5-1/3*(24*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3+4*A/(tan(1/2*f*x+1/2*e)+1)^
2)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(106) = 212.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{60\,Bc^2fx-(15\,Bc^2fx-(3\,A-43\,B)c^2)\cos(fx+e)^3-12\,(A-B)c^2-(45\,Bc^2fx-(9\,A+11\,B)c^2)}{15\left(a^3f\cos(fx+e)^3+3\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 1
2*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*
c^2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 -
(15*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9
*B)*c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f
*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3
*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.73.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 7.72 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.97

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*
*2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*A*c**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2
)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 15*B*c**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*c*
*2*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2
 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**2/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f
*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
+ f*x/2) + 15*a**3*f) + 75*B*c**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e...
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(106) = 212.

Time = 0.31 (sec) , antiderivative size = 1134, normalized size of antiderivative = 10.31

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*
a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*...
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{15\,(fx+e)Bc^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 170\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan
(1/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2
*f*x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.73.9 Mupad [B] (verification not implemented)

Time = 16.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{c^2\,(120\,B+150\,B\,(e+f\,x))}{15}-10\,B\,c^2\,(e+f\,x)\right)+\frac{c^2\,(46\,B-6\,A+15\,B\,(e+f\,x))}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{c^2\,(30\,B-}{}\right.$$

$$+\frac{B\,c^2\,x}{a^3}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x
)
```

output
```
(tan(e/2 + (f*x)/2)^3*((c^2*(120*B + 150*B*(e + f*x)))/15 - 10*B*c^2*(e +
f*x)) + (c^2*(46*B - 6*A + 15*B*(e + f*x)))/15 + tan(e/2 + (f*x)/2)^4*((c^
2*(30*B - 30*A + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) + tan(e/2 + (f*x
)/2)^2*((c^2*(340*B - 60*A + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f*x)) +
tan(e/2 + (f*x)/2)*((c^2*(200*B + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x))
 - B*c^2*(e + f*x))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5) + (B*c^2*x)/a^3
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

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3.74.1 Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{a(A-11B)c\cos(e+fx)}{15f\left(a^2+a^2\sin(e+fx)\right)^2} + \frac{(A+4B)c\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-2/5*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^3+1/15*a*(A-11*B)*c*cos(f*x+e)/
f/(a^2+a^2*sin(f*x+e))^2+1/15*(A+4*B)*c*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

3.74.2 Mathematica [A] (verified)

Time = 6.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-25B\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right)\right)}{30a^3f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[
(f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*
f*x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

3.74.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3336, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A + B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\downarrow 3336$$

$$ac\left(-\frac{\int \frac{a(A-6B)+5aB\sin(e+fx)}{(\sin(e+fx)a+a)^2}\,dx}{5a^3} - \frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3042$$

$$ac\left(-\frac{\int \frac{a(A-6B)+5aB\sin(e+fx)}{(\sin(e+fx)a+a)^2}\,dx}{5a^3} - \frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3229$$

$$ac\left(-\frac{\frac{1}{3}(A+4B)\int \frac{1}{\sin(e+fx)a+a}\,dx - \frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3} - \frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3042$$

$$ac\left(-\frac{\frac{1}{3}(A+4B)\int\frac{1}{\sin(e+fx)a+a}dx-\frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\downarrow 3127$$

$$ac\left(-\frac{-\frac{(A+4B)\cos(e+fx)}{3f(a\sin(e+fx)+a)}-\frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
a*c*((-2*(A - B)*Cos[e + f*x])/(5*a*f*(a + a*Sin[e + f*x])^3) - (-1/3*(a*(
A - 11*B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 4*B)*Cos[e + f*
x])/(3*f*(a + a*Sin[e + f*x])))/(5*a^3))
```

3.74.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.74.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
parallelrisch	$-\frac{2c\left(A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(5A-B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{(A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}+\frac{B}{15}\right)}{f a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f a^3}$
default	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f a^3}$
risch	$\frac{-\frac{10Bc e^{2i(fx+e)}}{3}+2iBc e^{3i(fx+e)}-\frac{2iBc e^{i(fx+e)}}{3}+2Bc e^{4i(fx+e)}-\frac{2iAc e^{i(fx+e)}}{3}+\frac{2Ac e^{2i(fx+e)}}{3}+\frac{2Ac}{15}+\frac{8Bc}{15}+2iAc e^{3i(f}}{f a^3\left(e^{i(fx+e)}+i\right)^5}$
norman	$\frac{-\frac{8Ac+2Bc}{15fa}-\frac{2Ac\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(23Ac-3Bc)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\frac{2(7Ac+7Bc)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{2(11Ac-Bc)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2 a^2\left(\tan\right.}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output

```
-2*c*(A*tan(1/2*f*x+1/2*e)^4+(A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A-B)*tan(1/
2*f*x+1/2*e)^2+1/3*(A+B)*tan(1/2*f*x+1/2*e)+4/15*A+1/15*B)/f/a^3/(tan(1/2*
f*x+1/2*e)+1)^5
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{(A+4B)c\cos(fx+e)^3 - (2A-7B)c\cos(fx+e)^2 + 3(A-B)c\cos(fx+e) + 6(A-B)c - ((A+4}{15\left(a^3 f\cos(fx+e)^3 + 3a^3 f\cos(fx+e)^2 - 2a^3 f\cos(fx+e) - 4a^3 f + \left(a^3 f\cos(f\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B
)*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c
*cos(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.74.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(97) = 194.

Time = 4.42 (sec) , antiderivative size = 1035, normalized size of antiderivative = 10.05

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output

```
Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*
x/2) + 15*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c
/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 +
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) + 10*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*
tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f
*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + ...
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(97) = 194$.

Time $= 0.21$ (sec) , antiderivative size $= 733$, normalized size of antiderivative $= 7.12$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{15\,a^3 f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*
B*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2
*f*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e)
+ 4*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{41Ac}{4}-\frac{Bc}{4}-\frac{11Ac\cos(e+fx)}{2}+\frac{Bc\cos(e+fx)}{2}+5Ac\sin(e+fx)+5Bc\sin(e+fx)-\frac{3A}{}\right.}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}c}{}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*cos(e + f*x))/2 + (B
*c*cos(e + f*x))/2 + 5*A*c*sin(e + f*x) + 5*B*c*sin(e + f*x) - (3*A*c*cos(
2*e + 2*f*x))/4 + (3*B*c*cos(2*e + 2*f*x))/4 - (5*A*c*sin(2*e + 2*f*x))/4
- (5*B*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*
e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

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3.75.1 Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} + \frac{2(3A+2B)\tan(e+fx)}{15a^3cf}$$

output
```
-1/5*(A-B)*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/15*(3*A+2*B)*sec(f*x+e)/c
/f/(a^3+a^3*sin(f*x+e))+2/15*(3*A+2*B)*tan(f*x+e)/a^3/c/f
```

3.75.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{\cos(e+fx)(-80B-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))+9A\cos(3(e+fx))+B\cos}{240a^3}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)),x]
```

output

```
(Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e
+ f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] -
 80*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*S
in[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])
*(1 + Sin[e + f*x])^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^2}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(\sin(e+fx)a+a)^2}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{\frac{(3A+2B)\int \frac{\sec^2(e+fx)}{\sin(e+fx)a+a}\,dx}{5a} - \frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(3A+2B)\int \frac{1}{\cos(e+fx)^2(\sin(e+fx)a+a)}\,dx}{5a} - \frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\downarrow 3151$$

$$\frac{(3A+2B)\left(\frac{2\int \sec^2(e+fx)dx}{3a}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}$$
$$ac$$
$$\downarrow 3042$$
$$\frac{(3A+2B)\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx}{3a}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}$$
$$ac$$
$$\downarrow 4254$$
$$\frac{(3A+2B)\left(-\frac{2\int 1d(-\tan(e+fx))}{3af}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}$$
$$ac$$
$$\downarrow 24$$
$$\frac{(3A+2B)\left(\frac{2\tan(e+fx)}{3af}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}$$
$$ac$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x])/(f*(a + a*Sin[e + f*x])^2) + ((3*A + 2*B)*(-1
/3*Sec[e + f*x]/(f*(a + a*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*f)))/(5*a
))/(a*c)
```

3.75.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.75.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4\left(-3iA-12A\,e^{i(fx+e)}+10B\,e^{3i(fx+e)}-2iB-8B\,e^{i(fx+e)}+10iB\,e^{2i(fx+e)}+15iA\,e^{2i(fx+e)}\right)}{15\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)a^3cf}$
parallelrisch	$\frac{-30A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A-30B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A-40B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-40B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(18A-}{15fca^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(-2B+2A)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{a^3cf}$
default	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(-2B+2A)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}}{a^3cf}$
norman	$\frac{\frac{12A-2B}{15cfa}-\frac{2(6A+7B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cfa}-\frac{2A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cfa}+\frac{2(9A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cfa}+\frac{2(2A-7B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cfa}-\frac{2(9A+4B)\left(}{3}$ $\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(-3*I*A-12*A*exp(I*(f*x+e))+10*B*exp(3*I*(f*x+e))-2*I*B-8*B*exp(I*(f
*x+e))+10*I*B*exp(2*I*(f*x+e))+15*I*A*exp(2*I*(f*x+e)))/(exp(I*(f*x+e))+I)
^5/(exp(I*(f*x+e))-I)/a^3/c/f
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$$
$$= \frac{4\,(3\,A+2\,B)\cos\left(fx+e\right)^2 + \left(2\,(3\,A+2\,B)\cos\left(fx+e\right)^2 - 9\,A - 6\,B\right)\sin\left(fx+e\right) - 6\,A - 9\,B}{15\,\left(a^3cf\cos\left(fx+e\right)^3 - 2\,a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,a^3cf\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
 6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*
x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

3.75.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. 2(85) = 170.

Time = 4.54 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

output

```
Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60
*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*
c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60
*A*tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 +
f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f
*x/2)**3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**
5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60
*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3
*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*t
an(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2
 + f*x/2) - 15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3
*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*t
an(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*ta
n(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)
**3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3
*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**...
```

3.75.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(96) = 192$.

Time = 0.21 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.15

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$$

$$= \frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\right.}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(f}}\right)}{15\,f}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) +
5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1)
- 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(
f*x + e)/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 5*a^3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx =$$
$$-\frac{\frac{15\,(A+B)}{a^3c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-1)} + \frac{105\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4-15\,B\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^4+270\,A\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+30\,B\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+360\,A\tan(\frac{1}{2}\,fx+}{a^3c(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)^5}}{60\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x
+ 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 +
30*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*
f*x + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) +
63*A + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.75.9 Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx =$$

$$-\frac{2\left(\frac{15\,A\cos(e+f\,x)}{4}-\frac{5\,B}{2}-\frac{5\,B\cos(e+f\,x)}{8}-\frac{15\,A\sin(e+f\,x)}{4}-\frac{5\,B\sin(e+f\,x)}{2}+3\,A\cos(2\,e+2\,f\,x)-\frac{3\,A\cos(3\,e+3\,f}{4}\right.}{15\,a^3\,c\,f\left(\frac{5\cos(e+f\,x)}{4}-\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

output
```
-(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e
+ f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3
*f*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2
*f*x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3
*f*x))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e
+ 2*f*x)))
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

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3.76.1 Optimal result

Integrand size = 36, antiderivative size = 90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))} + \frac{(4A+B)\tan(e+fx)}{5a^3c^2f} + \frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}$$

output
```
-1/5*(A-B)*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+1/5*(4*A+B)*tan(f*x+e)/
a^3/c^2/f+1/15*(4*A+B)*tan(f*x+e)^3/a^3/c^2/f
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(90) = 180.

Time = 1.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(240B+54(A-B)\cos(e+fx)$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^2),x]
```

output

```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A
*Cos[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*C
os[4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e +
 f*x)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e +
f*x)] + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 +
 Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)
```

3.76.3 Rubi [A] (verified)

Time $= 0.53$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 0.84$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used $= \{3042, 3446, 3042, 3338, 3042, 4254, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sin(e+fx)a+a}\,dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(\sin(e+fx)a+a)}\,dx}{a^2c^2}$$

$$\downarrow 3338$$

$$\frac{\frac{(4A+B)\int \sec^4(e+fx)dx}{5a} - \frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{(4A+B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4 dx}{5a} - \frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\downarrow 4254$$

$$\frac{-\frac{(4A+B)\int(\tan^2(e+fx)+1)d(-\tan(e+fx))}{5af}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\downarrow 2009$$

$$\frac{-\frac{(4A+B)(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx))}{5af}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x
]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^3)/(f*(a + a*Sin[e + f*x])) - ((4*A + B)*(-Tan
[e + f*x] - Tan[e + f*x]^3/3))/(5*a*f))/(a^2*c^2)
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.76.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4i\left(24iA\,e^{3i(fx+e)}+6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}-8A\,e^{2i(fx+e)}+2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}-4A-B\right)}{15\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)^3 f\,c^2a^3}$
parallelrisch	$\frac{-30A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-30A-30B)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10A-20B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(50A-10B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(}{15f\,c^2a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\right.}$
derivativedivides	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{B}{4}+\frac{A}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2B-2A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3c^2f}$
default	$\frac{-\frac{2\left(\frac{B}{4}+\frac{A}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{B}{4}+\frac{A}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2B-2A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3c^2f}$
norman	$\frac{-\frac{6A+4B}{10cfa}-\frac{4(4A+B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cfa}+\frac{A\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{(14A+16B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10cfa}-\frac{(6A+4B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2cfa}-\frac{(2A+8B)}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}\right.\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
4/15*I*(24*I*A*exp(3*I*(f*x+e))+6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+e
))+8*I*A*exp(I*(f*x+e))-8*A*exp(2*I*(f*x+e))+2*I*B*exp(I*(f*x+e))-2*B*exp(
2*I*(f*x+e))-4*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^3/f/c^2/a^3
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(4A+B\right)\cos\left(fx+e\right)^4 - \left(4A+B\right)\cos\left(fx+e\right)^2 - \left(2\left(4A+B\right)\cos\left(fx+e\right)^2 + 4A + B\right)\sin\left(fx+\right.}{15\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right) + a^3c^2f\cos\left(fx+e\right)^3\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B
)*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e
)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

3.76.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. $2(82) = 164$.

Time = 9.00 (sec) , antiderivative size = 2674, normalized size of antiderivative = 29.71

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

output

```
Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 -
 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 +
 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15
*a**3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)*
*8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)*
*6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)*
*3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2)
- 15*a**3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x
/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x
/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x
/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x
/2) - 15*a**3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 +
 f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 +
 f*x/2) - 15*a**3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e
/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e
/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e
/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*ta...
```

3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(84) = 168$.

Time $= 0.22$ (sec) , antiderivative size $= 650$, normalized size of antiderivative $= 7.22$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}} + \frac{}{a^3}}{15\,f}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^
3*c^2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
6*a^3*c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*
c^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e)
 + 1) + 9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2
+ 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time $= 0.47$ (sec) , antiderivative size $= 221$, normalized size of antiderivative $= 2.46$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx =$$

$$-\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A+7\,B\right)}{a^3c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,B\tan}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A
*tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(
tan(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/
2*f*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e
)^3 + 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*t
an(1/2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.76.9 Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\frac{8A}{15} + \frac{2B}{15} + \frac{16A\sin(e+fx)}{15} + \frac{4B\sin(e+fx)}{15}\right)\cos(e+fx)^2 + \frac{2A}{15} + \frac{8B}{15} + \frac{8A\sin(e+fx)}{15} + \frac{2B\sin(e+fx)}{15}}{a^3c^2f\left(2\cos(e+fx)^3\sin(e+fx) + 2\cos(e+fx)^3\right)}$$
$$- \frac{\frac{2A}{5} - \frac{2B}{5} + \frac{2A\sin(e+fx)}{5} - \frac{2B\sin(e+fx)}{5}}{a^3c^2f\left(2\sin(e+fx)+2\right)} - \frac{\cos(e+fx)\left(\frac{16A}{15} + \frac{4B}{15}\right)}{a^3c^2f\left(2\sin(e+fx)+2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x
)
```

output
```
((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos
(e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f
*x))/15))/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) -
 ((2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^
2*f*(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2
*f*(2*sin(e + f*x) + 2))
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

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3.77.1 Optimal result

Integrand size = 36, antiderivative size = 84

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}$$

output
```
1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3
/c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f
```

3.77.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{a^3c^3f}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^3),x]
```

output
```
(B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
 + Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

3.77.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3148$$

$$\frac{A\int \sec^6(e+fx)dx + \frac{B\sec^5(e+fx)}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^6 dx + \frac{B\sec^5(e+fx)}{5f}}{a^3c^3}$$

$$\downarrow 4254$$

$$\frac{\frac{B\sec^5(e+fx)}{5f} - \frac{A\int(\tan^4(e+fx)+2\tan^2(e+fx)+1)d(-\tan(e+fx))}{f}}{a^3c^3}$$

$$\downarrow 2009$$

$$\frac{\frac{B\sec^5(e+fx)}{5f} - \frac{A\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{f}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x
]
```

output
```
((B*Sec[e + f*x]^5)/(5*f) - (A*(-Tan[e + f*x] - (2*Tan[e + f*x]^3)/3 - Tan
[e + f*x]^5/5))/f)/(a^3*c^3)
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.77.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\frac{16iA\,e^{2i(fx+e)}}{3}+\frac{32B\,e^{5i(fx+e)}}{5}+\frac{32iA\,e^{4i(fx+e)}}{3}+\frac{16iA}{15}}{\left(e^{i(fx+e)}+i\right)^5\left(e^{i(fx+e)}-i\right)^5 f c^3 a^3}$
parallelrisch	$\frac{-2A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-2B\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{8A\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{116A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15}-4B\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{8A\left(\tan^3\left(\frac{f}{3}\right.\right.}{f c^3 a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{11A}{8}+\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3c^3f}$
default	$\frac{-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{11A}{8}+\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3c^3f}$
norman	$\frac{-\frac{2B}{5acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{acf}+\frac{2A\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{76A\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15acf}+\frac{2A\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3acf}-\frac{2A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}-\frac{76A\left(\tan^7\left(\frac{f}{15cf}\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
16/15*(5*I*A*exp(2*I*(f*x+e))+6*B*exp(5*I*(f*x+e))+10*I*A*exp(4*I*(f*x+e))
+I*A)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^5/f/c^3/a^3
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\left(8\,A\cos(fx+e)^4+4\,A\cos(fx+e)^2+3\,A\right)\sin(fx+e)+3\,B}{15\,a^3c^3f\cos(fx+e)^5}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/
(a^3*c^3*f*cos(f*x + e)^5)
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(78) = 156.

Time = 6.10 (sec) , antiderivative size = 1098, normalized size of antiderivative = 13.07

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10
- 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6
 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**
2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f
*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 +
 f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2
+ f*x/2)**2 - 15*a**3*c**3*f) - 116*A*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*
tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*
f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3
*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**3/(15*a*
*3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*
a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75
*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*A*tan(e/2 + f*x/2)
/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8
 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)*
*4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*B*tan(e/2 +
 f*x/2)**8/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 +
 f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2
 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 60*B
*tan(e/2 + f*x/2)**4/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**...
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\frac{\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)A}{a^3c^3} + \frac{3B}{a^3c^3\cos(fx+e)^5}}{15f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
 + 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+58\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+\right.}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*ta
n(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/
2*e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((
tan(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+15\,B\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-20\,A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+58\,A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+30\,B\tan\left(\frac{e}{2}+\right.\right.}{15\,a^3\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)^5}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x
)
```

output
```
-(2*(3*B + 15*A*tan(e/2 + (f*x)/2) - 20*A*tan(e/2 + (f*x)/2)^3 + 58*A*tan(
e/2 + (f*x)/2)^5 - 20*A*tan(e/2 + (f*x)/2)^7 + 15*A*tan(e/2 + (f*x)/2)^9 +
 30*B*tan(e/2 + (f*x)/2)^4 + 15*B*tan(e/2 + (f*x)/2)^8))/(15*a^3*c^3*f*(ta
n(e/2 + (f*x)/2)^2 - 1)^5)
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

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3.78.1 Optimal result

Integrand size = 36, antiderivative size = 121

$$\begin{aligned}&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx\\&=\frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}\\&\quad+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}\end{aligned}$$

output
```
1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a
^3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3
/c^4/f
```

3.78.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(121) = 242.

Time = 2.89 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-8960B+1500(A+B)\cos$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^4),x]
```

output
```
-1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e
+ f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(
e + f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*
(e + f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin
[e + f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e
+ f*x)] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*
(e + f*x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[
5*(e + f*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(a^3*c^4*f*
(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3)
```

3.78.3 Rubi [A] (verified)

Time $= 0.54$ (sec) , antiderivative size $= 89$, normalized size of antiderivative $= 0.74$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used $= \{3042, 3446, 3042, 3338, 3042, 4254, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4}\,dx$$
$$\downarrow 3446$$
$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$
$$\downarrow 3042$$
$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))}\,dx}{a^3c^3}$$
$$\downarrow 3338$$
$$\frac{\frac{(6A-B)\int \sec^6(e+fx)dx}{7c} + \frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{(6A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^6 dx}{7c} + \frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}$$
$$a^3c^3$$

$$\downarrow 4254$$

$$\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))} - \frac{(6A-B)\int (\tan^4(e+fx)+2\tan^2(e+fx)+1)d(-\tan(e+fx))}{7cf}$$
$$a^3c^3$$

$$\downarrow 2009$$

$$\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))} - \frac{(6A-B)\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}$$
$$a^3c^3$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])) - ((6*A - B)*(-Tan[e
+ f*x] - (2*Tan[e + f*x]^3)/3 - Tan[e + f*x]^5/5))/(7*c*f))/(a^3*c^3)
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.78.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{16i(120iA\,e^{5i(fx+e)}-20iB\,e^{5i(fx+e)}+70B\,e^{6i(fx+e)}+60iA\,e^{3i(fx+e)}+30A\,e^{4i(fx+e)}-10iB\,e^{3i(fx+e)}-5B\,e^{4i(fx+e)}}{105(e^{i(fx+e)}+i)^5(e^{i(fx+e)}-i)^7 f c^4 a^3}$
parallelrisch	$\frac{-210A\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A-210B)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(210A+140B)\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-630A-70B)\left(\tan^{8}\left(\frac{fx}{2}\right.\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
default	$\frac{-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{a}$
norman	$\frac{(-2B+2A)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cfa}-\frac{2A+2B}{7cfa}-\frac{152(6A-B)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}-\frac{2A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{afc}+\frac{4B\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cfa}+\frac{(312A-752B}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
-16/105*I*(120*I*A*exp(5*I*(f*x+e))-20*I*B*exp(5*I*(f*x+e))+70*B*exp(6*I*(
f*x+e))+60*I*A*exp(3*I*(f*x+e))+30*A*exp(4*I*(f*x+e))-10*I*B*exp(3*I*(f*x+
e))-5*B*exp(4*I*(f*x+e))+12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))-2*I*B
*exp(I*(f*x+e))-4*B*exp(2*I*(f*x+e))+6*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f
*x+e))-I)^7/f/c^4/a^3
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx =$$
$$-\frac{8\,(6\,A-B)\cos(fx+e)^6 - 4\,(6\,A-B)\cos(fx+e)^4 - (6\,A-B)\cos(fx+e)^2 + \left(8\,(6\,A-B)\cos(fx\right.}{105\left(a^3c^4f\cos(fx+e)^5\sin(fx+e) - a^3c^4\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B
)*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^
2 + 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f
*x + e) - a^3*c^4*f*cos(f*x + e)^5)
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6135 vs. 2(109) = 218.

Time = 33.49 (sec) , antiderivative size = 6135, normalized size of antiderivative = 50.70

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

output

```
Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**
12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/
2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 +
f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/
2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan
(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*t
an(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*
c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a
**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7
+ 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)*
*4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/
2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/
2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(
e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f
*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**
4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*
c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a*
*3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**
3*c**4*f) - 630*A*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)...
```

3.78.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(114) = 228$.

Time $= 0.24$ (sec) , antiderivative size $= 1019$, normalized size of antiderivative $= 8.42$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output

```
-2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 35*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3
*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f
*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12) - 3*A*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 105*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 35*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x...
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(114) = 228$.

Time $= 0.38$ (sec) , antiderivative size $= 333$, normalized size of antiderivative $= 2.75$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-280\,B\tan\left(\frac{1}{2}\,fx+\right.\right.}{a^3c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 5
40*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2
*f*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e
) - 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2
*e) + 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^
6 - 10080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945
*A*tan(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/
2*f*x + 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1
/2*e)^2 + 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 43
4*B*tan(1/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

3.78.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{\left(\frac{16A}{35}-\frac{8B}{105}-\frac{32A\sin(e+fx)}{35}+\frac{16B\sin(e+fx)}{105}\right)\cos(e+fx)^4+\left(\frac{4A}{35}-\frac{2B}{105}-\frac{16A\sin(e+fx)}{35}+\frac{8B\sin(e+fx)}{105}\right)\cos}{a^3c^4f\,\left(2\cos(e+fx)^5\sin(e+fx)-2\cos(e+fx)^5\right.}$$
$$-\frac{\frac{2A}{7}+\frac{2B}{7}-\frac{2A\sin(e+fx)}{7}-\frac{2B\sin(e+fx)}{7}}{a^3c^4f\,(2\sin(e+fx)-2)}-\frac{\cos(e+fx)\left(\frac{32A}{35}-\frac{16B}{105}\right)}{a^3c^4f\,(2\sin(e+fx)-2)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x
)
```

output
```
((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + c
os(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e
+ f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))
/35 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x)
- 2*cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*si
n(e + f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35
 - (16*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

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3.79.1 Optimal result

Integrand size = 36, antiderivative size = 162

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\,(c^5-c^5\sin(e+fx))}$$
$$+ \frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f} + \frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f} + \frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}$$

output

```
1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x
+e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*
(7*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f
```

3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(162) = 324.

Time = 3.75 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-184320B+1125(49A+13B$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^5),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*
(e + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A
*Cos[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 3
25*B*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)]
 - 1225*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x
] + 92160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*
x)] - 136192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4
*(e + f*x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*S
in[5*(e + f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168
*A*Sin[7*(e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + S
in[e + f*x])^5*(1 + Sin[e + f*x])^3)
```

3.79.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$\downarrow 3338$$

$$\frac{\frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}dx}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{(7A-2B)\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))}dx}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$$\downarrow 3151$$

$$\frac{\frac{(7A-2B)\left(\frac{6\int \sec^6(e+fx)dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{(7A-2B)\left(\frac{6\int \csc\left(e+fx+\frac{\pi}{2}\right)^6 dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$$\downarrow 4254$$

$$\frac{\frac{(7A-2B)\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\int \left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7cf}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$$\downarrow 2009$$

$$\frac{\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}+\frac{(7A-2B)\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}\right)}{9c}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B)*(Sec
[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])) - (6*(-Tan[e + f*x] - (2*Tan[e + f*
x]^3)/3 - Tan[e + f*x]^5/5))/(7*c*f)))/(9*c))/(a^3*c^3)
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.79.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{32\left(-2iB\,e^{2i(fx+e)}+2iB+315iA\,e^{6i(fx+e)}+133iA\,e^{4i(fx+e)}-38iB\,e^{4i(fx+e)}-90iB\,e^{6i(fx+e)}+28A\,e^{i(fx+e)}+180B\,e\right.}{315\left(e^{i(fx+e)}+i\right)^5\left(e^{i(f\right.}$
parallelrisch	$\frac{-630A\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(1260A-630B)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-420A+840B)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3360A-840B)\left(\tan\right.}{}$
derivativedivides	$-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{\frac{51A}{16}+\frac{21}{1}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
default	$-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{\frac{51A}{16}+\frac{21}{1}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
norman	$\frac{14A-40B}{252cfa}+\frac{2(602A-697B)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}-\frac{A\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2cfa}-\frac{(1036A-296B)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{105cfa}+\frac{(476A-136B)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{315cfa}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-32/315*(-2*I*B*exp(2*I*(f*x+e))+2*I*B+315*I*A*exp(6*I*(f*x+e))+133*I*A*ex
p(4*I*(f*x+e))-38*I*B*exp(4*I*(f*x+e))-90*I*B*exp(6*I*(f*x+e))+28*A*exp(I*
(f*x+e))+180*B*exp(7*I*(f*x+e))-8*B*exp(I*(f*x+e))+7*I*A*exp(2*I*(f*x+e))+
112*A*exp(3*I*(f*x+e))+140*A*exp(5*I*(f*x+e))-40*B*exp(5*I*(f*x+e))-32*B*e
xp(3*I*(f*x+e))-7*I*A)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^9/f/c^5/a^3
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{32\left(7A-2B\right)\cos\left(fx+e\right)^6-16\left(7A-2B\right)\cos\left(fx+e\right)^4-4\left(7A-2B\right)\cos\left(fx+e\right)^2-\left(16\left(7A-\right.\right.}{315\left(a^3c^5f\cos\left(fx+e\right)^7+2a^3c^{\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*
(7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*
B)*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x +
 e) - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*
sin(f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

3.79.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8396 vs. $2(150) = 300$.

Time = 59.13 (sec) , antiderivative size = 8396, normalized size of antiderivative = 51.83

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x
/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2
 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**
5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3
*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a
**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1260*A*tan(e/2 + f*x/2)**
12/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/
2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 +
 f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan
(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5
*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*
c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a*
*3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a*
*3*c**5*f) - 420*A*tan(e/2 + f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*
x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*
tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3...
```

3.79.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. 2(154) = 308.

Time = 0.26 (sec) , antiderivative size = 1201, normalized size of antiderivative = 7.41

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 - 483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)
^11/(cos(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25
)/(a^3*c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 19*a^3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 - 45*a^3*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - 16*a^3*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e
)^12/(cos(f*x + e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^
13 - a^3*c^5*sin(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 516*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos...
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(154) = 308.

Time = 0.37 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.40

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx =$$

$$-\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-940\,B\tan\left(\frac{1}{2}\right.\right.}{a^3c^5\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4
+ 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*ta
n(1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x +
 1/2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*
x + 1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x
+ 1/2*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e
)^7 + 546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 9
61380*A*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A
*tan(1/2*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/
2*f*x + 1/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x +
 1/2*e)^2 - 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e)
 + 6732*B*tan(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) - 1)^9))/f
```

3.79.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{\left(\frac{128\,B}{315} - \frac{64\,A}{45} + \frac{32\,A\,\sin(e+f\,x)}{45} - \frac{64\,B\,\sin(e+f\,x)}{315}\right)\cos(e+f\,x)^6 + \left(\frac{8\,A\,\sin(e+f\,x)}{9} - \frac{20\,B}{63} - \frac{8\,A}{9} + \frac{20\,B\,\sin(e+f\,x)}{63} -\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x
)
```

output
```
((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - c
os(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e +
f*x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^
2*((8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) +
 cos(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*s
in(e + f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e
+ f*x))/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e
+ f*x) - 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

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3.80.1 Optimal result

Integrand size = 36, antiderivative size = 205

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}$$
$$+ \frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f} + \frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f} + \frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}$$

output
```
1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*
x+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6
-c^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*
x+e)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f
```

3.80.2 Mathematica [A] (verified)

Time = 5.41 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.96

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{1013760B - 3850(107A-3B)\cos(e+fx) + 135168(8A-3B)\cos(2(e+fx)) - 127330A\cos(3(e+fx)$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^6),x]
```

output
```
(1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
 + f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*
Cos[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] -
1050*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f
*x)] + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(
e + f*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Si
n[e + f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*
A*Sin[3*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)]
 - 5250*B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e
+ f*x)] + 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin
[7*(e + f*x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B
*Sin[8*(e + f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^11*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

3.80.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6}\,dx$$
$$\downarrow 3446$$
$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$
$$\downarrow 3042$$
$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$\downarrow 3338$$

$$\frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}dx}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{(8A-3B)\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))^2}dx}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 3151$$

$$\frac{(8A-3B)\left(\frac{7\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}dx}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{(8A-3B)\left(\frac{7\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))}dx}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 3151$$

$$\frac{(8A-3B)\left(\frac{7\left(\frac{6\int \sec^6(e+fx)dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{(8A-3B)\left(\frac{7\left(\frac{6\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 4254$$

$$\frac{(8A-3B)\left(\frac{7\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\int\left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7cf}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\downarrow 2009$$

$$\frac{\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}+\frac{(8A-3B)\left(\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}+\frac{7\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}\right)}{9c}\right)}{11c}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^3) + ((8*A - 3*B)*(Se
c[e + f*x]^5/(9*f*(c - c*Sin[e + f*x])^2) + (7*(Sec[e + f*x]^5/(7*f*(c - c
*Sin[e + f*x])) - (6*(-Tan[e + f*x] - (2*Tan[e + f*x]^3)/3 - Tan[e + f*x]^
5/5))/(7*c*f)))/(9*c)))/(11*c))/(a^3*c^3)
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.80.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

method	result
risch	$\frac{32i\left(-48iA\,e^{i(fx+e)}+30iB\,e^{3i(fx+e)}+495B\,e^{8i(fx+e)}-102iB\,e^{5i(fx+e)}+528A\,e^{6i(fx+e)}+18iB\,e^{i(fx+e)}-198B\,e^{6i(fx+}\right.}{495(}$
parallelrisch	$\frac{-990A\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2970A-990B)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-3630A+1980B)\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4950A-2970B)}{}$
derivativedivides	$-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{217A}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$
default	$-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{217A}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
32/495*I*(-48*I*A*exp(I*(f*x+e))+30*I*B*exp(3*I*(f*x+e))+495*B*exp(8*I*(f*
x+e))-102*I*B*exp(5*I*(f*x+e))+528*A*exp(6*I*(f*x+e))+18*I*B*exp(I*(f*x+e)
)-198*B*exp(6*I*(f*x+e))+272*I*A*exp(5*I*(f*x+e))+400*A*exp(4*I*(f*x+e))+8
80*I*A*exp(7*I*(f*x+e))-150*B*exp(4*I*(f*x+e))-330*I*B*exp(7*I*(f*x+e))+80
*A*exp(2*I*(f*x+e))-80*I*A*exp(3*I*(f*x+e))-30*B*exp(2*I*(f*x+e))-8*A+3*B)
/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^11/f/c^6/a^3
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.08

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{16\,(8\,A-3\,B)\cos(fx+e)^8 - 72\,(8\,A-3\,B)\cos(fx+e)^6 + 30\,(8\,A-3\,B)\cos(fx+e)^4 + 7\,(8\,A-3\,B)}{495\left(3\,a^3c^6f\cos(fx+e)^7 - 4\,a^3\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*
(8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B
)*cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x
+ e)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e
)^7 - 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f
*cos(f*x + e)^5)*sin(f*x + e))
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11011 vs. 2(187) = 374.

Time = 100.07 (sec) , antiderivative size = 11011, normalized size of antiderivative = 53.71

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

output

```
Piecewise((-990*A*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**
16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e
/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6
*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a*
*3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16
830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**
4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/
2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*A*tan(
e/2 + f*x/2)**14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*
tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c
**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 1683
0*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**1
0 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*
x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/
2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6
*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 3630*A*tan(e/2 + f*x/2)**13/(495*
a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 +
 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x...
```

3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(196) = 392$.

Time $= 0.27$ (sec) , antiderivative size $= 1387$, normalized size of antiderivative $= 6.77$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output

```
-2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 821
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1
)^12 - 1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(
cos(f*x + e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a
^3*c^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 50*a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 110*a^3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 34*a^3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x
 + e)^12/(cos(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 10*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin
(f*x + e)^15/(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e)
 + 1)^16) + 3*B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^...
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(196) = 392$.

Time $= 0.38$ (sec) , antiderivative size $= 445$, normalized size of antiderivative $= 2.17$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx =$$

$$-\frac{33\left(555A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-315B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+1920A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-1020B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2710A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1410B\tan\right.}{a^3c^6\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output

```
-1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4
+ 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*t
an(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x
 + 1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*
f*x + 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2
*f*x + 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x +
1/2*e)^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)
^8 - 6652800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 +
10407474*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 1143
5424*A*tan(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270
*A*tan(1/2*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*ta
n(1/2*f*x + 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2
*f*x + 1/2*e)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x +
 1/2*e) + 56364*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1
/2*f*x + 1/2*e) - 1)^11))/f
```

3.80.9 Mupad [B] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.31

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2\left(\frac{165\,B\,\sin(e+f\,x)}{4} - \frac{6875\,A\,\cos(e+f\,x)}{64} - \frac{825\,B\,\cos(e+f\,x)}{64} - 110\,A\,\sin(e+f\,x) - \frac{495\,B}{8} - 66\,A\,\cos(2\,e+2\,f\,x) -\right.}{}$$

input

```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x
)
```

output

```
(2*((165*B*sin(e + f*x))/4 - (6875*A*cos(e + f*x))/64 - (825*B*cos(e + f*x
))/64 - 110*A*sin(e + f*x) - (495*B)/8 - 66*A*cos(2*e + 2*f*x) - (2125*A*c
os(3*e + 3*f*x))/64 - 50*A*cos(4*e + 4*f*x) + (625*A*cos(5*e + 5*f*x))/64
 - 10*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x)
 + (99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e
 + 4*f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (4
5*B*cos(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*
f*x))/64 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin
(5*e + 5*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*
A*sin(8*e + 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*
f*x))/4 + (375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*
sin(6*e + 6*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/
128))/(495*a^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 -
 (55*cos(e + f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32
 + (25*sin(4*e + 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/6
4))
```

3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

3.81.1 Optimal result 804
3.81.2 Mathematica [A] (verified) 805
3.81.3 Rubi [A] (verified) 805
3.81.4 Maple [A] (verified) 808
3.81.5 Fricas [A] (verification not implemented) 808
3.81.6 Sympy [F(-1)] 809
3.81.7 Maxima [F] 809
3.81.8 Giac [A] (verification not implemented) 810
3.81.9 Mupad [F(-1)] 810

3.81.1 Optimal result

Integrand size = 36, antiderivative size = 198

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}$$

output

```
256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/99*a*(11
*A-5*B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*
(c-c*sin(f*x+e))^(5/2)/f+64/1155*a*(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(
f*x+e))^(1/2)+8/231*a*(11*A-5*B)*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

3.81.2 Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{ac^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-35332A+27085B+60(121A-202B)\cos$$

$$13860f\left(c$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e +
 f*x]]*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*C
os[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[
3*(e + f*x)] + 2870*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]))
```

3.81.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx$$

$$\downarrow 3335$$

$$ac\left(\frac{1}{11}(11A-5B)\int \cos^2(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{11}(11A-5B)\int \cos(e+fx)^2(c-c\sin(e+fx))^{5/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}\right)$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)-\frac{2B\,c}{}\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\int \cos(e+fx)^2(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)-\frac{2B\,c}{}\right.$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)+\frac{2c\,co}{}\right.\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)+\frac{2c\,co}{}\right.\right.$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin}}{7f}\right.\right.\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin}}{7f}\right.\right.\right.$$

$$\downarrow 3152$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e}}{7f}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f) + ((11*A - 5*
B)*((2*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f) + (4*c*((2*c*Cos
[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + (8*c*((8*c^2*Cos[e + f*x]^3)
/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*
Sin[e + f*x]])))/7))/3))/11)
```

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.81.4 Maple [A] (verified)

Time = 7.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^2a(315B(\cos^4(fx+e))+(-385A+1435B)(\cos^2(fx+e))\sin(fx+e)+(1815A-3345B)(\cos^2(fx+e)}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$
parts	$\frac{2aA(\sin(fx+e)-1)c^4(1+\sin(fx+e))(5(\sin^3(fx+e))-27(\sin^2(fx+e))+71\sin(fx+e)-177)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}+\frac{2Ba(\sin(fx+e)-1)c^4(1+\sin(fx+e}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3465*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*(315*B*cos(f*x+e)^4+(-385*A+1
435*B)*cos(f*x+e)^2*sin(f*x+e)+(1815*A-3345*B)*cos(f*x+e)^2+(3916*A-4300*B
)*sin(f*x+e)-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e$$
$$+fx))^{7/2}\,dx = \frac{2\left(315\,Bac^3\cos\left(fx+e\right)^6-35\left(11\,A-32\,B\right)ac^3\cos\left(fx+e\right)^5+5\left(209\,A-221\,B\right)ac^3\cos\left(fx\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
 + 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(
f*x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3
*cos(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*
(11*A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)
^3 - 96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x
 + e) - 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(7/2), x)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{\sqrt{2}\left(6930\,Bac^3\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-315\,Bac^3\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
-1/55440*sqrt(2)*(6930*B*a*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 315*B*a*c^3*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 48510*(2*A*a*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) - B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 693*(16*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 5*B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/
2*e) + 495*(10*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 9*B*a*c^3*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 385*(2*
A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a*c^3*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (A + B\sin(e+fx))\,(a+a\sin(e+fx))\,(c-c\sin(e+fx))^{7/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),
x)
```

3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.82.1 Optimal result

Integrand size = 36, antiderivative size = 157

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}} + \frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

output

```
64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/9*a*B*c*cos(f
*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f+16/105*a*(3*A-B)*c^3*cos(f*x+e)^3/f/(c-c*
sin(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

3.82.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{ac^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-942A+664B+30(3A-8B)\cos(2(e+fx$$
$$630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f
*x]]*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*S
in[e + f*x] + 35*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]))
```

3.82.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3446$$
$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$
$$\downarrow 3042$$
$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$
$$\downarrow 3335$$

$$ac\left(\frac{1}{3}(3A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}dx - \frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{3}(3A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))^{3/2}dx - \frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx + \frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right) - \frac{2B\cos^3(e+\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx + \frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right) - \frac{2B\cos^3(e+\right.$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx + \frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right) + \frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx}\right.\right.$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx + \frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right) + \frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx}\right.\right.$$

$$\downarrow 3152$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}} + \frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right) + \frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f) + ((3*A - B)*(
(2*c*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + (8*c*((8*c^2*Cos[e +
 f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]^3)/(5*f*Sqr
t[c - c*Sin[e + f*x]])))/7))/3)
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.82.4 Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^2a(-35B(\cos^2(fx+e))\sin(fx+e)+(-45A+120B)(\cos^2(fx+e))+(-162A+194B)\sin(fx+e)+$ $315\cos(fx+e)\sqrt{c-c\sin(fx+e)}f$
parts	$-\frac{2aA(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}-\frac{2Ba(\sin(fx+e)-1)c^3(1+\sin(fx+e))(35(\sin^4(fx+e$ $315\cos(fx-$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e
)+(-45*A+120*B)*cos(f*x+e)^2+(-162*A+194*B)*sin(f*x+e)+258*A-226*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.55

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{2\left(35\,Bac^2\cos(fx+e)^5+5\,(9\,A-10\,B)ac^2\cos(fx+e)^4+(117\,A-109\,B)ac^2\cos(fx+e)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (
117*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 3
2*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x
+ e)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x
+ e)^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e
))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.82.6 Sympy [F]

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = a\left(\int Ac^2\sqrt{-c\sin(e+fx)+c}\,dx + \int \left(-Ac^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\right)dx + \int \left(-Ac^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int Ac^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int \left(-Bc^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int \left(-Bc^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
a*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c**2*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c**2*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
, x) + Integral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + In
tegral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*
c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x))
```

3.82.7 Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2), x)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(35\,Bac^2\cos\left(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 630\left(5\,Aac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/2520*sqrt(2)*(35*B*a*c^2*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 630*(5*A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 2*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 210*(A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 126*(3*A*a*
c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*A*a*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 3*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos
(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (A + B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),
x)
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.83.1 Optimal result

Integrand size = 36, antiderivative size = 116

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}} + \frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}} - \frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

output

```
8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c
^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f
*x+e))^(1/2)/f
```

3.83.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{ac\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(98A-59B+15B\cos(2(e+fx))+(-42A+66B)\right)}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
(a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e
+ f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.83.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3335$$

$$ac\left(\frac{1}{7}(7A-B)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{7}(7A-B)\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\downarrow 3153$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{4}{5}c\int\frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{4}{5}c\int\frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\downarrow 3152$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + ((7*A - B)*((8
*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]
^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])))/7)
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.83.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^2a(-15B(\cos^2(fx+e))+\sin(fx+e)(21A-33B)-49A+37B)}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2aA(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba(\sin(fx+e)-1)c^2(1+\sin(fx+e))(15(\sin^3(fx+e))-39(\sin^2(fx+e))+5}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(-15*B*cos(f*x+e)^2+sin(f*x+e)
*(21*A-33*B)-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{2\left(15\,Bac\cos(fx+e)^4-3\,(7\,A-6\,B)ac\cos(fx+e)^3+(7\,A-B)ac\cos(fx+e)^2-4\,(7\,A-B)ac\cos(}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A
 - B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c -
  (15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a
*c*cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.83.6 Sympy [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = a\left(\int Ac\sqrt{-c\sin(e+fx)+c}\,dx\right.$$
$$+\int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)\,dx$$
$$+\int Bc\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$\left.+\int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x), x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**
3, x))
```

3.83.7 Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{2}\left(15\,Bac\cos\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-105\left(4\,Aac\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi\right.\right.\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 105*(4*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) - 35*(2*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (A + B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),
x)
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+f$

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3.84.1 Optimal result

Integrand size = 36, antiderivative size = 73

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}} - \frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

output
```
2/15*a*(5*A+B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/5*a*B*c*cos(f*x
+e)^3/f/(c-c*sin(f*x+e))^(1/2)
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(73) = 146.

Time = 1.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.62

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(32B-30\sqrt{2}A\sqrt{1+\cos(e+fx)}\right)+\sqrt{2}\sqrt{1+\cos(e+fx)}\left(5(2A+B)\cos\left(\frac{3}{2}(e+\right.\right.\right.}{30\sqrt{2}f\sqrt{1+\cos(e}}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
-1/30*(a*(Cos[(e + f*x)/2]*(32*B - 30*Sqrt[2]*A*Sqrt[1 + Cos[e + f*x]]) +
Sqrt[2]*Sqrt[1 + Cos[e + f*x]]*(5*(2*A + B)*Cos[(3*(e + f*x))/2] + 3*B*Cos
[(5*(e + f*x))/2] - 2*(20*A + B + 2*(5*A + B)*Cos[e + f*x] - 3*B*Cos[2*(e
+ f*x)])*Sin[(e + f*x)/2]))*Sqrt[c - c*Sin[e + f*x]])/(Sqrt[2]*f*Sqrt[1 +
Cos[e + f*x]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.84.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3335$$

$$ac\left(\frac{1}{5}(5A+B)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{1}{5}(5A+B)\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3152$$

$$ac\left(\frac{2c(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

output
```
a*c*((2*(5*A + B)*c*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2
*B*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]]))
```

3.84.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.84.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^2a(3B\sin(fx+e)+5A-2B)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2aA(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f} - \frac{2Ba(\sin(fx+e)-1)c(1+\sin(fx+e))(3(\sin^2(fx+e))-4\sin(fx+e)+8)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f} - \frac{2a(A+B)(\sin($

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\left(3\,Ba\cos(fx+e)^3+(5\,A+4\,B)a\cos(fx+e)^2-(5\,A+B)a\cos(fx+e)-2\,(5\,A+B)a+\left(3\,Ba\c\right.}{15\,(f\cos(fx+e)-f\sin$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
-2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*c
os(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x
+ e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x +
 e) - f*sin(f*x + e) + f)
```

3.84.6 Sympy [F]

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= a\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx + \int A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx \right.$$
$$+ \int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$\left. + \int B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

3.84.7 Maxima [F]

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)\sqrt{-c\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c), x)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{\sqrt{2}\left(30\,Aa\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Ba\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\operatorname{sgn}(\text{si}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-1/30*sqrt(2)*(30*A*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*B*a*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 5*(2*A*a*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(c)/f
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),
x)
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.85.1 Optimal result 832
3.85.2 Mathematica [A] (verified) 832
3.85.3 Rubi [A] (verified) 833
3.85.4 Maple [A] (verified) 836
3.85.5 Fricas [B] (verification not implemented) 836
3.85.6 Sympy [F] 837
3.85.7 Maxima [F] 837
3.85.8 Giac [B] (verification not implemented) 838
3.85.9 Mupad [F(-1)] 838

3.85.1 Optimal result

Integrand size = 36, antiderivative size = 122

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}a(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

output

```
2*a*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2
^(1/2)/f/c^(1/2)-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a
*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f
```

3.85.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sqrt{-c(1+\sin(e+fx))}+\right.}{3\sqrt{c}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sq
rt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x])
)] + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])
))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x
]])
```

3.85.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3337, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3337$$

$$ac\left(\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}-\frac{2\int -\frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{2\sqrt{c-c\sin(e+fx)}}dx}{3c^2}\right)$$

$$\downarrow 27$$

$$ac\left(\frac{\int \frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{\int \frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\downarrow 3230$$

$$ac\left(\frac{6c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{6c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\downarrow 3128$$

$$ac\left(\frac{-\frac{12c(A+B)\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\downarrow 219$$

$$ac\left(\frac{\frac{6\sqrt{2}\sqrt{c}(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a*c*((2*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c^2*f) + ((6*Sqrt[2]*(
A + B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/f - (2*(3*A + 5*B)*c*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]]))/
(3*c^2))
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3337
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Simp[1/(b^2*(m + 3))   I
nt[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
 a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.85.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a\left(3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A+3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-B(c(1}{3c^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{aA(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{Ba(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(3c^{\frac{3}{2}}\sqrt{2}\,\text{arcta}\right.}{3c^2\cos(fx+e)\sqrt{c}}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(3*c^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+3*c^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-B*(c*(1+sin(f*x+e)))^(3/2)-3*(c*(1+sin(f*x+e)))^(1/2)*A*c-3*(c*(1+sin(f*x+e)))^(1/2)*B*c)/c^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.08

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$=\frac{3\sqrt{2}((A+B)ac\cos(fx+e)-(A+B)ac\sin(fx+e)+(A+B)ac)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{c}}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
 B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2
)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*
cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e
) - (3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

3.85.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = a\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt
(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) +
 c), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

3.85.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
 c), x)
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(105) = 210$.

Time $= 0.33$ (sec) , antiderivative size $= 300$, normalized size of antiderivative $= 2.46$

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$

$$= \frac{\frac{3\sqrt{2}(Aa\sqrt{c}+Ba\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4\sqrt{2}\left(3\,Aa\sqrt{c}+5\,Ba\sqrt{c}-\frac{6\,Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6\,Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+}}{c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)^3\,s}}{3f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
1/3*(3*sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))) - 4*sqrt(2)*(3*A*a*sqrt(c) + 5*B*a*sqrt(c) - 6*A*a*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 - 6*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) + 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c*((cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^3*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))}{\sqrt{c - c\,\sin(e + f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2)
, x)
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

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3.86.1 Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a(A+5B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output
```
a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*(A+5*B)*arctanh(1/2*cos(
f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)+2*a*B*cos
(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\sec(e+fx)\left(\sqrt{2}(A+5B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)(\cos$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(S
qrt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin
[e + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B
- 2*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

3.86.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^3}+\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{2c^3}\right)$$

$$\downarrow 3230$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{c(A+5B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{c(A+5B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\downarrow 3128$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{-\frac{2c(A+5B)\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\downarrow 219$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\frac{\sqrt{2}\sqrt{c}(A+5B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(c*f*(c - c*Sin[e + f*x])^(3/2)) - ((Sqrt[2]*(
A + 5*B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/f - (4*B*c*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]]))/(2*c^3))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 1.69 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

method	result
default	$\frac{a\left(A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+5B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx}{2\sqrt{c}}\right.\right.}{2}$
parts	$\frac{aA\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\sin(fx+e)+2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\right)\sqrt{c(1+\sin(f}{4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(5/2)*a*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.77

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left((A+5B)ac\cos(fx+e)^2-(A+5B)ac\cos(fx+e)-2(A+5B)ac+((A+5B)ac\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) -
2*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x +
 e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sq
rt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(
f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) +
(A + B)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x
 + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*
cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.86.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right.$$
$$+\int \frac{A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$\left.+\int \frac{B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*
sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/
(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)),
 x) + Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
) + c*sqrt(-c*sin(e + f*x) + c)), x))
```

3.86.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(3/2), x)
```

3.86.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(102) = 204$.

Time $= 0.36$ (sec) , antiderivative size $= 390$, normalized size of antiderivative $= 3.39$

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$

$$\frac{2\sqrt{2}\left(Aa\sqrt{c}+5\,Ba\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(\frac{Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(Aa\sqrt{}\right.}{}$$

$$-\frac{}{8f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
-1/8*(2*sqrt(2)*(A*a*sqrt(c) + 5*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 28*B*a*sq
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1) - A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^2 - 5*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c^2*((cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - (cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2)
, x)
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

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3.87.1 Optimal result

Integrand size = 36, antiderivative size = 126

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(A-7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

output
```
1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*(A+9*B)*cos(f*x+e)/c
/f/(c-c*sin(f*x+e))^(3/2)-1/16*a*(A-7*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^
(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 4.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(\sqrt{2}(A-7B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(e+fx))}\right)}{16c^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[
Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1
+ Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A
- 5*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))
/(c^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x
]])
```

3.87.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^3}+\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int \frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}dx}{4c^3}\right)$$

$$\downarrow 3229$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{1}{4}(A-7B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{1}{4}(A-7B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

$$\downarrow 3128$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A-7B)\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2f}}{4c^3}\right)$$

$$\downarrow 219$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{(A-7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\sqrt{c}f}+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) - (((A - 7*
B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(2*
Sqrt[2]*Sqrt[c]*f) + ((A + 9*B)*c*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^
(3/2)))/(4*c^3))
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(107) = 214$.

Time = 2.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

method	result
default	$-\frac{a\left(\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^2(A-7B)(\cos^2(fx+e))+2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^2(A-7B)+2A(c+c\,s\right.}{}$
parts	$\frac{aA\left(-3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2+6\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\sin(fx+e)+6\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\right.}{32c^{\frac{9}{2}}(\sin(fx+e)-1)\cos(fx+e)\sqrt{c-c\sin(}}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/16/c^(9/2)*a*(arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)*c^2*(A-7*B)*cos(f*x+e)^2+2*sin(f*x+e)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)*c^2*(A-7*B)+2*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+
4*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e)
)^(1/2)*2^(1/2)/c^(1/2))*c^2+18*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-28*B*(c+c
*sin(f*x+e))^(1/2)*c^(3/2)+14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)
*2^(1/2)/c^(1/2))*c^2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.13

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left((A-7\,B)a\cos\left(fx+e\right)^3+3\,(A-7\,B)a\cos\left(fx+e\right)^2-2\,(A-7\,B)a\cos\left(fx+e\right)-4\,(A-7\,B)a-\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2
- 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2
 - 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-
(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x +
 e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*
x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) -
4*(A + B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c
^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
 - 4*c^3*f)*sin(f*x + e))
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.87.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(5/2), x)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(107) = 214$.

Time = 0.38 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$\frac{4\sqrt{2}(Aa-7Ba)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\sqrt{2}\left(Aa\sqrt{c}+Ba\sqrt{c}+\frac{16\,Ba\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{6\,Aa\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}+\right.}{c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/128*(4*sqrt(2)*(A*a - 7*B*a)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))) + sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) + 16*B*a*sqrt(c)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 6*A*a*sq
rt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^2 + 42*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^
3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) - (16*sqrt(2)*B*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + sqrt(2
)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + sqrt(2)*B*a*c^(7/2
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/c^6)/f
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2)
, x)
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

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3.88.1 Optimal result

Integrand size = 36, antiderivative size = 163

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(A-3B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

output

```
1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*(A+13*B)*cos(f*x+e)
/c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e
))^(3/2)-1/64*a*(A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*
x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 6.37 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(3\sqrt{2}(A-3B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin(e+fx))}\right)}{192c^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
-1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcT
an[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*
(1 + Sin[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*
A - 13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^7))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

3.88.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3336$$

$$ac\left(\frac{\int -\frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}+\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}\right)$$

$$\downarrow 25$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\int\frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\int\frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}\right)$$

$$\downarrow 3229$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\downarrow 3129$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\downarrow 3042$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\downarrow 3128$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)+\frac{c(A+13B)}{4f(c-c\sin(}{6c^3}\right)$$

$$\downarrow 219$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx}{4f(c-c\sin(e+fx))^{5/}}}{6c^3}\right.$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(3*c*f*(c - c*Sin[e + f*x])^(7/2)) - (((A + 13
*B)*c*Cos[e + f*x])/(4*f*(c - c*Sin[e + f*x])^(5/2)) + (3*(A - 3*B)*(ArcTa
nh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c
^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/8)/(6*c^3))
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(140) = 280.

Time = 2.97 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.16

method	result
default	$-\frac{a\left(3\arctanh\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^4(A-3B)(\cos^2(fx+e))\sin(fx+e)-9\arctanh\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^4(A-3B)(\cos^2(f$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/192/c^(15/2)*a*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^4*(A-3*B)*cos(f*x+e)^2*sin(f*x+e)-9*arctanh(1/2*(c+c*sin(f*x+e))^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*cos(f*x+e)^2-12*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*sin(f*x+e)+6*A*(c
+c*sin(f*x+e))^(5/2)*c^(3/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-24*A*(c+c
*sin(f*x+e))^(1/2)*c^(7/2)-18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-32*B*(c+c*s
in(f*x+e))^(3/2)*c^(5/2)+72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+12*A*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-36*B*2^(1/2)*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/
2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(140) = 280$.

Time $= 0.28$ (sec) , antiderivative size $= 490$, normalized size of antiderivative $= 3.01$

$$\int \frac{(a + a\sin(e+fx))(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\big((A-3B)a\cos(fx+e)^4 - 3(A-3B)a\cos(fx+e)^3 - 8(A-3B)a\cos(fx+e)^2 + 4(A-3B)a$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)
^3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*
B)*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A -
 3*B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*
x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin
(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) +
2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)
) - 4*(3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(1
1*A - B)*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2
*(5*A + 17*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x
 + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f
*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4
*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(7/2), x)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(140) = 280$.

Time $= 0.45$ (sec) , antiderivative size $= 628$, normalized size of antiderivative $= 3.85$

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
-1/1536*(12*sqrt(2)*(A*a*sqrt(c) - 3*B*a*sqrt(c))*log(-(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c) + B*a*sqrt(c) - 3*A*a*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
1) + 9*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1) - 3*A*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 22*A*a*sqrt(c
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^3 - 66*B*a*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3/(c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
- sqrt(2)*(3*A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) + 3*B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*a*c^(17/2)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 9*B*a*c^
(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2 - A*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^3 - B*a*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)/(c^12*sgn(sin(-1/4*pi...
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2)
, x)
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

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3.89.1 Optimal result

Integrand size = 38, antiderivative size = 210

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f} - \frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}$$

output

```
256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003
*a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^2*B*c^2*c
os(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/f+8/429*a^2*(13*A-3*B)*c^4*cos(f*x+e)^5
/f/(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f
*x+e))^(1/2)/f
```

3.89.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(210) = 420$.

Time $= 10.72$ (sec) , antiderivative size $= 1355$, normalized size of antiderivative $= 6.45$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^
(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x
))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (
(A - 4*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos
[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4) + ((7*A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Si
n[e + f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin...
```

3.89.3 Rubi [A] (verified)

Time $= 1.09$ (sec) , antiderivative size $= 189$, normalized size of antiderivative $= 0.90$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used $= \{3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\downarrow 3335$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\int \cos(e+fx)^4(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\right)$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)-\frac{2B\cos}{}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\int \cos(e+fx)^4\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)-\frac{2B\cos}{}\right.$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(}}{11f}\right.\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\int\frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(}}{11f}\right.\right.$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx}{9f\sqrt{c-c\sin(e+}}\right.\right.\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx}{9f\sqrt{c-c\sin(e+}}\right.\right.\right.$$

$$\downarrow 3152$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f) + ((13*A
- 3*B)*((2*c*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f) + (12*c*((2*c
*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + (8*c*((8*c^2*Cos[e + f*x
]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e + f*x]^5)/(7*f*(c - c*
Sin[e + f*x])^(3/2))))/9))/11))/13)
```

3.89.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.89.4 Maple [A] (verified)

Time = 36.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^3a^2(1155B(\cos^4(fx+e))+(-1365A+4935B)(\cos^2(fx+e))\sin(fx+e)+(5915A-10605B)(\cos^2(f$
	$15015\cos(fx+e)\sqrt{c-c\sin(fx+e)}f$
parts	$\frac{2Aa^2(\sin(fx+e)-1)c^4(1+\sin(fx+e))(5(\sin^3(fx+e))-27(\sin^2(fx+e))+71\sin(fx+e)-177)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}+\frac{2Ba^2(\sin(fx+e)-1)c^4(1+\sin(fx$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*(1155*B*cos(f*x+e)^4+(-136
5*A+4935*B)*cos(f*x+e)^2*sin(f*x+e)+(5915*A-10605*B)*cos(f*x+e)^2+(11180*A
-11820*B)*sin(f*x+e)-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{2\left(1155\,Ba^2c^3\cos(fx+e)^7+105\,(13\,A-14\,B)a^2c^3\cos(fx+e)^6+35\,(91\,A-87\,B)a^2c^3\cos\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
 + e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^
3*cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B
)*a^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13
*A - 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2
*c^3*cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A
- 3*B)*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 +
256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x
+ e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(-c\sin(fx + e) + c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c
)^(7/2), x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.78

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = -\frac{\sqrt{2}\left(10010\,Aa^2c^3\cos\left(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 1155\,Ba^2c^3\cos\left(-\frac{13}{4}\pi + \frac{13}{2}fx -\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
-1/480480*sqrt(2)*(10010*A*a^2*c^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 1155*B*a^2*c^3*cos(-13/4*pi + 13/2*f*x + 13/
2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 60060*(7*A*a^2*c^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(4*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*p
i + 3/2*f*x + 3/2*e) - 3003*(22*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) - 7*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f
*x + 5/2*e) + 4290*(A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^
2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e)
- 1365*(2*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c
)/f
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2\,(c - c\,\sin(e + f\,x))^{7/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2)
, x)
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.90.1 Optimal result

Integrand size = 38, antiderivative size = 167

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

output

```
64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*
(11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(
f*x+e))^(1/2)/f
```

3.90.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1173 vs. $2(167) = 334$.

Time $= 12.62$ (sec) , antiderivative size $= 1173$, normalized size of antiderivative $= 7.02$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5
/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*
(c - c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/
2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A
 + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(
5/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])
^2*(c - c*Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a
 + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B
)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin
[(3*(e + f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(5/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - ...
```

3.90.3 Rubi [A] (verified)

Time $= 0.90$ (sec) , antiderivative size $= 152$, normalized size of antiderivative $= 0.91$, number of steps used $= 10$, number of rules used $= 10$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used $= \{3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3335$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\int \cos(e+fx)^4\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+f$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\int \frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+f$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{4}{7}c\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{4}{7}c\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3152$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f) + ((11*A -
B)*((2*c*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + (8*c*((8*c^2*Cos
[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e + f*x]^5)/(7*f
*(c - c*Sin[e + f*x])^(3/2))))/9))/11)
```

3.90.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.90.4 Maple [A] (verified)

Time = 36.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^3a^2(-315B(\cos^2(fx+e))\sin(fx+e)+(-385A+980B)(\cos^2(fx+e))+(-1210A+1370B)\sin(fx}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$
parts	$-\frac{2Aa^2(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}-\frac{2Ba^2(\sin(fx+e)-1)c^3(1+\sin(fx+e))(63(\sin^5(fx}$

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-2/3465*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f
*x+e)+(-385*A+980*B)*cos(f*x+e)^2+(-1210*A+1370*B)*sin(f*x+e)+1562*A-1402*
B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(151) = 302.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{2\left(315\,Ba^2c^2\cos(fx+e)^6-35\,(11\,A-10\,B)a^2c^2\cos(fx+e)^5+5\,(11\,A-B)a^2c^2\cos(fx+e)^4-8\,(11$$

input

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
 e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x
 + e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos
(f*x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(1
1*A - B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 4
8*(11*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) +
 128*(11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

3.90.6 Sympy [F]

$$\int (a + a\sin(e+fx))^2 (A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = a^2\left(\int Ac^2\sqrt{-c\sin(e+fx)+c}\,dx + \int \left(-2Ac^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int Ac^2\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int \left(-2Bc^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
a**2*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*A*c**2*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x), x) + Integral(-2*B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
 + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5
, x))
```

3.90.7 Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(151) = 302$.

Time $= 0.62$ (sec) , antiderivative size $= 340$, normalized size of antiderivative $= 2.04$

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(315\,Ba^2c^2\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6930\left(6\,Aa^2c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/55440*sqrt(2)*(315*B*a^2*c^2*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6930*(6*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 2310*(4*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 693*(8*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 495*(2*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) + 385*(2*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^2\,(c - c\sin(e + fx))^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
, x)
```

3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.91.1 Optimal result

Integrand size = 38, antiderivative size = 120

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

output
```
8/315*a^2*(9*A+B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/63*a^2*(9*A+
B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/9*a^2*B*c^2*cos(f*x+e)^5/f/
(c-c*sin(f*x+e))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 9.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{a^2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(162A-87B+35B\cos(2(e+fx))+(-90A+130$$
$$315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*
(e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.91.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3335$$

$$a^2c^2\left(\frac{1}{9}(9A+B)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{9}(9A+B)\int \frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3153$$

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{4}{7}c\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{4}{7}c\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3152$$

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + ((9*A + B)
*((8*c^2*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e +
f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))))/9)
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.91.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^3a^2(-35B(\cos^2(fx+e))+\sin(fx+e)(45A-65B)-81A+61B)}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2Aa^2(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba^2(\sin(fx+e)-1)c^2(1+\sin(fx+e))(35(\sin^4(fx+e))-85(\sin^3(fx+e))}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/315*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(-35*B*cos(f*x+e)^2+sin(f*x+
e)*(45*A-65*B)-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(108) = 216.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(35\,Ba^2c\cos(fx+e)^5+5\,(9\,A+8\,B)a^2c\cos(fx+e)^4-(9\,A+B)a^2c\cos(fx+e)^3+2\,(9\,A+B)a^2c\,c\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (
9*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
+ B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4
- 5*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*
(9*A + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.91.6 Sympy [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = a^2\left(\int Ac\sqrt{-c\sin(e+fx)+c}\,dx\right.$$
$$+\int Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$+\int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)\,dx$$
$$+\int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)\,dx$$
$$+\int Bc\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$+\int Bc\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx$$
$$+\int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)\,dx$$
$$\left.+\int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\right)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*si
n(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
)**3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integ
ral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e +
f*x) + c)*sin(e + f*x)**4, x))
```

3.91.7 Maxima [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(-c\sin(fx + e) + c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c
)^(3/2), x)
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(108) = 216$.

Time $= 0.65$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 1.98$

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = -\frac{\sqrt{2}\left(1890\,Aa^2c\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 35\,Ba^2c\cos\left(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e\right)\right.}{\text{sg}}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
-1/2520*sqrt(2)*(1890*A*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 35*B*a^2*c*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 210*(3*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 
1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f
*x + 3/2*e) - 126*(A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 45*(2
*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2\,(c - c\,\sin(e + f\,x))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
, x)
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

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3.92.1 Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

output
```
2/35*a^2*(7*A+3*B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/7*a^2*B*c^2
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(7A-2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.92.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3335$$

$$a^2c^2\left(\frac{1}{7}(7A+3B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{1}{7}(7A+3B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3152$$

$$a^2c^2\left(\frac{2c(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a^2*c^2*((2*(7*A + 3*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*B*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2)))
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.92.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^3a^2(5B\sin(fx+e)+7A-2B)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$
parts	$-\frac{2Aa^2(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}f} - \frac{2Ba^2(\sin(fx+e)-1)c(1+\sin(fx+e))(5(\sin^3(fx+e))-6(\sin^2(fx+e))+8\sin(fx+e)-16}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.38

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= \frac{2\left(5\,Ba^2\cos(fx+e)^4 - (7\,A+8\,B)a^2\cos(fx+e)^3 - (21\,A+19\,B)a^2\cos(fx+e)^2 + 2\,(7\,A+3\,B)a^2\,\mathrm{co}}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19
*B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^
2 - (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3
*B)*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x +
e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.92.6 Sympy [F]

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= a^2\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx + \int 2A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx\right.$$
$$+ \int A\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx + \int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$+ \int 2B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx$$
$$\left. + \int B\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) +
Integral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

3.92.7 Maxima [F]

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^2\sqrt{-c\sin(fx+e) + c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
 + c), x)
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(73) = 146$.

Time $= 0.54$ (sec) , antiderivative size $= 200$, normalized size of antiderivative $= 2.47$

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}\left(5\,Ba^2\cos\left(-\frac{7}{4}\pi + \frac{7}{2}fx + \frac{7}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 35\left(4\,Aa^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/140*sqrt(2)*(5*B*a^2*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 35*(4*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 35*
(2*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 7*(2*A*a^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 3*B*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-
5/4*pi + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx)) \sqrt{c - c\sin(e + fx)}\, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2\,\sqrt{c - c\,\sin(e + f\,x)}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
, x)
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

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3.93.1 Optimal result

Integrand size = 38, antiderivative size = 161

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{4\sqrt{2}a^2(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$
$$- \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

output
```
-2/5*a^2*B*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/3*a^2*(A+B)*c*cos(f
*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2
)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*(A+B)*cos(f*x+e)
/f/(c-c*sin(f*x+e))^(1/2)
```

3.93.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \$\text{Aborted}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
$Aborted
```

3.93.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3339, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3339$$

$$a^2c^2\left((A+B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left((A+B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}\,dx-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left((A+B)\left(\frac{2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left((A+B)\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\right.$$

$$\downarrow 3128$$

$$a^2c^2\left((A+B)\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{}{5f}\right.$$

$$\downarrow 219$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos}{5f(c-c\sin}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2)) + (A + B)*
((-2*Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*A
rcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2
)*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]]))))/c))
```

3.93.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.93.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^2\left(30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A+30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-3E\right.}{15c^3\cos(fx+e)\sqrt{}}$
parts	$-\frac{A\,a^2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{B\,a^2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(15c^{\frac{5}{2}}\sqrt{2}\,a\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(30*c^(5/2)*2^(1/2)*arct
anh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+30*c^(5/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-3*B*(c*(1+sin(f*x+e))
)^(5/2)-5*A*(c*(1+sin(f*x+e)))^(3/2)*c-5*B*(c*(1+sin(f*x+e)))^(3/2)*c-30*(
c*(1+sin(f*x+e)))^(1/2)*A*c^2-30*(c*(1+sin(f*x+e)))^(1/2)*B*c^2)/c^3/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(142) = 284.

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= 2\left(\frac{15\sqrt{2}((A+B)a^2c\cos(fx+e)-(A+B)a^2c\sin(fx+e)+(A+B)a^2c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx}\right.\right.$$
$$\sqrt{c}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e)
+ (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt
(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^
2*cos(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 +
(3*B*a^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*
a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

3.93.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= a^2\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)
/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e
+ f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) +
Integral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*si
n(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x))
```

3.93.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
 + c), x)
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(142) = 284$.

Time $= 0.48$ (sec) , antiderivative size $= 480$, normalized size of antiderivative $= 2.98$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= 2\left(\frac{15\sqrt{2}\left(Aa^2\sqrt{c}+Ba^2\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{8\sqrt{2}\left(10Aa^2\sqrt{c}+13Ba^2\sqrt{c}-\frac{35Aa^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-\frac{35Ba^2\sqrt{c}\left(\right.\right.}{\cos\left(\right.}\right.$$
$$=$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
2/15*(15*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - 8*sqrt(2)*(10*A*a^2*sqrt(c) + 13*B*a^2*sqrt(c) - 35
*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 35*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 55*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 85*B*a^2*sqrt(c
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2 - 45*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^3 - 45*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*a^2*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4
 + 30*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^4)/(c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/
f
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/
2), x)
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

3.94.1 Optimal result . . . 901
3.94.2 Mathematica [C] (verified) . . . 901
3.94.3 Rubi [A] (verified) . . . 902
3.94.4 Maple [A] (verified) . . . 906
3.94.5 Fricas [B] (verification not implemented) . . . 906
3.94.6 Sympy [F] . . . 907
3.94.7 Maxima [F] . . . 908
3.94.8 Giac [B] (verification not implemented) . . . 908
3.94.9 Mupad [F(-1)] . . . 909

3.94.1 Optimal result

Integrand size = 38, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}a^2(3A+7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/2*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+1/6*a^2*(3*A+7*B)*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-a^2*(3*A+7*B)*arctanh(1/2*cos(f*x+e)
*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+a^2*(3*A+7*B)*c
os(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*
ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^4*(c - c*Sin[e + f*x])^(3/2))
```

3.94.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{4c}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\downarrow 3128$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e\right.}{3cf(c-c\sin(}\right.$$

$$\Big\downarrow 219$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}\right.}{4c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])^(7/2)) - ((3*A
 + 7*B)*((-2*Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*S
qrt[2]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(c^(3/2)*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/(4*c)
)
```

3.94.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.94.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.60

method	result
default	$-\frac{a^2\left(\sin(fx+e)\left(6A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-9A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+2B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{c}+18B\sqrt{c+c\sin(fx+e)}\right.\right.}{}$
parts	$-\frac{A\,a^2\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\sin(fx+e)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2-2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\right)\sqrt{c(1+\sin}$
	$4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/3/c^(7/2)*a^2*(sin(f*x+e)*(6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-9*A*2^(1/
2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+2*B*(c+c*sin(f*
x+e))^(3/2)*c^(1/2)+18*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-21*B*2^(1/2)*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)-12*A*(c+c*sin(f*x+e))^
(1/2)*c^(3/2)+9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2-2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-24*B*(c+c*sin(f*x+e))^(1/2)*c^(
3/2)+21*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)
*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.19

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}\left((3A+7B)a^2c\cos(fx+e)^2-(3A+7B)a^2c\cos(fx+e)-2(3A+7B)a^2c+(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f
*x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A +
 7*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e)
 + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A
+ 10*B)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2
+ (B*a^2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f
*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.94.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a^2\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right.$$
$$+\int \frac{2A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{A\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{2B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$\left.+\int \frac{B\sin^3(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**2*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*si
n(e + f*x) + c)), x) + Integral(2*A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(A*sin(e +
f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x
) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(2*B*sin(e + f*x)**2/(
-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)),
x) + Integral(B*sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + c*sqrt(-c*sin(e + f*x) + c)), x))
```

3.94.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(3/2), x)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(157) = 314$.

Time $= 0.52$ (sec) , antiderivative size $= 566$, normalized size of antiderivative $= 3.22$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
-1/12*(6*sqrt(2)*(3*A*a^2*sqrt(c) + 7*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^2*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^2*sqrt(c) + B*a^2*
sqrt(c) + 6*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) + 14*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)/(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))) - 16*sqrt(2)*(3*A*a^2*sqrt(c) + 11*B*a^2*sqrt(c) - 6*A*a^2*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e
) + 1) - 18*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) + 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*B*a^2*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(c
^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/
2), x)
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

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3.95.1 Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3a^2(A+9B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-1/8*a^2*(A+9*B)*co
s(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)+3/8*a^2*(A+9*B)*arctanh(1/2*cos(f*x+e)
*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-3/8*a^2*(A+9*B)
*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output

```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 (3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*
(5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*
B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f
*x])^2)/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^
(5/2))
```

3.95.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{8c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{7/2}}dx}{8c}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3128$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+f}{cf\sqrt{c-c\sin(}}\right.}{2c^2}\right.}{8c}\right.$$

$$\downarrow 219$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right)}{8c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x])^(9/2)) - ((A +
 9*B)*(Cos[e + f*x]^3/(c*f*(c - c*Sin[e + f*x])^(5/2)) - (3*((2*Sqrt[2]*Ar
cTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)
*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/(2*c^2)))/(8*c))
```

3.95.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(152) = 304$.

Time = 3.30 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.21

method	result
default	$-\frac{a^2\left(3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2+27B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^2(fx+e))c^2-6A\sqrt{2}\,\operatorname{arctan}\right.$
parts	Expression too large to display

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x
+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e)))^(1/2)*
c^(3/2)*sin(f*x+e)^2-54*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(
1/2)/c^(1/2))*sin(f*x+e)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*A*2^(
1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+26*B*(c*(1+
sin(f*x+e)))^(3/2)*c^(1/2)+32*(c*(1+sin(f*x+e)))^(1/2)*B*c^(3/2)*sin(f*x+e
)+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-1
2*A*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(152) = 304$.

Time $= 0.28$ (sec) , antiderivative size $= 449$, normalized size of antiderivative $= 2.57$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3\sqrt{2}((A+9B)a^2\cos(fx+e)^3+3(A+9B)a^2\cos(fx+}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x +
e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos
(f*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e)
)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*c
os(f*x + e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos
(f*x + e)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))
*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2
 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f)*sin(f*x + e))
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.95.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(5/2), x)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(152) = 304$.

Time $= 0.67$ (sec) , antiderivative size $= 601$, normalized size of antiderivative $= 3.43$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
-1/64*(256*sqrt(2)*B*a^2/(c^(5/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)) - 12*sqrt(2)*(A*a^2*sqrt(c) + 9*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 8*A*a^2
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 24*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) + 18*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 162*B*a^2*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (8*sqrt(2)*A*a^2*c^(7/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 24*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 + sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2)/c^6)/f
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/
2), x)
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

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3.96.1 Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2(A-11B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}$$

output
```
1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)+1/24*a^2*(A-11*B)
*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(A-11*B)*cos(f*x+e)/c^2/f/
(c-c*sin(f*x+e))^(3/2)+1/32*a^2*(A-11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^
(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.88 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.95

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3 + 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^
(1/4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*
A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A
 + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e +
 f*x])^(7/2))
```

3.96.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(A-11B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A-11B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{9/2}}dx}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\downarrow 3128$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{c^2f}+\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos}{6f(c-c\sin(e}\right.$$

$$\downarrow 219$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{5/2}f}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+f}{6f(c-c\sin(e+fx)}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(6*f*(c - c*Sin[e + f*x])^(11/2)) + ((A
- 11*B)*(Cos[e + f*x]^3/(2*c*f*(c - c*Sin[e + f*x])^(7/2)) - (3*(-(ArcTanh
[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]*c^(5/
2)*f)) + Cos[e + f*x]/(c*f*(c - c*Sin[e + f*x])^(3/2))))/(4*c^2)))/(12*c))
```

3.96.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.

Time = 3.26 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.02

method	result
default	$\frac{a^2\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^3(A-11B)(\cos^2(fx+e))\sin(fx+e)-9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^3(A-11B)(\cos^2(f$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/96/c^(13/2)*a^2*(3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^3*(A-11*B)*cos(f*x+e)^2*sin(f*x+e)-9*2^(1/2)*arctanh(1/2*(c+c*sin
(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*cos(f*x+e)^2-12*2^(1/2)*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*sin(f*x+e)+6*A
*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-24*A*(
c+c*sin(f*x+e))^(1/2)*c^(5/2)+126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-352*B*(
c+c*sin(f*x+e))^(3/2)*c^(3/2)+264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+12*A*2^
(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-132*B*2^(1/2
)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+e
)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(152) = 304$.

Time = 0.27 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\big((A-11B)a^2\cos(fx+e)^4-3(A-11B)a^2\cos(fx+e)^3-8(A-11B)a^2\cos(fx+e)^2+4(A-$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
-1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*
x + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e)
 + 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*co
s(f*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*
sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A
+ 13*B)*a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*
a^2 + (3*(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e)
- 32*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x
+ e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x
 + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f
*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(7/2), x)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(152) = 304$.

Time = 0.74 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.76

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
1/768*(12*sqrt(2)*(A*a^2*sqrt(c) - 11*B*a^2*sqrt(c))*log(-(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^2*sqrt(c) + B*a^2*sqrt(c) + 3*A*
a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1) + 15*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 93*B*a^2*sqrt(c)*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
2 - 22*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^3 + 242*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 93*B*a^2*c^(17/2)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*
A*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^2 - 15*B*a^2*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - A*a^2*c^(17/2)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - B*a^2*c^
(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + ...
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2}{(c - c\,\sin(e + f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/
2), x)
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

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3.97.1 Optimal result

Integrand size = 38, antiderivative size = 222

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2(3A-13B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}$$

output

```
1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)+1/48*a^2*(3*A-13*
B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*a^2*(3*A-13*B)*cos(f*x+e)/c^
2/f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B)*cos(f*x+e)/c^3/f/(c-c*sin(
f*x+e))^(3/2)+1/512*a^2*(3*A-13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/
(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2}{\cdots}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(9/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Co
s[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 79
1*B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f
*x))/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*
I)*(-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] +
 1517*B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e +
f*x))/2] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Si
n[(7*(e + f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

3.97.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$\downarrow 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$\downarrow 3338$$

$$a^2c^2\left(\frac{(3A-13B)\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-13B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{11/2}}dx}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3159$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\downarrow 3129$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5}{8f(c-c\sin(e}\right.$$

$$\downarrow 3042$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5}{8f(c-c\sin(e}\right.$$

$$\downarrow 3128$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}\right.$$

$$\downarrow 219$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(8*f*(c - c*Sin[e + f*x])^(13/2)) + ((3*
A - 13*B)*(Cos[e + f*x]^3/(3*c*f*(c - c*Sin[e + f*x])^(9/2)) - (Cos[e + f*
x]/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) - (ArcTanh[(Sqrt[c]*Cos[e + f*x])/(S
qrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*
f*(c - c*Sin[e + f*x])^(3/2)))/(4*c^2))/(2*c^2)))/(16*c))
```

3.97.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(195) = 390$.

Time = 3.72 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.98

method	result
default	$-\frac{a^2\left(3\arctanh\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)(\cos^4(fx+e))+12\arctanh\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)(\cos^2(fx+e$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/1536*a^2*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
*c^5*(3*A-13*B)*cos(f*x+e)^4+12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2*sin(f*x+e)-24*arctanh(1/2*(c
+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2-
24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13
*B)*sin(f*x+e)-18*A*(c+c*sin(f*x+e))^(7/2)*c^(3/2)+132*A*(c+c*sin(f*x+e))^
(5/2)*c^(5/2)+264*A*(c+c*sin(f*x+e))^(3/2)*c^(7/2)-144*A*(c+c*sin(f*x+e))^
(1/2)*c^(9/2)+78*B*(c+c*sin(f*x+e))^(7/2)*c^(3/2)+452*B*(c+c*sin(f*x+e))^(
5/2)*c^(5/2)-1144*B*(c+c*sin(f*x+e))^(3/2)*c^(7/2)+624*B*(c+c*sin(f*x+e))^
(1/2)*c^(9/2)+72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*c^5-312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))
*c^5)*(c*(1+sin(f*x+e)))^(1/2)/c^(19/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(195) = 390$.

Time $= 0.29$ (sec) , antiderivative size $= 654$, normalized size of antiderivative $= 2.95$

$$\int \frac{(a + a\sin(e+fx))^2(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{9/2}}\,dx =$$

$$-\frac{3\sqrt{2}\big((3\,A - 13\,B)a^2\cos(fx+e)^5 + 5\,(3\,A - 13\,B)a^2\cos(fx+e)^4 - 8\,(3\,A - 13\,B)a^2\cos(fx+e)^3 - 20$$

input

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="fricas")
```

output
```
-1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*c
os(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*co
s(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3
*A - 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*
A - 13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A -
 13*B)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f
*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*
cos(f*x + e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2
*cos(f*x + e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3
*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 1
2*(19*A + 3*B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*s
in(f*x + e) + c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f
*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*
f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)
^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.97.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(9/2), x)
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(195) = 390$.

Time $= 0.54$ (sec) , antiderivative size $= 745$, normalized size of antiderivative $= 3.36$

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="giac")
```

output
```
1/24576*(24*sqrt(2)*(3*A*a^2*sqrt(c) - 13*B*a^2*sqrt(c))*log(-(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^5*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*a^2*sqrt(c) + 3*B*a^2*sqrt(c
) + 32*B*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) - 24*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 72*B*a^2*sqrt(c)*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 96*B*
a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^3 + 150*A*a^2*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 650*B*a^2*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (96*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) + 24*sqrt(2)*A*a^2*c^(31/2)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^2 - 72*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 - 32*sqrt(2)*B*a^2*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1...
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/
2), x)
```

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

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3.98.1 Optimal result

Integrand size = 38, antiderivative size = 210

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}} + \frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}} + \frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}} - \frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}$$

output

```
256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a
^3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c
^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)
^7/f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(
1/2)/f
```

3.98.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(210) = 420$.

Time $= 11.61$ (sec) , antiderivative size $= 1569$, normalized size of antiderivative $= 7.47$

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
(5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x
])^3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
6) - (3*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin
[e + f*x])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a +
a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)
*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/
(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6) + ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c
 - c*Sin[e + f*x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*
Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*S
in[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*...
```

3.98.3 Rubi [A] (verified)

Time $= 1.08$ (sec) , antiderivative size $= 189$, normalized size of antiderivative $= 0.90$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used $= \{3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\downarrow 3335$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\int \cos(e+fx)^6\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right)$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\int \frac{\cos(e+fx)^6}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+\right.$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+f\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+f\right.\right.$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+f\right.\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+f\right.\right.\right.$$

$$\downarrow 3152$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f) + ((15*A -
B)*((2*c*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]]) + (12*c*((2*c*Cos
[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + (8*c*((8*c^2*Cos[e + f*x]
^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Cos[e + f*x]^7)/(9*f*(c - c*S
in[e + f*x])^(5/2))))/11))/13))/15)
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.98.4 Maple [A] (verified)

Time = 167.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^4a^3(3003B(\cos^4(fx+e))+(-3465A+12243B)(\cos^2(fx+e))\sin(fx+e)+(14175A-24969B)(\cos^2($ $45045\cos(fx+e)\sqrt{c-c\sin(fx+e)}f$
parts	$\frac{2Aa^3(\sin(fx+e)-1)c^4(1+\sin(fx+e))(5(\sin^3(fx+e))-27(\sin^2(fx+e))+71\sin(fx+e)-177)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}+\frac{2Ba^3(\sin(fx+e)-1)c^4(1+\sin(fx$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*(3003*B*cos(f*x+e)^4+(-346
5*A+12243*B)*cos(f*x+e)^2*sin(f*x+e)+(14175*A-24969*B)*cos(f*x+e)^2+(24780
*A-25676*B)*sin(f*x+e)-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(190) = 380$.

Time $= 0.28$ (sec) , antiderivative size $= 405$, normalized size of antiderivative $= 1.93$

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{2\left(3003\,Ba^3c^3\cos(fx+e)^8 - 231\,(15\,A - 14\,B)a^3c^3\cos(fx+e)^7 + 21\,(15\,A - B)a^3c^3\cos(fx+e)^6 - 28\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
-2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*
x + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*co
s(f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^
3*cos(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*
a^3*c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x +
 e)^7 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos
(f*x + e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c
^3*cos(f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)
*a^3*c^3*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(190) = 380$.

Time $= 0.56$ (sec) , antiderivative size $= 458$, normalized size of antiderivative $= 2.18$

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\sqrt{2}\left(3003\,Ba^3c^3\cos\left(-\frac{15}{4}\,\pi+\frac{15}{2}\,fx+\frac{15}{2}\,e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-225225\left(8\,Aa^3c^3\mathrm{sg}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
1/2882880*sqrt(2)*(3003*B*a^3*c^3*cos(-15/4*pi + 15/2*f*x + 15/2*e)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 225225*(8*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 75075*(6*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) + B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x
+ 3/2*e) + 27027*(10*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a
^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e)
 + 19305*(4*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c^3*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 5005*(
12*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a^3*c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) - 4095*(2*A*a^3*c
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e) + 3465*(2*A*a^3*c^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))*cos(-13/4*pi + 13/2*f*x + 13/2*e))*sqrt(c)/f
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\, dx = \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^3\,(c - c\sin(e + fx))^{7/2}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2)
, x)
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.99.1 Optimal result

Integrand size = 38, antiderivative size = 161

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

output

```
64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3
*(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*
sin(f*x+e))^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^3c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}(-949}{18018f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/18018*(a^3*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x]
)^2*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]*(-9490*A + 6200*B + 126*
(13*A - 32*B)*Cos[2*(e + f*x)] + (9464*A - 9667*B)*Sin[e + f*x] + 693*B*Si
n[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

3.99.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3335$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\int\frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\int\frac{\cos(e+fx)^6}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3}}\right.\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3}}\right.\right.$$

$$\downarrow 3152$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]]) + ((13*A +
B)*((2*c*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + (8*c*((8*c^2*
Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Cos[e + f*x]^7)/(
9*f*(c - c*Sin[e + f*x])^(5/2))))/11))/13)
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.99.4 Maple [A] (verified)

Time = 168.77 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4a^3(-693B(\cos^2(fx+e))\sin(fx+e)+(-819A+2016B)(\cos^2(fx+e))+(-2366A+2590B)\sin(f}{9009\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2Aa^3(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3(\sin^2(fx+e))-14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f} - \frac{2Ba^3(\sin(fx+e)-1)c^3(1+\sin(fx+e))(3465(\sin^6($

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/9009*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f
*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+(-2366*A+2590*B)*sin(f*x+e)+2782*A-2558
*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{2\left(693\,Ba^3c^2\cos(fx+e)^7 + 63\,(13\,A+12\,B)a^3c^2\cos(fx+e)^6 - 7\,(13\,A+B)a^3c^2\cos(fx+e)^5 + 10\,(13}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x +
 e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*
x + e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*co
s(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^
2 + (693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 -
 70*(13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)
^3 - 96*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x
 + e) - 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c
)^(5/2), x)
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(145) = 290$.

Time $= 0.59$ (sec) , antiderivative size $= 372$, normalized size of antiderivative $= 2.31$

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{\sqrt{2}\left(180180\,Aa^3c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+693\,Ba^3c^2\cos\left(-\frac{13}{4}\pi+\frac{13}{2}fx+\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
-1/288288*sqrt(2)*(180180*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 693*B*a^3*c^2*cos(-13/4*pi + 13/2*f*x + 13/
2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15015*(4*A*a^3*c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*c
os(-3/4*pi + 3/2*f*x + 3/2*e) - 9009*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi +
 5/2*f*x + 5/2*e) - 2574*(5*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
+ 2*B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x +
 7/2*e) + 2002*(A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^3*c^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) + 81
9*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^3\,(c - c\,\sin(e + f\,x))^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
, x)
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.100.1 Optimal result

Integrand size = 38, antiderivative size = 124

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}} + \frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

output
```
8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(1
1*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^(3/2)
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1157 vs. 2(124) = 248.

Time = 12.55 (sec) , antiderivative size = 1157, normalized size of antiderivative = 9.33

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3
/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a +
a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Co
s[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c -
c*Sin[e + f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e
 + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((8*A + 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e
 + f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)*Sin[(5*(e + f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]...
```

3.100.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3335$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3153$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{4}{9}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{4}{9}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\downarrow 3152$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + ((11*A
+ 3*B)*((8*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Co
s[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))))/11)
```

3.100.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.100.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^4a^3(-63B(\cos^2(fx+e))+\sin(fx+e)(77A-105B)-121A+93B)}{693\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2A\,a^3(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2B\,a^3(\sin(fx+e)-1)c^2(1+\sin(fx+e))(15(\sin^5(fx+e))-35(\sin^4(fx+e))-}{165\cos(fx+e)\sqrt{c-c\sin($

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(-63*B*cos(f*x+e)^2+sin(f*x+
e)*(77*A-105*B)-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(112) = 224.

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.31

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{2\left(63\,Ba^3c\cos\left(fx+e\right)^6-7\left(11\,A+12\,B\right)a^3c\cos\left(fx+e\right)^5-\left(187\,A+177\,B\right)a^3c\cos\left(fx+e\right.\right.}{+fx))^{3/2}\,dx =}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 -
(187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3
 - 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e
) + 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a
^3*c*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*
B)*a^3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A + 
3*B)*a^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*si
n(f*x + e) + f)
```

3.100.6 Sympy [F]

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\, dx = a^3 \left(\int Ac\sqrt{-c\sin(e + fx) + c}\, dx + \int 2Ac\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\, dx + \int \left(-2Ac\sqrt{-c\sin(e + fx) + c}\sin^3(e + fx)\right) dx + \int \left(-Ac\sqrt{-c\sin(e + fx) + c}\sin^4(e + fx)\right) dx + \int Bc\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\, dx + \int 2Bc\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\, dx + \int \left(-2Bc\sqrt{-c\sin(e + fx) + c}\sin^4(e + fx)\right) dx + \int \left(-Bc\sqrt{-c\sin(e + fx) + c}\sin^5(e + fx)\right) dx \right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**3*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*c*sqrt(-c*
sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-2*A*c*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x)**3, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e +
 f*x)**4, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + I
ntegral(2*B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-2*
B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(-B*c*sqrt(-c*
sin(e + f*x) + c)*sin(e + f*x)**5, x))
```

3.100.7 Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(112) = 224$.

Time $= 0.49$ (sec) , antiderivative size $= 294$, normalized size of antiderivative $= 2.37$

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\sqrt{2}\left(693\,Ba^3c\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-63\,Ba^3c\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/11088*sqrt(2)*(693*B*a^3*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 63*B*a^3*c*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 1386*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 462*(8*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 99*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 77*(2*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\, dx = \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^3\,(c - c\sin(e + fx))^{3/2}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
, x)
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

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3.101.1 Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

output
```
2/63*a^3*(9*A+5*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/9*a^3*B*c^3
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)
```

3.101.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(9A-2B+7B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.101.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3335$$

$$a^3c^3\left(\frac{1}{9}(9A+5B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{1}{9}(9A+5B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\downarrow 3152$$

$$a^3c^3\left(\frac{2c(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a^3*c^3*((2*(9*A + 5*B)*c*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*B*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2)))
```

3.101.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.101.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^4a^3(7B\sin(fx+e)+9A-2B)}{63\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$
parts	$-\frac{2Aa^3(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}-\frac{2Ba^3(\sin(fx+e)-1)c(1+\sin(fx+e))(35(\sin^4(fx+e))-40(\sin^3(fx+e))+48(\sin^2(fx+$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.86

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2\left(7\,Ba^3\cos(fx+e)^5+(9\,A+26\,B)a^3\cos(fx+e)^4-(27\,A+29\,B)a^3\cos(fx+e)^3-4\,(18\,A+17\,B)a$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 2
9*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*
B)*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A +
 19*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5
*B)*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x +
e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

3.101.6 Sympy [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= a^3\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx+\int 3A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx\right.$$
$$+\int 3A\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx$$
$$+\int A\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx+\int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx$$
$$+\int 3B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx$$
$$+\int 3B\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx$$
$$\left.+\int B\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3,
x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**4, x))
```

3.101.7 Maxima [F]

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^3 \sqrt{-c\sin(fx+e) + c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
 + c), x)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(73) = 146$.

Time $= 0.44$ (sec) , antiderivative size $= 253$, normalized size of antiderivative $= 3.12$

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}\left(7\,Ba^3\cos\left(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 126\left(5\,Aa^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/504*sqrt(2)*(7*B*a^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 126*(5*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*
a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) +
42*(9*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(A*a^3*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*co
s(-5/4*pi + 5/2*f*x + 5/2*e) + 9*(2*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) + 5*B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x +
 7/2*e))*sqrt(c)/f
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
, x)
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

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3.102.1 Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{8\sqrt{2}a^3(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$
$$- \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

output

```
-2/7*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/5*a^3*(A+B)*c^2*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*(A+B)*c*cos(f*x+e)^3/f/(c-c*sin
(f*x+e))^(3/2)+8*a^3*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin
(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e)
)^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left((6720+6720i)\sqrt[4]{-1}(A+B)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right.\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
-1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((6
720 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
 + f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B +
 6*(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin
[3*(e + f*x)])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin
[e + f*x]])
```

3.102.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3339, 3042, 3158, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$
$$\downarrow 3446$$
$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3339$$

$$a^3c^3\left((A+B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left((A+B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{7/2}}dx-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left((A+B)\left(\frac{2\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left((A+B)\left(\frac{2\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+\right.$$

$$\downarrow 3042$$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+\right.$$

$$\downarrow 3158$$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\right.$$

$\downarrow$ 3042

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\right.$$

$\downarrow$ 3128

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(-\frac{4\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(}{5cf(c-c\sin}\right.\right.$$

$\downarrow$ 219

3.102. $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+f\right.\right.\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2)) + (A + B)*
((-2*Cos[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^(5/2)) + (2*((-2*Cos[e +
f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTanh[(Sqrt[
c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*Cos
[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/c))
```

3.102.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.102.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16

method	result
default	$\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^3\left(-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B+1$
parts	$-\frac{A\,a^3(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{B\,a^3(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(105c^{\frac{7}{2}}\sqrt{2}\right.$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(-420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-420*c^(7/2)*2^(1/2)*
arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+15*B*(c*(1+sin(f*x
+e)))^(7/2)+21*A*(c*(1+sin(f*x+e)))^(5/2)*c+21*B*(c*(1+sin(f*x+e)))^(5/2)*
c+70*A*(c*(1+sin(f*x+e)))^(3/2)*c^2+70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2+420*
(c*(1+sin(f*x+e)))^(1/2)*A*c^3+420*(c*(1+sin(f*x+e)))^(1/2)*B*c^3)/c^4/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= 2\left(\frac{210\sqrt{2}((A+B)a^3c\cos(fx+e)-(A+B)a^3c\sin(fx+e)+(A+B)a^3c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(f}\right.\right.$$
$$\left.\left.\sqrt{c}\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e
) + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e)
+ 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sq
rt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*
B)*a^3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 14
8*B)*a^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 +
 3*(7*A + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*
(161*A + 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x
 + e) - c*f*sin(f*x + e) + c*f)
```

3.102.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= a^3\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx \right.$$
$$+ \int \frac{3A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+ \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$\left. + \int \frac{3B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^4(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)
/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(
e + f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c),
x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*
sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)*
*3/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(
e + f*x) + c), x))
```

3.102.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
 + c), x)
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(177) = 354$.

Time = 0.39 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.24

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
4/105*(105*(sqrt(2)*A*a^3*sqrt(c) + sqrt(2)*B*a^3*sqrt(c))*log(-(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(161*A*a^3*sqrt(c) + 191*B*a^3*
sqrt(c) - 812*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) - 812*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 2121*A*a^3*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 +
 2751*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^2 - 3080*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 3080*B*a^3*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3
+ 2555*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^4 + 3605*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1260*A*a^3*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5
 - 1260*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^5 + 315*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)^6/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6 + 525*B*a^3*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^6/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^6)
/(c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*...
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^3}{\sqrt{c - c\,\sin(e + f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/
2), x)
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

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3.103.1 Optimal result

Integrand size = 38, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}a^3(5A+9B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}$$
$$+\frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}$$
$$+\frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/2*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(9/2)+1/10*a^3*(5*A+9*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+1/3*a^3*(5*A+9*B)*cos(f*x+e)^3/f/
(c-c*sin(f*x+e))^(3/2)-2*a^3*(5*A+9*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1
/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+2*a^3*(5*A+9*B)*cos(f*x+e)/c
/f/(c-c*sin(f*x+e))^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A +
B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A +
 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

3.103.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2c}{5cf(c-}\right.\right.$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e-}{3cf(c-c\sin(e}\right.\right.$$

$$\downarrow 219$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{4c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(2*f*(c - c*Sin[e + f*x])^(9/2)) - ((5*A
 + 9*B)*((-2*Cos[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^(5/2)) + (2*((-2*
Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTan
h[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f)
 - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/c))/(4*c))
```

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.103.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.62

method	result
default	$\frac{2a^3\left(\sin(fx+e)\left(-60A\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}-5A(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}-120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}-15B(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}-3B(c+$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/15*a^3*(sin(f*x+e)*(-60*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-5*A*(c+c*sin(f*
x+e))^(3/2)*c^(3/2)-120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-15*B*(c+c*sin(f*x
+e))^(3/2)*c^(3/2)-3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+75*A*2^(1/2)*arctanh
(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+135*B*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)+90*A*(c+c*sin(f*x+e))^(1/2)*
c^(5/2)+5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+150*B*(c+c*sin(f*x+e))^(1/2)*c^
(5/2)+15*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/
2)-75*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-13
5*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1
+sin(f*x+e)))^(1/2)/c^(9/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(195) = 390$.

Time $= 0.29$ (sec) , antiderivative size $= 430$, normalized size of antiderivative $= 1.97$

$$\int \frac{(a + a\sin(e+fx))^3(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}\Big((5A+9B)a^3c\cos(fx+e)^2-(5A+9B)a^3c\cos(fx+e)-2(5A+9B)a^3c+}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos
(f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A
 + 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin
(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x +
e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (
5*A + 18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3
*A + 5*B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5
*A + 21*B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)
*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2
*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(3/2), x)
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(195) = 390$.

Time $= 0.43$ (sec) , antiderivative size $= 730$, normalized size of antiderivative $= 3.35$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
-1/30*(30*sqrt(2)*(5*A*a^3*sqrt(c) + 9*B*a^3*sqrt(c))*log(-(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*a^3*sqrt(c)*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(
c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*a^3*sqrt(c) + B*a
^3*sqrt(c) + 10*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 18*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)/(c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) - 16*sqrt(2)*(35*A*a^3*sqrt(c) + 81*B*a^3*sqrt(c) - 130*
A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1) - 270*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 200*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 480*B*a^3*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 - 150*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 330*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 45*A*a^3*sqrt(c)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e)...
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/
2), x)
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

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3.104.1 Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5a^3(3A+11B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^3*(3*A+11*B
)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a^3*(3*A+11*B)*cos(f*x+e)^3
/c/f/(c-c*sin(f*x+e))^(3/2)+5/4*a^3*(3*A+11*B)*arctanh(1/2*cos(f*x+e)*c^(1
/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-5/4*a^3*(3*A+11*B)*c
os(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

3.104.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 +
 I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ 24*(A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4*Sin[(3*(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c -
 c*Sin[e + f*x])^(5/2))
```

3.104.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{8c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{9/2}}dx}{8c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3158$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2}{3cf(c-}\right.\right.}{2c^2}\right)}{8c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2}{3cf(c-\right.}{2c^2}\right.}{8c}\right.$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2}{cf\sqrt{}}\right.\right.}{c}}{2c^2}\right.}{8c}\right.$$

$$\downarrow 219$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}+\frac{2\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right)}{8c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(4*f*(c - c*Sin[e + f*x])^(11/2)) - ((3*
A + 11*B)*(Cos[e + f*x]^5/(c*f*(c - c*Sin[e + f*x])^(7/2)) - (5*((-2*Cos[e
 + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTanh[(Sq
rt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*
Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/(2*c^2)))/(8*c))
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(198) = 396.

Time = 3.98 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

method	result
default	$-\frac{a^3\left(\left(24A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-45A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+8B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{c}+120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-16\right.\right.$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/12/c^(9/2)*a^3*((24*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-45*A*2^(1/2)*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+8*B*(c+c*sin(f*x+e))^(3
/2)*c^(1/2)+120*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2)*cos(f*x+e)^2+sin(f*x+e)*(48*
A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))
^(1/2)*2^(1/2)/c^(1/2))*c^2+16*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+240*B*(c+c
*sin(f*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*c^2)+54*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-132*A*(c+c*sin
(f*x+e))^(1/2)*c^(3/2)+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(
1/2)/c^(1/2))*c^2+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-420*B*(c+c*sin(f*x+e
))^(1/2)*c^(3/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))*c^2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(198) = 396.

Time = 0.29 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.24

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{15\sqrt{2}\big((3A+11B)a^3\cos(fx+e)^3+3(3A+11B)a^3\cos$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos
(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A
 + 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 1
1*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
 + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^
4 - 4*(3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2
 + 3*(13*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x +
e)^3 + 12*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e)
+ 12*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x
+ e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*
cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(5/2), x)
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(198) = 396.

Time = 0.45 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.46

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
1/96*(60*sqrt(2)*(3*A*a^3*sqrt(c) + 11*B*a^3*sqrt(c))*log(-(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^3*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 1
6*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1) + 32*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 90*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 330*B*a^3*sqrt
(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/(c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 3*(16*sqrt(2)*A*a
^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 32*sqrt(2)*B*a^3*c^(7/2)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + sqrt(2)*A*a^3*c^(7/2)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)^2 + sqrt(2)*B*a^3*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2)/c^6 - 128*sqrt(2)*(3*A*a^3*sqrt(c) + 17*B*a^3*sqrt(c) - 6
*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 30*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/...
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/
2), x)
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

3.105.1 Optimal result . . . 998
3.105.2 Mathematica [C] (verified) . . . 999
3.105.3 Rubi [A] (verified) . . . 999
3.105.4 Maple [B] (verified) . . . 1005
3.105.5 Fricas [B] (verification not implemented) . . . 1005
3.105.6 Sympy [F(-1)] . . . 1006
3.105.7 Maxima [F] . . . 1006
3.105.8 Giac [B] (verification not implemented) . . . 1007
3.105.9 Mupad [F(-1)] . . . 1008

3.105.1 Optimal result

Integrand size = 38, antiderivative size = 217

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{5a^3(A+13B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(13/2)-1/24*a^3*(A+13*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^3*(A+13*B)*cos(f*x+e)^3/c/
f/(c-c*sin(f*x+e))^(5/2)-5/16*a^3*(A+13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)+5/16*a^3*(A+13*B)*cos(f*
x+e)/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.66 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.94

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\left(\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)
*(-1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A +
 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A
+ 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])
^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/
2))
```

3.105.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{12c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{11/2}}dx}{12c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{4c^2}\right)}{12c}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{7/2}}dx}{4c^2}\right)}{12c}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{4c^2}\right)}{12c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{4c^2}\right)}{12c}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2c}{cf\sqrt{c}}\right.}{2c^2}\right.}{4c^2}\right.}{12c}\right.$$

$$\Big\downarrow \text{3042}$$

3.105. $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2}{cf\sqrt{c}}\right.}{2c^2}\right.}{4c^2}\right.}{12c}\right.$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\right.}{cf}\right.}{4c^2}\right.}{12c}\right.$$

$$\downarrow 219$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+f}{\sqrt{2}\sqrt{c-c\sin(e}}\right.\right.\right.\right.}{c^{3/2}f}}{2c^2}}{4c^2}}{12c}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(6*f*(c - c*Sin[e + f*x])^(13/2)) - ((A
+ 13*B)*(Cos[e + f*x]^5/(2*c*f*(c - c*Sin[e + f*x])^(9/2)) - (5*(Cos[e + f
*x]^3/(c*f*(c - c*Sin[e + f*x])^(5/2)) - (3*((2*Sqrt[2]*ArcTanh[(Sqrt[c]*C
os[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*Cos[e +
 f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/(2*c^2)))/(4*c^2)))/(12*c))
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(190) = 380.

Time = 4.36 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.41

method	result
default	$\frac{a^3\left(15A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^3(fx+e))c^3+195B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(\sin^3(fx+e))c^3-45A\sqrt{2}\,\text{arcta}\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^
(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin
(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^
(1/2)*c^(5/2)*sin(f*x+e)^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2
)+45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f
*x+e)*c^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^
(1/2)*c^(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)
-15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-92
8*B*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-288*(c*(1+sin(f*x+e)))^(1/2)*B*c^(5/2
)*sin(f*x+e)-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^
(1/2))*c^3+120*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))
^(1/2)*c^(5/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(190) = 380.

Time = 0.29 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{15\sqrt{2}\big((A+13\,B)a^3\cos(fx+e)^4-3\,(A+13\,B)a^3\cos(f}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
 + e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e)
+ 8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos
(f*x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
 e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)
*a^3*cos(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)
*a^3*cos(f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A +
79*B)*a^3*cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a
^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4
*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*
f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
- 8*c^4*f)*sin(f*x + e))
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(7/2), x)
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(190) = 380$.

Time $= 0.52$ (sec) , antiderivative size $= 719$, normalized size of antiderivative $= 3.31$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
1/384*(1536*sqrt(2)*B*a^3/(c^(7/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) - 60*sqrt(2)*(A*a^3*sqrt(c) + 13*B*a^3*sqrt(c))*log(-(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^4*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*a^3*sqrt(c) + B*a^3*sqrt(c) + 9*A*a
^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) + 21*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 45*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 237*B*a^3*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^2 + 110*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^3 + 1430*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^3/(c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(45*A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 237*B*a^3*c^(17/2)
*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
 + 9*A*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^2 + 21*B*a^3*c^(17/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + A*a^3*c^(17/2)*(cos(-...
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/
2), x)
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

3.106.1 Optimal result . . . 1009
3.106.2 Mathematica [C] (verified) . . . 1010
3.106.3 Rubi [A] (verified) . . . 1010
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3.106.9 Mupad [F(-1)] . . . 1019

3.106.1 Optimal result

Integrand size = 38, antiderivative size = 217

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{5a^3(A-15B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}$$
$$+\frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}}$$
$$-\frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}$$

output

```
1/8*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(15/2)+1/48*a^3*(A-15*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/192*a^3*(A-15*B)*cos(f*x+e)^3/
c/f/(c-c*sin(f*x+e))^(7/2)+5/128*a^3*(A-15*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*
x+e))^(3/2)-5/256*a^3*(A-15*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c
*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.41 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.64

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(9/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Co
s[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 270
3*B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e +
f*x))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 +
 120*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e +
f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2
] + 405*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e
 + f*x))/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15
*A*Sin[(7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

3.106.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(A-15B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}dx}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A-15B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{13/2}}dx}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{9/2}}dx}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right.$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+}{8f(c-}\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{c^2f}+\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right.}{4c^2}\right.}{6c^2}\right.}{16c}\right.$$

$$\downarrow 219$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{5/2}f}\right)}{4c^2}\right)}{6c^2}\right)}{16c}-$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(8*f*(c - c*Sin[e + f*x])^(15/2)) + ((A
 - 15*B)*(Cos[e + f*x]^5/(3*c*f*(c - c*Sin[e + f*x])^(11/2)) - (5*(Cos[e +
 f*x]^3/(2*c*f*(c - c*Sin[e + f*x])^(7/2)) - (3*(-(ArcTanh[(Sqrt[c]*Cos[e +
  f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]*c^(5/2)*f)) + Cos[e +
 f*x]/(c*f*(c - c*Sin[e + f*x])^(3/2))))/(4*c^2)))/(6*c^2)))/(16*c))
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(190) = 380.

Time = 4.52 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.99

method	result
default	$-\frac{a^3\left(-15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)(\cos^4(fx+e))-60\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)(\cos^2(fx+e$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/768/c^(17/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^4-60*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^2*sin(f*x+e)+120*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+
e)^2+120*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(
A-15*B)*sin(f*x+e)+30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+292*A*(c+c*sin(f*x+
e))^(5/2)*c^(3/2)-440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)+240*A*(c+c*sin(f*x+
e))^(1/2)*c^(7/2)+1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2)-4380*B*(c+c*sin(f*
x+e))^(5/2)*c^(3/2)+6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-3600*B*(c+c*sin(
f*x+e))^(1/2)*c^(7/2)-120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(
1/2)/c^(1/2))*c^4+1800*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(190) = 380$.

Time $= 0.30$ (sec) , antiderivative size $= 633$, normalized size of antiderivative $= 2.92$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx =$$

$$-\frac{15\sqrt{2}\big((A - 15\,B)a^3\cos(fx + e)^5 + 5\,(A - 15\,B)a^3\cos(fx + e)^4 - 8\,(A - 15\,B)a^3\cos(fx + e)^3 - 20\,(A$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="fricas")
```

output
```
-1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(
f*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x +
 e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^
3*cos(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos
(f*x + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*
sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (19
1*A - 561*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 1
2*(21*A - 59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*
cos(f*x + e)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3
*cos(f*x + e) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/
(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 -
20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f
*x + e) + 16*c^5*f)*sin(f*x + e))
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(9/2), x)
```

3.106.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(190) = 380$.

Time $= 0.49$ (sec) , antiderivative size $= 928$, normalized size of antiderivative $= 4.28$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="giac")
```

output
```
-1/12288*(120*sqrt(2)*(A*a^3 - 15*B*a^3)*log(-(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^(9/2)*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^3*sqrt(c) + 3*B*a^3*sqrt(c) + 16*A*a^3
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 48*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) + 24*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 312*B*a^3*sqrt(c)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
 - 48*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^3 + 1392*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 250*A*a^3*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 +
 3750*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1)^4)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4/(c^5*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (
48*sqrt(2)*A*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1392*sqrt(
2)*B*a^3*c^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 24*sqrt(2)*A*a^3*c
^(31/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*...
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/
2), x)
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

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3.107.1 Optimal result

Integrand size = 38, antiderivative size = 266

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{a^3(3A-17B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}}$$
$$+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}$$

output

```
1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(17/2)+1/80*a^3*(3*A-17
*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-1/96*a^3*(3*A-17*B)*cos(f*x+e
)^3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3*A-17*B)*cos(f*x+e)/c^3/f/(c-c*
sin(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(f*x+e)/c^4/f/(c-c*sin(f*x+e))^(
3/2)-1/1024*a^3*(3*A-17*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin
(f*x+e))^(1/2))/c^(11/2)/f*2^(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.20 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.54

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(11/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(56370*A*C
os[(e + f*x)/2] + 38970*B*Cos[(e + f*x)/2] - 31140*A*Cos[(3*(e + f*x))/2]
 - 38580*B*Cos[(3*(e + f*x))/2] - 10404*A*Cos[(5*(e + f*x))/2] - 12724*B*Co
s[(5*(e + f*x))/2] + 435*A*Cos[(7*(e + f*x))/2] + 7775*B*Cos[(7*(e + f*x))
/2] - 45*A*Cos[(9*(e + f*x))/2] + 255*B*Cos[(9*(e + f*x))/2] + (240 + 240*
I)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10 + 56370*A*Sin[(e + f*x)/2]
 + 38970*B*Sin[(e + f*x)/2] + 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(
3*(e + f*x))/2] - 10404*A*Sin[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))
/2] - 435*A*Sin[(7*(e + f*x))/2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[
(9*(e + f*x))/2] + 255*B*Sin[(9*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2))
```

3.107.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3446$$

$$a^3c^3\int\frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\downarrow 3042$$

$$a^3c^3\int\frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\downarrow 3338$$

$$a^3c^3\left(\frac{(3A-17B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}dx}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-17B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{15/2}}dx}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{11/2}}dx}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/}}\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/}}\right.$$

$$\downarrow 3159$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}\right)}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A-}{10f(c}\right.$$

$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}\right)}{8c^2}\right)}{20c}+\frac{(A-$$

$$\downarrow 3129$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/}}\right)}{4c^2}\right)}{8c^2}\right)}{20c}$$

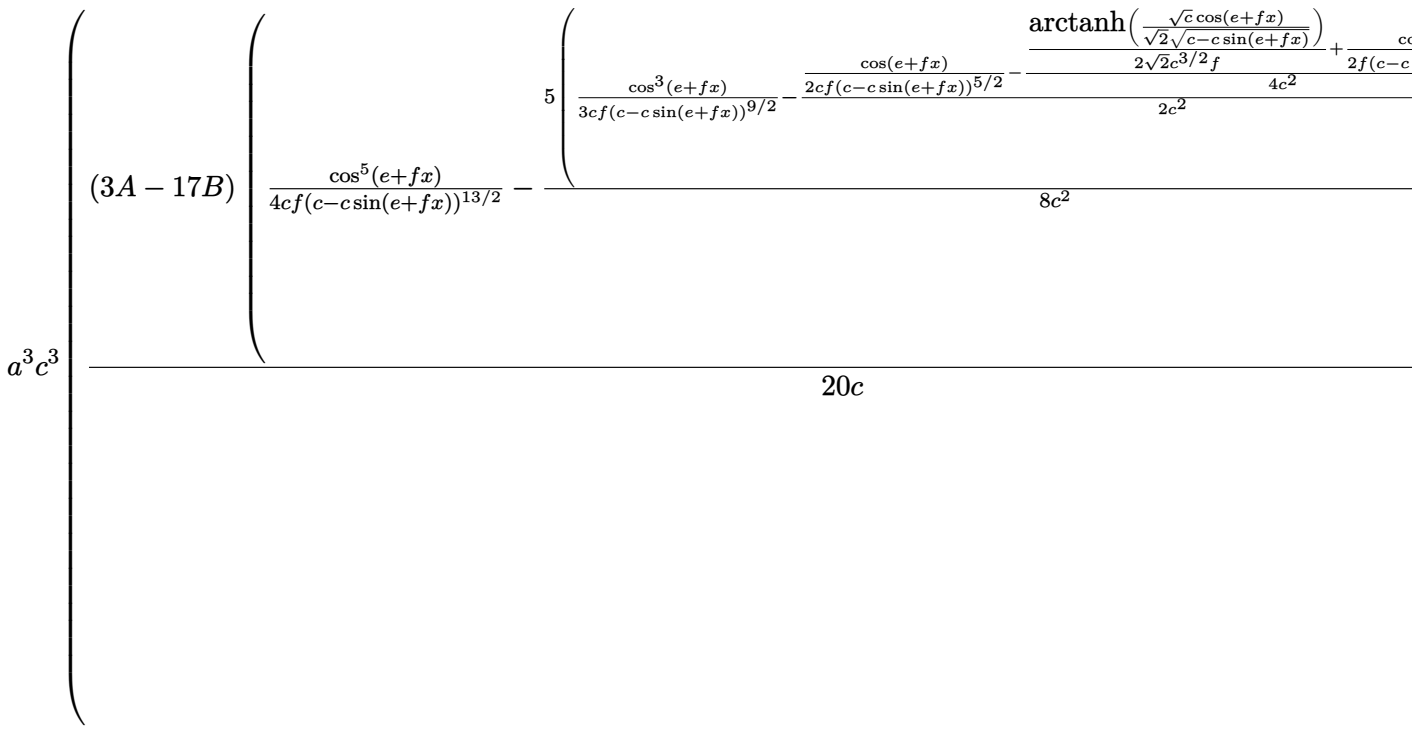
$$\downarrow 3042$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/}}}{2c^2}\right)}{8c^2}\right)}{20c}\right.$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}}{4c^2}}{2c^2}\right)}{8c^2}\right)}{20c}\right.$$

$$\downarrow 219$$



input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11
/2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(10*f*(c - c*Sin[e + f*x])^(17/2)) + ((3
*A - 17*B)*(Cos[e + f*x]^5/(4*c*f*(c - c*Sin[e + f*x])^(13/2)) - (5*(Cos[e
 + f*x]^3/(3*c*f*(c - c*Sin[e + f*x])^(9/2)) - (Cos[e + f*x]/(2*c*f*(c - c
*Sin[e + f*x])^(5/2)) - (ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e +
 f*x])^(3/2)))/(4*c^2))/(2*c^2)))/(8*c^2)))/(20*c))
```

3.107.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(235) = 470$.

Time = 5.05 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.98

method	result
default	$\frac{a^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)(\cos^4(fx+e))\sin(fx+e)-75\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)(\mathrm{co}\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,method=_
RETURNVERBOSE)
```

output
```
1/15360*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*c^6*(3*A-17*B)*cos(f*x+e)^4*sin(f*x+e)-75*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4-180*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^
2*sin(f*x+e)+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2
))*c^6*(3*A-17*B)*cos(f*x+e)^2+240*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin(f*x+e)-90*A*(c+c*sin(f*x+e))^(9/2)
*c^(3/2)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2
)*c^(7/2)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1
/2)*c^(11/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^
(7/2)*c^(5/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+
e))^(3/2)*c^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arc
tanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctan
h(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2
)/c^(23/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(235) = 470$.

Time = 0.29 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.86

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$

$$-\frac{15\sqrt{2}\big((3A-17B)a^3\cos(fx+e)^6-5(3A-17B)a^3\cos(fx+e)^5-18(3A-17B)a^3\cos(fx+e)^4+}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="fricas")
```

output
```
-1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3
*cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3
*cos(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3
*cos(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6
*(3*A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32
*(3*A - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(
3*A - 17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*
cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
+ (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B
)*a^3*cos(f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 38
9*B)*a^3*cos(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*
A + 43*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f
*x + e)^4 + 80*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*co
s(f*x + e)^2 - 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(
f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*
cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5
+ 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)
^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(11/2), x)
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(235) = 470$.

Time $= 0.56$ (sec) , antiderivative size $= 990$, normalized size of antiderivative $= 3.72$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="giac")
```

output
```
-1/245760*(120*sqrt(2)*(3*A*a^3*sqrt(c) - 17*B*a^3*sqrt(c))*log(-(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(c^6*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(6*A*a^3*sqrt(c) + 6*B*a^3*sqr
t(c) + 15*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1) + 75*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 30*A*a^3*sqrt(c)*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 290*B*
a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)^2 - 120*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 360*B*a^3*sqrt(c)*(cos(-1/4*pi +
 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 60*A*a^3
*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4 - 900*B*a^3*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 822*A*a^3*sqrt(c)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 4658*B*a^3*
sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)^5)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5/(c^6*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(60*A
*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 900*B*a^3*c^(49/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)...
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{11/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11
/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11
/2), x)
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

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3.108.1 Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}-\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}$$
$$-\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}$$
$$-\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af}$$
$$-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

output

```
-12/35*(7*A-9*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-1/7*(7*A-9*B)*c
*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(
9/2)/a/c/f-128/35*(7*A-9*B)*c^4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-32/3
5*(7*A-9*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

3.108.2 Mathematica [A] (verified)

Time = 14.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(4900A-6125B+196(A-2B)\cos(2(e+fx)}{140af\left(\cos\left(\frac{1}{2}(e+fx)\right)-si}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
-1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]
*(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)]
+ 2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B
*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x]))
```

3.108.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3446$$
$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}dx}{ac}$$
$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(7A-9B)\int(c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(7A-9B)\int(c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\int(c-c\sin(e+fx))^{5/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\int(c-c\sin(e+fx))^{5/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\int(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\int(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+}{ac}$$

$$\downarrow 3125$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+fx)(}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/f) - ((7*A - 9*B)*c*(
(2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*f) + (12*c*((2*c*Cos[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*f) + (8*c*((8*c^2*Cos[e + f*x])/(3*f*S
qrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*
f)))/5))/7))/2)/(a*c)
```

3.108.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.108.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(5B(\cos^4(fx+e))+(-7A+29B)(\cos^2(fx+e))\sin(fx+e)+(49A-103B)(\cos^2(fx+e))+(308A-436B)\sin(fx+e)}{35a\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/35*c^4/a*(sin(f*x+e)-1)*(5*B*cos(f*x+e)^4+(-7*A+29*B)*cos(f*x+e)^2*sin(f
*x+e)+(49*A-103*B)*cos(f*x+e)^2+(308*A-436*B)*sin(f*x+e)+588*A-716*B)/cos(
f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{2\left(5\,Bc^3\cos(fx+e)^4+(49\,A-103\,B)c^3\cos(fx+e)^2+4\,(147\,A-179\,B)c^3-\left((7\,A-29\,B)c^3\cos(fx+\right.\right.}{35\,af\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147
*A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(182) = 364.

Time = 0.34 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = 2\left(\frac{7\left(91\,c^{\frac{7}{2}}+\frac{86\,c^{\frac{7}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{336\,c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{266\,c^{\frac{7}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{490\,c^{\frac{7}{2}}}{(\cos(f}\right.\right.$$
$$\left.\left.\left(a+\frac{a\sin(fx}{\cos(fx+}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c
^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*
x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(
7/2) + 407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 2030*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(182) = 364.

Time = 0.51 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.88

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
16/35*sqrt(2)*sqrt(c)*(35*(A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (77*A*c^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 109*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
504*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 728*B*c^3*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) + 1337*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
 2009*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1680*A*c^3*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2800*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^3 + 1015*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1015*B*c^
3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 280*A*c^3*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^5 + 280*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*...
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
, x)
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

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3.109.1 Optimal result

Integrand size = 38, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}$$
$$-\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}$$
$$-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}$$

output
```
-1/5*(5*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-(A-B)*sec(f*x+e)*(c
-c*sin(f*x+e))^(7/2)/a/c/f-32/15*(5*A-7*B)*c^3*cos(f*x+e)/a/f/(c-c*sin(f*x
+e))^(1/2)-8/15*(5*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

3.109.2 Mathematica [A] (verified)

Time = 11.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(450A-600B+2(5A-16B)\cos(2(e+fx))+}{30af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
-1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e +
 f*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*
(1 + Sin[e + f*x]))
```

3.109.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^{5/2}(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^{5/2}(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(5A-7B)\int (c - c\sin(e+fx))^{5/2}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(5A-7B)\int (c - c\sin(e+fx))^{5/2}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\int(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\int(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B}{}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B}{}}{ac}$$

$$\downarrow 3125$$

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)(c}{f}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/f) - ((5*A - 7*B)*c*(
(2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*f) + (8*c*((8*c^2*Cos[e +
 f*x])/(3*f*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e
 + f*x]])/(3*f)))/5))/2)/(a*c)
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.109.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2c^3(\sin(fx+e)-1)\left(-3B\left(\cos^2(fx+e)\right)\sin(fx+e)+(-5A+16B)\left(\cos^2(fx+e)\right)+(-50A+82B)\sin(fx+e)-110A+142B\right)}{15a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	95

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-2/15*c^3/a*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-5*A+16*B)*cos(f
*x+e)^2+(-50*A+82*B)*sin(f*x+e)-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{2\left((5A-16B)c^2\cos(fx+e)^2+2(55A-71B)c^2+\left(3Bc^2\cos(fx+e)^2+2(25A-41B)c^2\right)\sin(fx+\right.}{15af\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*co
s(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c
)/(a*f*cos(f*x + e))
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(145) = 290$.

Time = 0.32 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.43

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}}{a + a\sin(e+fx)}\,dx = 2\left(\frac{5\left(23c^{\frac{5}{2}} + \frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{65c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{40c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{65c^{\frac{5}{2}}\sin}{(\cos(fx+}\right.\right.$$
$$\left.\left.\frac{}{\left(a + \frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)}\right.}\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 17
0*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7
9*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos
(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(145) = 290$.

Time = 0.44 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.60

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}}{a + a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
8/15*sqrt(2)*sqrt(c)*(15*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (25*A*c^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 41*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 11
0*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 190*B*c^2*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 160*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 32
0*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*A*c^2*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^3 + 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
3 + 15*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*c^2*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5))/f
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
, x)
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

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3.110.1 Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

output
```
-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/c/f-4/3*(3*A-5*B)*c^2*cos(f*x+e
)/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(
1/2)/a/f
```

3.110.2 Mathematica [A] (verified)

Time = 9.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-18A+27B+B$$
$$3af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)
] + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

3.110.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(3A-5B)\int (c - c\sin(e+fx))^{3/2}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(3A-5B)\int (c - c\sin(e+fx))^{3/2}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\downarrow 3126$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{4}{3}c\int \sqrt{c - c\sin(e+fx)}dx + \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\downarrow 3125$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/f) - ((3*A - 5*B)*c*(
(8*c^2*Cos[e + f*x])/(3*f*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sq
rt[c - c*Sin[e + f*x]])/(3*f)))/2)/(a*c)
```

3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.110.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2c^2(\sin(fx+e)-1)(-B(\cos^2(fx+e))+\sin(fx+e)(3A-7B)+9A-13B)}{3a\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$	73

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/3*c^2/a*(sin(f*x+e)-1)*(-B*cos(f*x+e)^2+sin(f*x+e)*(3*A-7*B)+9*A-13*B)/c
os(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(Bc\cos(fx+e)^2-(3A-7B)c\sin(fx+e)-(9A-\right.}{3af\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqr
t(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

3.110.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{Ac\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx + \int \left(-\frac{Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\right)}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(
-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integ
ral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + In
tegral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1),
x))/a
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(108) = 216$.

Time $= 0.33$ (sec) , antiderivative size $= 294$, normalized size of antiderivative $= 2.49$

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = 2\left(\frac{3\left(3c^{\frac{3}{2}}+\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e}{(\cos(fx+e)+1}\right.\right.$$
$$\left.\left.\frac{}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f
*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2
)) - 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(
f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/
2)))/f
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(108) = 216$.

Time = 0.42 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.99

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}}{a + a\sin(e+fx)}\,dx = \frac{4\sqrt{2}\sqrt{c}\left(\frac{3\,(A\mathrm{csgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\mathrm{csgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}+1\right)}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
4/3*sqrt(2)*sqrt(c)*(3*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (3*A*c*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - 7*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) + 18*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*c*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)^2)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 1)^3))/f
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}}{a + a\sin(e+fx)}\,dx = \int \frac{(A + B\,\sin(e+f\,x))\,(c - c\,\sin(e+f\,x))^{3/2}}{a + a\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
, x)
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

3.111.1 Optimal result . . . 1053
3.111.2 Mathematica [A] (verified) . . . 1053
3.111.3 Rubi [A] (verified) . . . 1054
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3.111.5 Fricas [A] (verification not implemented) . . . 1056
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3.111.1 Optimal result

Integrand size = 38, antiderivative size = 73

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

output
```
-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/c/f-(A-3*B)*c*cos(f*x+e)/a/f/(c
-c*sin(f*x+e))^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\sec(e+fx)(-A+2B+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{af}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x]),x]
```

output
```
(2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f
)
```

3.111.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3334, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sqrt{c - c\sin(e+fx)}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sqrt{c - c\sin(e+fx)}dx - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

$$\downarrow 3125$$

$$\frac{-\frac{c^2(A-3B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x
]
```

output
```
(-(((A - 3*B)*c^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*S
ec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/f)/(a*c)
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.111.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(-B\sin(fx+e)+A-2B)}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	53

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\,(B\sin(fx+e)-A+2\,B)\sqrt{-c\sin(fx+e)+c}}{af\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

3.111.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{A\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx + \int \frac{B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(69) = 138.

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.38

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}-\frac{A\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
-2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*s
qrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*s
qrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(69) = 138$.

Time $= 0.35$ (sec) , antiderivative size $= 175$, normalized size of antiderivative $= 2.40$

$$\int \frac{(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}}{a + a\sin(e+fx)}\,dx =$$
$$-\frac{2\sqrt{2}\left(A\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 3B\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \frac{A(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)\operatorname{sgn}(\sin}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}}\right.}{af\left(\frac{(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)^2}{(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)^2} - 1\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
-2*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) - A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - B*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1))*sqrt(c)/(a*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$$

$$= \frac{2\sqrt{-c\,(\sin(e+f\,x)-1)}\left(2\,B\,\sin(2\,e+2\,f\,x)-2\,A\,\sin(2\,e+2\,f\,x)-4\,A\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+7\,B\right.}{a\,f\,\left(4\sin(e+f\,x)^2+\sin(e+f\,x)+\sin(3\,e+3\,f\,x)-\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
,x)
```

output
```
(2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f
*x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1)
+ B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*
f*x) + 4*sin(e + f*x)^2 - 4))
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

3.112.1 Optimal result . . . 1059
3.112.2 Mathematica [C] (verified) . . . 1059
3.112.3 Rubi [A] (verified) . . . 1060
3.112.4 Maple [A] (verified) . . . 1062
3.112.5 Fricas [A] (verification not implemented) . . . 1062
3.112.6 Sympy [F] . . . 1063
3.112.7 Maxima [F] . . . 1063
3.112.8 Giac [A] (verification not implemented) . . . 1064
3.112.9 Mupad [F(-1)] . . . 1064

3.112.1 Optimal result

Integrand size = 38, antiderivative size = 91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

output
```
1/2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a
/f*2^(1/2)/c^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+B-(1+i)\sqrt[4]{-1}(A+B\right.}{af(1+\sin(e+fx))\sqrt{c-c\sin}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[
e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

3.112.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3446, 3042, 3334, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)\sqrt{c - c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)\sqrt{c - c\sin(e+fx)}}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}\,dx}{ac}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$$\downarrow 3128$$

$$\frac{-\frac{c(A+B)\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}\,d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$$\downarrow 219$$

$$\frac{\frac{\sqrt{c}(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x
]
```

output
```
(((A + B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(Sqrt[2]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
f)/(a*c)
```

3.112.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.112.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result
default	$-\frac{(\sin(fx+e)-1)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,A+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,B\right.}{2a\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)
*B)/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.78

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}(A+B)\sqrt{c}\cos(fx+e)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx\right.}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}}{4acf\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x
+ e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e)
 + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) +
 c)*(A - B))/(a*c*f*cos(f*x + e))
```

3.112.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
(Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
 + c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*
x) + sqrt(-c*sin(e + f*x) + c)), x))/a
```

3.112.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)), x)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\frac{\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\sqrt{2}(A\sqrt{c}-B\sqrt{c})}{ac\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{4f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
1/4*(sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))) + 4*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c*((cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))
, x)
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

3.113.1 Optimal result . . . 1065
3.113.2 Mathematica [C] (verified) . . . 1065
3.113.3 Rubi [A] (verified) . . . 1066
3.113.4 Maple [A] (verified) . . . 1068
3.113.5 Fricas [A] (verification not implemented) . . . 1069
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3.113.8 Giac [B] (verification not implemented) . . . 1070
3.113.9 Mupad [F(-1)] . . . 1071

3.113.1 Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(3A-B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+1/8*(3*A-B)*arctanh(1/2*
cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-(A-
B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.09

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\text{si}\right.}{}$$

input

```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 +
I)*(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

3.113.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 3446, 3042, 3334, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{2}c(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx - \frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx - \frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow 3129$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow 3128$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
,x]
```

output
```
(-(((A - B)*Sec[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])) + ((3*A - B)*c*(Ar
cTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2
]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2)/(a*c)
```

3.113.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.113.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\sin(fx+e)\left(3A\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c-B\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c-6A\,c\right.}{8c^{\frac{5}{2}}a\cos(fx$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^
(3/2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/
2)*2^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.70

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}((3\,A-B)\cos(fx+e)\sin(fx+e)-(3\,A-B)\cos(fx+e))\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{}}{\cos(fx+}\right.}{16\,(ac^2f\cos(fx+e}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
 e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c
*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x +
 e))
```

3.113.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{}{-c\sqrt{-c}}}{a}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(
e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

3.113.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)
^(3/2)), x)
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(119) = 238$.

Time $= 0.36$ (sec) , antiderivative size $= 405$, normalized size of antiderivative $= 2.98$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}\left(3A\sqrt{c}-B\sqrt{c}\right)\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/32*(2*sqrt(2)*(3*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^2*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) + sqrt(2)*(A + B + 14*A*(cos(-1/4*pi + 1/2*f*x + 1/2
*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 14*B*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 3*A*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + B*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a
*c^(3/2)*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + (cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))
, x)
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

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3.114.1 Optimal result

Integrand size = 38, antiderivative size = 180

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3(5A-3B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
3/32*(5*A-3*B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A+B)*sec(f*x+e
)/a/c/f/(c-c*sin(f*x+e))^(3/2)+3/64*(5*A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/
2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(5/2)/f*2^(1/2)-1/8*(5*A-3*B)*sec(f
*x+e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\text{si}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*
A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + S
in[e + f*x])*(c - c*Sin[e + f*x])^(5/2))
```

3.114.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$\downarrow 3338$$

$$\frac{\frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8c} + \frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(5A-3B)\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3166$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3129$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 3128$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\downarrow 219$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\arctanh\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))
,x]
```

output
```
(((A + B)*Sec[e + f*x])/(4*f*(c - c*Sin[e + f*x])^(3/2)) + ((5*A - 3*B)*(-
(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e
 + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e
 + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2))/(8*c))/(a*c)
```

3.114.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(157) = 314.

Time = 1.01 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(-15A\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2+30Ac^{\frac{5}{2}}+9B\sqrt{c+c\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^2-$

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/64/c^(9/2)/a*((-15*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+30*A*c^(5/2)+9*B*(c+c*sin(f*x+e))^(1/
2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*c^
(5/2))*cos(f*x+e)^2+sin(f*x+e)*(-30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*A*c^(5/2)+18*B*(c+c*
sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2-24*B*c^(5/2))+30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c
*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-24*A*c^(5/2)-18*B*(c+c*sin(f*x+e))
^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*
B*c^(5/2))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.57

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((5A-3B)\cos(fx+e)^3 + 2(5A-3B)\cos(fx+e)\sin(fx+e) - 2(5A-3B)\cos(fx+e)\right)\sqrt{c}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)
*sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2
 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e
) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(co
s(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3
*(5*A - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sq
rt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*
sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)
^(5/2)), x)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(157) = 314$.

Time $= 0.62$ (sec) , antiderivative size $= 504$, normalized size of antiderivative $= 2.80$

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{12\sqrt{2}(5A\sqrt{c}-3B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{c}+B\sqrt{c}-\frac{1}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
1/512*(12*sqrt(2)*(5*A*sqrt(c) - 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a*c^3*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 16*A*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 90
*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2 - 54*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2/
(a*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))) + 128*sqrt(2)*(A*sqrt(c) - B*sqrt(c))/(a*c^3*((cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - (16*sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - sqrt(2)*A*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
 sqrt(2)*B*a*c^(7/2)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^6))/f
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))
, x)
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

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3.115.1 Optimal result

Integrand size = 38, antiderivative size = 242

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f}$$
$$-\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f}$$
$$-\frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}$$
$$-\frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f}$$
$$-\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f}$$
$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}$$

output

```
-512/105*(7*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*
A-13*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*se
c(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(
f*x+e))^(9/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2
/f+2048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

3.115.2 Mathematica [A] (verified)

Time = 12.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{c^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx}}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
(c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(95550
*A - 179340*B - 72*(203*A - 402*B)*Cos[2*(e + f*x)] + 6*(7*A - 38*B)*Cos[4
*(e + f*x)] + 119952*A*Sin[e + f*x] - 219618*B*Sin[e + f*x] + 784*A*Sin[3*
(e + f*x)] - 2131*B*Sin[3*(e + f*x)] + 15*B*Sin[5*(e + f*x)]))/(840*a^2*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

3.115.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{6}c(7A-13B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{11/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(7A-13B)\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-\frac{(A-B)\sec^3(e+fx)(c-cs}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)+\frac{2c\sec(e+fx)(c-cs}{7f}\right.}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-(A-}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx)(}{a^2c^2}\right.\right.}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)+\right.}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+}{a^2}\right.\right.\right.}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int\frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right.\right.\right.}{a^2c^2}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/}}{3f}\right.\right.\right.}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/f - ((7*A - 13*
B)*c*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(7*f) + (16*c*((2*c*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*f) + (12*c*((2*c*Sec[e + f*x]*(c
 - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[e + f*x]*Sqrt[c - c*Si
n[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/f))/3))/5))
/7))/6)/(a^2*c^2)
```

3.115.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.115.4 Maple [A] (verified)

Time = 20.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(15B(\cos^4(fx+e))\sin(fx+e)+(21A-114B)(\cos^4(fx+e))+(196A-544B)(\cos^2(fx+e))\sin(fx+e)+(-1848A}{105a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output

```
-2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*cos(f*x+e)^4*sin(f*x+e)
+(21*A-114*B)*cos(f*x+e)^4+(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(-1848*A+
3732*B)*cos(f*x+e)^2+(7448*A-13592*B)*sin(f*x+e)+6888*A-13032*B)/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\left(7A-38B\right)c^4\cos\left(fx+e\right)^4-12\left(154A-311B\right)c^4\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x
+ e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136
*B)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(218) = 436.

Time = 0.33 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.15

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 53
70*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(
9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 723*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1)^(9/2)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e)
 + 1) + 35250*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 13801
2*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)
^8 + 68549*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 4707*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/...
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(218) = 436$.

Time $= 0.84$ (sec) , antiderivative size $= 978$, normalized size of antiderivative $= 4.04$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{9/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-16/105*sqrt(2)*sqrt(c)*(35*(11*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 17*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*c^4*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) - 36*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*c
^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^4*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1) + 1)^3) - (511*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 1069*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3262*A*c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 6958*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 8421*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 18459*B*c^
4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 10780*A*c^4*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^3 + 24220*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1...
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{9/2}}{(a+a\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^2, x)
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

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3.116.1 Optimal result

Integrand size = 38, antiderivative size = 201

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}$$

output

```
-32/15*(5*A-11*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-4/15*(5*A-11
*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/15*(5*A-11*B)*sec(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)
/a^2/c^2/f+128/15*(5*A-11*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

3.116.2 Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-2100A+4725B+12(25A-62B)\cos(2(e+$$
$$60a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
-1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x
)] - 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 4
6*B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + S
in[e + f*x])^2)
```

3.116.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow 3446$$
$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}dx}{a^2c^2}$$
$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{6}c(5A-11B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(5A-11B)\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right) - \frac{(A-B)\sec^3(e+fx)(c-cs}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right) - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right) + \frac{2c\sec(e+fx)(c-c\sin}{5f}\right.}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right) + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right) - \frac{(A-}{}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right) + \frac{2c\sec(e+fx)(c-}{3}\right.\right.}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right) + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right) + \right.}{a^2c^2}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/f - ((5*A - 11*
B)*c*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*f) + (12*c*((2*c*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[e + f*x]*
Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2)
)/f))/3))/5))/6)/(a^2*c^2)
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.116.4 Maple [A] (verified)

Time = 20.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(3B(\cos^4(fx+e))+(-5A+23B)(\cos^2(fx+e))\sin(fx+e)+(75A-189B)(\cos^2(fx+e))+(-340A+724B)\sin(fx+e)}{15a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^4+(-5*A+23*B)*c
os(f*x+e)^2*sin(f*x+e)+(75*A-189*B)*cos(f*x+e)^2+(-340*A+724*B)*sin(f*x+e)
-300*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(3Bc^3\cos(fx+e)^4+3(25A-63B)c^3\cos(fx+e)^2-12(25A-57B)c^3-\left((5A-23B)c^3\cos(fx+\right.\right.}{15\left(a^2f\cos(fx+e)\sin(fx+e)+a^2f\cos(fx+\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(2
5*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e)
+ a^2*f*cos(f*x + e))
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(181) = 362$.

Time = 0.34 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.33

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c
^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812
*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 2
85*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)
*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 2896*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 3612*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/
2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(181) = 362$.

Time $= 0.75$ (sec) , antiderivative size $= 774$, normalized size of antiderivative $= 3.85$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-16/15*sqrt(2)*sqrt(c)*(5*(4*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7
*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 6*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 1)^3) - (20*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) - 53*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 85*A*c^3*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 235*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1
25*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 365*B*c^3*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 75*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^3 + 165*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi ...
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^2, x)
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

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3.117.1 Optimal result

Integrand size = 38, antiderivative size = 154

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}$$

output

```
-8/3*(A-3*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-3*B)*sec(f*x
+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(
9/2)/a^2/c^2/f+32/3*(A-3*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

3.117.2 Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-50A+160B+6(A-4B)\cos(2(e+fx))+}{6a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx)}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
-1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x
] + B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
 Sin[e + f*x])^2)
```

3.117.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\se}{3f}}{a^2c^2}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/f - ((A - 3*B)*c
*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[
e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/f))/3))/2)/(a^2*c^2)
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.117.4 Maple [A] (verified)

Time = 19.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{2c^3(\sin(fx+e)-1)(-B(\cos^2(fx+e))\sin(fx+e)+(-3A+12B)(\cos^2(fx+e))+(18A-50B)\sin(fx+e)+14A-46B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$	105

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(-3*A+12*B)*cos(f*x+e)^2+(18*A-50*B)*sin(f*x+e)+14*A-46*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(3\left(A-4\,B\right)c^2\cos\left(fx+e\right)^2-2\left(7\,A-23\,B\right)c^2+\left(Bc^2\cos\left(fx+e\right)^2-2\left(9\,A-25\,B\right)c^2\right)\sin\left(fx+e\right)\right)\sqrt{}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x + e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(138) = 276$.

Time = 0.33 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.75

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-2\left(\frac{\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{90\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{108\,c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{56\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36\,c^{\frac{5}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{11}{(c}\right.\right.$$
$$\left.\left.\left(a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output

```
-2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
 - 2*(17*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
)/f
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(138) = 276$.

Time $= 0.61$ (sec) , antiderivative size $= 293$, normalized size of antiderivative $= 1.90$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}}{(a + a\sin(e+fx))^2}\,dx = \frac{32\sqrt{2}\left(Ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 3Bc^2\mathrm{sgn}\left(\sin\right.\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
32/3*sqrt(2)*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^2 + 9*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2*A*c^2*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^3)*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 1)^3)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{(a+a\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^2, x)
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

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3.118.2 Mathematica [A] (verified) 1104
3.118.3 Rubi [A] (verified) 1105
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3.118.5 Fricas [A] (verification not implemented) 1107
3.118.6 Sympy [F(-1)] 1108
3.118.7 Maxima [B] (verification not implemented) 1108
3.118.8 Giac [B] (verification not implemented) 1109
3.118.9 Mupad [B] (verification not implemented) 1110

3.118.1 Optimal result

Integrand size = 38, antiderivative size = 115

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}$$

output
```
-1/3*(A-7*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^
3*(c-c*sin(f*x+e))^(7/2)/a^2/c^2/f+4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e
))^(1/2)/a^2/f
```

3.118.2 Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(2A-23B+3B\cos$$
$$3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin($$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)
] + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{6}c(A-7B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(A-7B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{6}c(A-7B)\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right) - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{6}c(A-7B)\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx + \frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right) - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{6}c(A-7B)\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f - ((A - 7*B)*c
*((-8*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c
- c*Sin[e + f*x])^(3/2))/f))/6)/(a^2*c^2)
```

3.118.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.118.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2c^2(\sin(fx+e)-1)(3B(\cos^2(fx+e))+\sin(fx+e)(3A-15B)+A-13B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	81

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^2+sin(f*x+e)*(3
*A-15*B)+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\,Bc\cos(fx+e)^2+3\,(A-5\,B)c\sin(fx+e)+(A-\right.}{3\,(a^2 f\cos(fx+e)\sin(fx+e)+a^2 f\,c}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqr
t(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x +
e))
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(103) = 206.

Time = 0.35 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{2\left(\frac{\left(c^{\frac{3}{2}}+\frac{6c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)A}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2\left(5c^{\frac{3}{2}}+\frac{15c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}\right.}{(a^2}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x
 + e)/(cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 5*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)^(3/2)))/f
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(103) = 206$.

Time $= 0.54$ (sec) , antiderivative size $= 283$, normalized size of antiderivative $= 2.46$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}}{(a + a\sin(e+fx))^2}\,dx =$$

$$4\sqrt{2}\sqrt{c}\left(\frac{3\,B c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)} + \frac{A c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 4\,B c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + \frac{3\,Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.}}{}\right.$$

$$-\frac{}{3}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-4/3*sqrt(2)*sqrt(c)*(3*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^2*((cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)
) + (A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*c*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 9*B*c*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 1)^3))/f
```

3.118.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.28

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{2\,B\,c}{a^2\,f}-\frac{B\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{a^2\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}}$$

$$+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{2\,B\,c}{3\,a^2\,f}-\frac{c\,(2\,A-3\,B)}{3\,a^2\,f}-\frac{2\,c\,(3\,A-2\,B)}{3\,a^2\,f}+\frac{c\,(A\,2\mathrm{i}-B\,3\mathrm{i})\,1\mathrm{i}}{3\,a^2\,f}+\frac{c\,(A\,3\mathrm{i}-B\,2\mathrm{i})\,2\mathrm{i}}{3\,a^2\,f}\right)}{(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i})\,(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i})^3}$$

$$-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{c\,(A-B)\,4\mathrm{i}}{a^2\,f}+\frac{c\,(A\,1\mathrm{i}-B\,2\mathrm{i})}{a^2\,f}+\frac{c\,(A\,1\mathrm{i}+B\,2\mathrm{i})}{3\,a^2\,f}\right)}{(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i})\,(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i})^2}$$

$$-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(\frac{4\,B\,c}{a^2\,f}+\frac{c\,(A\,1\mathrm{i}-B\,2\mathrm{i})\,4\mathrm{i}}{a^2\,f}\right)}{(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i})\,(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i})}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3
*A - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2
i)/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (
(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*B*c)/(a^2*f) - (B*c*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i)
 - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^
2*f) + (c*(A*1i + B*2i))/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i
+ f*x*1i) + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)
*4i)/(a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

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3.119.1 Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}$$

output
```
-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^2/c^2/f-1/3*(A+5*B)*sec(f
*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

3.119.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2(A+2B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^2,x]
```

output
```
(-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

3.119.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3334, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{6}c(A+5B)\int \sec^2(e+fx)(c - c\sin(e+fx))^{3/2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(A+5B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

$$\downarrow 3152$$

$$\frac{-\frac{c^2(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2
,x]
```

output
```
(-1/3*((A + 5*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f - ((A - B)*S
ec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*f))/(a^2*c^2)
```

3.119.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.119.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(3B\sin(fx+e)+A+2B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	63

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\,(3\,B\sin(fx+e)+A+2\,B)\sqrt{-c\sin(fx+e)+c}}{3\,(a^2 f\cos(fx+e)\sin(fx+e)+a^2 f\cos(fx+e))}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
 + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

3.119.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{A\sqrt{-c\sin(e+fx)+c}}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx + \int \frac{B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
(Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) +
1), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**
2 + 2*sin(e + f*x) + 1), x))/a**2
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.40

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)))/f
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(70) = 140$.

Time $= 0.74$ (sec) , antiderivative size $= 223$, normalized size of antiderivative $= 2.86$

$$\int \frac{(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}}{(a + a\sin(e+fx))^2}\,dx$$

$$= \frac{\sqrt{2}\left(A\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 5\,B\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{12\,B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+}\right.}{3\,a^2 f\left(\frac{\cos\left(-\frac{1}{4}\right.}{\cos\left(-\frac{1}{4}\right.}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/3*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) + 12*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 3*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2)*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 1)^3)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$$

$$=\frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\left(B\,3\mathrm{i}+2\,A\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+4\,B\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}\right)}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^3\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
(4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i
) - B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*
1i + f*x*1i)*1i + 1))
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

3.120.1 Optimal result 1117
3.120.2 Mathematica [C] (verified) 1117
3.120.3 Rubi [A] (verified) 1118
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3.120.5 Fricas [A] (verification not implemented) 1121
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3.120.7 Maxima [F] 1122
3.120.8 Giac [B] (verification not implemented) 1122
3.120.9 Mupad [F(-1)] 1123

3.120.1 Optimal result

Integrand size = 38, antiderivative size = 135

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}$$

output
```
-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*(A+B)*arctanh
(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1
/2)-1/2*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(-A+B)-3(A+B)\left(\cos\left(\right.\right.\right.\right.}{6a^2}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e +
f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 +
3*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^
2*Sqrt[c - c*Sin[e + f*x]])
```

3.120.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{2}c(A+B)\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}dx - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\downarrow 3154$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\downarrow 3128$$

$$\frac{\frac{1}{2}c(A+B)\left(-\frac{c\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f + ((A + B)*c*(
(Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])
])/(Sqrt[2]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f))/2)/(a^2*c^2)
```

3.120.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.120.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

method	result
default	$-\frac{(\sin(fx+e)-1)\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{3}{2}}cA+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)\right.}{12a^2c^{\frac{5}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/12/a^2*(sin(f*x+e)-1)/c^(5/2)/(1+sin(f*x+e))*(3*2^(1/2)*arctanh(1/2*(c*
(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A+3*2^(1
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)
))^(3/2)*c*B-10*A*c^(5/2)-6*A*c^(5/2)*sin(f*x+e)-2*B*c^(5/2)-6*B*c^(5/2)*s
in(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{3\sqrt{2}((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(}{\cos(fx+e)}\right.}{24\,(a^2cf\cos(fx+e}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt
(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*si
n(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e)
)
```

3.120.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(}}}{a^2}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
(Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e +
 f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(
e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x))/a**2
```

3.120.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^2\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e
) + c)), x)
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(116) = 232$.

Time $= 0.41$ (sec) , antiderivative size $= 259$, normalized size of antiderivative $= 1.92$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{3\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{8\sqrt{2}\left(2A\sqrt{c}+B\sqrt{c}+\frac{3A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{3B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\right.}{a^2c\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.\right.} \Big/ 24f$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/24*(3*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))) + 8*sqrt(2)*(2*A*sqrt(c) + B*sqrt(c) + 3*A*sqrt(c)*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*B*s
qrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e
) + 1) + 3*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))))/f
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^2\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2
)), x)
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

3.121.1 Optimal result 1124
3.121.2 Mathematica [C] (verified) 1124
3.121.3 Rubi [A] (verified) 1125
3.121.4 Maple [A] (verified) 1128
3.121.5 Fricas [A] (verification not implemented) 1128
3.121.6 Sympy [F(-1)] 1129
3.121.7 Maxima [F(-1)] 1129
3.121.8 Giac [B] (verification not implemented) 1130
3.121.9 Mupad [F(-1)] 1130

3.121.1 Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(5A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}$$

output

```
1/8*(5*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+1/16*(5*A+B)*arctanh(1
/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2
)-1/6*(5*A+B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(1/2)/a^2/c^2/f
```

3.121.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+s\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-
1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(3/2))
```

3.121.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3334, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{6}c(5A+B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(5A+B)\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3166$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3129$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\downarrow 3128$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{}}{3f}}{a^2c^2}$$

$$\downarrow 219$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e}}{3f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2
)),x]
```

3.121. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/f + ((5*A + B)*c*(
-(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[
e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[
e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2))/6)/(a^2*c^2)
```

3.121.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.121.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.60

method	result
default	$-\frac{15A(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+30Ac^{\frac{5}{2}}(\cos^2(fx+e))+3B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{}\right.$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/48/c^(7/2)/a^2*(15*A*(c+c*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+30*A*c^(5/2)*cos(f*x+e)^2+3*
B*(c+c*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*sin(f*x+e)*c+6*B*c^(5/2)*cos(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c
+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A-20*A*c^(5
/2)*sin(f*x+e)-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2
))*(c+c*sin(f*x+e))^(3/2)*c*B-4*B*c^(5/2)*sin(f*x+e)-4*A*c^(5/2)-20*B*c^(5
/2))/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}(5A+B)\sqrt{c}\cos(fx+e)^3\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c}}{}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f
*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5
*A + B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*si
n(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.121.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(152) = 304$.

Time = 0.45 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.85

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{6\sqrt{2}(5A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(\frac{A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.\right.}{\cos\left(-\frac{1}{4}\pi+\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
1/192*(6*sqrt(2)*(5*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) + 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 10*A*sqr
t(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1) - 2*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*c^2*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 16*sqr
t(2)*(7*A*sqrt(c) - B*sqrt(c) + 12*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*sqrt(c)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(
c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^2)/(a^2*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^2\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2
)), x)
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

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3.122.2 Mathematica [C] (verified) . . . 1132
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3.122.1 Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5(7A-B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+1/24*(7*A-B)*sec(f*
x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+5/128*(7*A-B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-5/48*(7*A-B)*
sec(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2
/f/(c-c*sin(f*x+e))^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\text{s}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1
/4)*(7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
+ 24*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*
(11*A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(5/2))
```

3.122.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3334, 3042, 3160, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{6}c(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(7A-B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}dx - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3160$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3166$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3129$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

3.122. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

$$\downarrow 3128$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-}{a^2c^2}$$

$$\downarrow 219$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec}{3f\sqrt{c-c}}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2
)),x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3)/(f*Sqrt[c - c*Sin[e + f*x]]) + ((7*A - B)*c
*(Sec[e + f*x]/(4*f*(c - c*Sin[e + f*x])^(3/2)) + (5*(-(Sec[e + f*x]/(f*Sq
rt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*S
qrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c
*Sin[e + f*x])^(3/2))))/2))/(8*c)))/6)/(a^2*c^2)
```

3.122.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3160
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Simp[(m + p + 1)/(a*(2*m + p + 1))   Int[
(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &
& IntegersQ[2*m, 2*p]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(198) = 396.

Time = 0.92 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.89

method	result
default	$-\frac{-210A(\sin^3(fx+e))c^{\frac{7}{2}}+30B(\sin^3(fx+e))c^{\frac{7}{2}}-46B\sin(fx+e)c^{\frac{7}{2}}+70A(\sin^2(fx+e))c^{\frac{7}{2}}-10B(\sin^2(fx+e))c^{\frac{7}{2}}+322A\sin(fx+e)c$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/384/c^(11/2)/a^2*(-210*A*sin(f*x+e)^3*c^(7/2)+30*B*sin(f*x+e)^3*c^(7/2)
-46*B*sin(f*x+e)*c^(7/2)+70*A*sin(f*x+e)^2*c^(7/2)-10*B*sin(f*x+e)^2*c^(7/
2)+322*A*sin(f*x+e)*c^(7/2)-15*B*sin(f*x+e)^2*2^(1/2)*arctanh(1/2*(c*(1+si
n(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c^2-210*A*sin(f
*x+e)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+
sin(f*x+e)))^(3/2)*c^2+30*B*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e
)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c^2+105*A*sin(f*x+e)^2
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f
*x+e)))^(3/2)*c^2-86*A*c^(7/2)+122*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)))^(3/2
)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-15*B*(
c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*c^2)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))
^(1/2)/f
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{15\sqrt{2}\left((7A-B)\cos(fx+e)^3\sin(fx+e)-(7A-B)\cos(fx+e)^3\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\sqrt{2}\sqrt{-c\sin(fx+}}{c}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(
f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e
) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*
(7*A - B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*
sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(
f*x + e)^3)
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. 2(198) = 396.

Time = 0.57 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.33

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
1/3072*(60*sqrt(2)*(7*A*sqrt(c) - B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^2*c^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 24*A*sqrt(c)*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 8*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1) + 210*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^2/(a^2*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(5*A*sqrt(c) - 2*B*sqrt(c) + 9*
A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) - 3*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1) + 6*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*sqrt(c)*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)/(a^2*c^3*((co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1
)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*(24*sqrt(2)*A*a^2*c^(7/2)*(co
s(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 8*sqrt(2)*B*a^2*c^(7/2)*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi ...
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^2\,(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2
)), x)
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

3.123.1 Optimal result 1141
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3.123.1 Optimal result

Integrand size = 38, antiderivative size = 242

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx &= \\ -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} &\\ +\frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} &\\ -\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} &\\ -\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} &\\ -\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} &\\ -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} &\end{aligned}$$

output

```
-2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*
B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e)
)^(9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5
*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f
```

3.123.2 Mathematica [A] (verified)

Time = 9.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{c^4(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}(11298A-33516B-40(137A-402B)\cos(2(e+fx))-10(A$$

$$120a^3f\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 3351
6*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)]
 + 15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1
335*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

3.123.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\int \frac{(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right) - \frac{(A-B)\sec^5(e+fx)(c-cs}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(}{5f}\right)}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right) - \frac{(A-}{}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-}{3f}\right)\right)}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) + \frac{2}{}\right)}{a^3c^3}$$

$$\downarrow 3153$$

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+\right.\right.\right.$$

$$\downarrow 3042$$

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right.\right.\right.$$

$$a^3c^3$$

$$\downarrow 3152$$

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))}{f}\right.\right.\right.$$

$$a^3c^3$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/f - ((A - 3*B)*
c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*f) + (16*c*((2*c*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(3*f) + 4*c*((2*c*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e + f*x]^3*(c - c*Sin[e +
f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/f))))
/5))/2)/(a^3*c^3)
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.123.4 Maple [A] (verified)

Time = 120.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)(3B(\cos^4(fx+e))\sin(fx+e)+(5A-30B)(\cos^4(fx+e))+(100A-336B)(\cos^2(fx+e))\sin(fx+e)+(680A-1980}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(3*B*cos(f*x+e)^4*sin(f*x+e)
+(5*A-30*B)*cos(f*x+e)^4+(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(680*A-1980
*B)*cos(f*x+e)^2+(-1000*A+3048*B)*sin(f*x+e)-1048*A+3096*B)/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(5\left(A-6\,B\right)c^4\cos\left(fx+e\right)^4+20\left(34\,A-99\,B\right)c^4\cos\left(fx+e\right)^2-8\left(131\,A-387\,B\right)c^4+\left(3\,Bc^4\cos\left(fx\right.\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^
2 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*
cos(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(218) = 436.

Time = 0.33 (sec) , antiderivative size = 945, normalized size of antiderivative = 3.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c
^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 30200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5
301*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x +
 e)^13/(cos(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) +
 1)^14)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/
2) + 905*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 10521*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 20772*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(
9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9...
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{1024\sqrt{2}\left(Ac^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3Bc^4\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{5Ac^4(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-}{(\cos(-\frac{1}{4}\pi+}\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1024/15*sqrt(2)*(A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 + 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*A*c^4*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 30*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4 - 6*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 6*B*c^4*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1)^5)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{9/2}}{(a+a\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^3, x)
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

3.124.1 Optimal result 1149
3.124.2 Mathematica [A] (verified) 1150
3.124.3 Rubi [A] (verified) 1150
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3.124.5 Fricas [A] (verification not implemented) 1153
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3.124.8 Giac [B] (verification not implemented) 1155
3.124.9 Mupad [F(-1)] 1156

3.124.1 Optimal result

Integrand size = 38, antiderivative size = 209

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx &= \\ -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} &\\ +\frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} &\\ -\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} &\\ -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} &\\ -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} &\end{aligned}$$

output

```
-128/15*(3*A-13*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+32/5*(3*A
-13*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-4/5*(3*A-13*B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-1/15*(3*A-13*B)*sec(f*x+e)^3*(c-c*sin(f*
x+e))^(9/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(13/2)/a^3/c^3
/f
```

3.124.2 Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(1092A-4557B+(-540A+2200B)\cos(2(e-$$
$$60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x
)] + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 1
70*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
Sin[e + f*x])^3)
```

3.124.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3446$$
$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}dx}{a^3c^3}$$
$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{10}c(3A-13B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(3A-13B)\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-cs}{3f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right) - (A-}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) + \frac{2c\sec^3(e+fx}{a^3c^3}\right.$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)\right.}{a^3c^3}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/}}{f}\right.}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/f - ((3*A - 13*
B)*c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(3*f) + 4*c*((2*c*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e + f*x]^3*(c
 - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])
^(5/2))/f))))/10)/(a^3*c^3)
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.124.4 Maple [A] (verified)

Time = 121.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2c^4(\sin(fx+e)-1)(5B(\cos^4(fx+e))+(-15A+85B)(\cos^2(fx+e))\sin(fx+e)+(-135A+545B)(\cos^2(fx+e))+(180A-820B)\sin(fx}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*cos(f*x+e)^4+(-15*A+85*B
)*cos(f*x+e)^2*sin(f*x+e)+(-135*A+545*B)*cos(f*x+e)^2+(180*A-820*B)*sin(f*
x+e)+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\left(5\,Bc^3\cos(fx+e)^4 - 5\,(27\,A-109\,B)c^3\cos(fx+e)^2}{15\left(a^3 f\cos}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51
*A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*c
os(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(189) = 378.

Time = 0.33 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.09

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 122
0*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
+ 1065*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 + 110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(
f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(7/2)) - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1
) + 1992*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 8370*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 6605*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7
/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(co
s(f*x + e) + 1)^10 + 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 ...
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(189) = 378$.

Time $= 0.56$ (sec) , antiderivative size $= 774$, normalized size of antiderivative $= 3.70$

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-4/15*sqrt(2)*sqrt(c)*(5*(3*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 19
*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 42*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) - 1)^3) - (33*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) - 113*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*A*c^3*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 490*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
+ 240*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 740*B*c^3*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^3 - 390*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*...
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^3, x)
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

3.125.1 Optimal result 1157
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3.125.9 Mupad [B] (verification not implemented) 1164

3.125.1 Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}$$

output

```
-32/15*(A-11*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+8/5*(A-11*B)*s
ec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-1/5*(A-11*B)*sec(f*x+e)^3*(c-c*si
n(f*x+e))^(7/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3
/c^3/f
```

3.125.2 Mathematica [A] (verified)

Time = 7.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(58A-488B-30(A-8B)\cos(2(e+fx))+5$$
$$30a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx)$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*
x] + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*
(1 + Sin[e + f*x])^3)
```

3.125.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$
$$\downarrow 3446$$
$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}dx}{a^3c^3}$$
$$\downarrow 3042$$
$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{-\frac{1}{10}c(A-11B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(A-11B)\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\si}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c}{f}\right)}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) - (A}{a^3c^3}$$

$$\downarrow 3152$$

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f} - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) + \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right) -}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/f - ((A - 11*B)
*c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e
+ f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(5/2))/f)))/10)/(a^3*c^3)
```

3.125. $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

3.125.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.125.4 Maple [A] (verified)

Time = 118.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2c^3(\sin(fx+e)-1)(15B(\cos^2(fx+e))\sin(fx+e)+(-15A+120B)(\cos^2(fx+e))+(10A-170B)\sin(fx+e)+22A-182B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	105

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)
+(-15*A+120*B)*cos(f*x+e)^2+(10*A-170*B)*sin(f*x+e)+22*A-182*B)/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\left(A-8\,B\right)c^2\cos\left(fx+e\right)^2-2\left(11\,A-91\,B\right)c^2-5\left(3\,Bc^2\cos\left(fx+e\right)^2+2\left(A-17\,B\right)c^2\right)\sin\left(fx+\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*
cos(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f
*x + e))
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(144) = 288$.

Time = 0.31 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.76

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output

```
2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 +
5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 680*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 1030*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 155*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2
)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos
(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1...
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(144) = 288$.

Time $= 0.51$ (sec) , antiderivative size $= 449$, normalized size of antiderivative $= 2.81$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}}{(a + a\sin(e+fx))^3}\,dx = 4\sqrt{2}\sqrt{c}\left(\frac{15\,Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)} + \frac{4\,Ac^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.\right.\right.$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
4/15*sqrt(2)*sqrt(c)*(15*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^3*((
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 1)) + (4*A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 29*B*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) + 20*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
130*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 40*A*c^2*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)^2 - 200*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
 - 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 15*B*c^2*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5))/f
```

3.125.9 Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 904, normalized size of antiderivative = 5.65

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^3,x)
```

output

```
((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(2*B*c^2)/(a^3*f) - (B*c^2*exp(e*1i + f*x*1i)*2i)/(a^3*f)))/(exp(e*1i + f*
x*1i) - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*7i)*1i)/(3*a^3*f) - (2*c^2
*(7*A - 12*B))/(3*a^3*f) + (c^2*(A*23i - B*28i)*2i)/(3*a^3*f) - (c^2*(42*A
 - 67*B))/(15*a^3*f) + (2*B*c^2)/(3*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^
3*f) + (4*B*c^2)/(a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
+ 1i)) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B
*3i))/(2*a^3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)
/(4*a^3*f) + (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)
*(exp(e*1i + f*x*1i) + 1i)^4) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i))/(a^
3*f) + (c^2*(A*5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((exp(
e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) + (exp(e*1i + f*x*1i)*(c
 - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^
2*(A*2i - B*5i)*1i)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*
B))/(5*a^3*f) + (c^2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5...
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

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3.126.1 Optimal result

Integrand size = 38, antiderivative size = 121

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}$$

output
```
4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x
+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e
))^(9/2)/a^3/c^3/f
```

3.126.2 Mathematica [A] (verified)

Time = 6.83 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2A+27B-15$$
$$15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*
x)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

3.126.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{9/2}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{10}c(A+9B)\int \sec^4(e+fx)(c - c\sin(e+fx))^{7/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(A+9B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\downarrow 3153$$

$$\frac{\frac{1}{10}c(A+9B)\left(-4c\int \sec^4(e+fx)(c - c\sin(e+fx))^{5/2}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(A+9B)\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx - \frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\downarrow 3152$$

$$\frac{\frac{1}{10}c(A+9B)\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/f + ((A + 9*B)*c
*((8*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f
*x]^3*(c - c*Sin[e + f*x])^(5/2))/f))/10)/(a^3*c^3)
```

3.126.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.126.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{2c^2(\sin(fx+e)-1)(-15B(\cos^2(fx+e))+\sin(fx+e)(5A+15B)-A+21B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	83

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(-15*B*cos(f*x+e)^2+sin(f*x+
e)*(5*A+15*B)-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\left(15\,Bc\cos\left(fx+e\right)^2-5\left(A+3\,B\right)c\sin\left(fx+e\right)+(A-\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*s
qrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(
f*x + e) - 2*a^3*f*cos(f*x + e))
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(109) = 218.

Time = 0.31 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
3/2)) - 6*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 + 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)^(3/2)))/f
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(109) = 218$.

Time $= 0.45$ (sec) , antiderivative size $= 373$, normalized size of antiderivative $= 3.08$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}}{(a + a\sin(e+fx))^3}\,dx =$$

$$-\frac{2\sqrt{2}\left(Ac\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 9\,Bc\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{5\,Ac(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)\mathrm{sgn}(s}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-2/15*sqrt(2)*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) + 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 45*
B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2 + 75*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*A*c*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*B*c*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5)
```

3.126.9 Mupad [B] (verification not implemented)

Time $= 19.60$ (sec) , antiderivative size $= 683$, normalized size of antiderivative $= 5.64$

$$\int \frac{(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}}{(a + a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^3,x)
```

output

```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*
(3*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)
*4i)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5) -
 (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (
2*c*(10*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((exp(e*
1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c -
 c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(2
*A - 3*B)*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))
/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (exp(e*1i + f*x
*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*((c*(A - B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a
^3*f) + (c*(A*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((exp(e*1i
 + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*exp(e*1i + f*x*1i)*
(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(
a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

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3.127.1 Optimal result

Integrand size = 38, antiderivative size = 85

$$\begin{aligned}\int &\frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx \\ &= -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf} \\ &\quad -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}\end{aligned}$$

output
```
-1/15*(3*A+7*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c/f-1/5*(A-B)*sec(
f*x+e)^5*(c-c*sin(f*x+e))^(7/2)/a^3/c^3/f
```

3.127.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\begin{aligned}\int &\frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx \\ &= -\frac{2(3A+2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^3,x]
```

output
```
(-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
)
```

3.127.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3334, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{10}c(3A+7B)\int \sec^4(e+fx)(c - c\sin(e+fx))^{5/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(3A+7B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

$$\downarrow 3152$$

$$\frac{-\frac{c^2(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3
,x]
```

output
```
(-1/15*((3*A + 7*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f - ((A
 - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*f))/(a^3*c^3)
```

3.127.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.127.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(5B\sin(fx+e)+3A+2B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\,(5\,B\sin(fx+e)+3\,A+2\,B)\sqrt{-c\sin(fx+e)+c}}{15\,\left(a^3 f\cos(fx+e)^3-2\,a^3 f\cos(fx+e)\sin(fx+e)-2\,a^3 f\cos(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(77) = 154$.

Time $= 0.31$ (sec) , antiderivative size $= 505$, normalized size of antiderivative $= 5.94$

$$\int \frac{(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}}{(a + a\sin(e+fx))^3}\,dx$$

$$= \frac{2\left(\frac{2B\left(\sqrt{c}+\frac{5\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5\sqrt{c}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}} + \frac{}{\left(a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10}{(}\right.}}{15\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*
sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5
*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(77) = 154$.

Time $= 0.41$ (sec) , antiderivative size $= 368$, normalized size of antiderivative $= 4.33$

$$\int \frac{(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}}{(a + a\sin(e+fx))^3}\,dx$$

$$= \frac{\sqrt{2}\left(3\,A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+7\,B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{20\,B\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)\mathrm{sgn}(\sin}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/30*sqrt(2)*(3*A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 20*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*A*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*B*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2 + 60*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 15*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^4)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 1)^5)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 18.18 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.64

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{8\,B}{5\,a^3\,f}-\frac{16\,A-8\,B}{10\,a^3\,f}+\frac{(A\,16i-B\,8i)\,1i}{10\,a^3\,f}\right)}{(e^{e\,1i+f\,x\,1i}-i)\,(e^{e\,1i+f\,x\,1i}+1i)^5}$$

$$-\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{4\,B}{3\,a^3\,f}-\frac{16\,A-16\,B}{30\,a^3\,f}+\frac{(A\,80i-B\,120i)\,1i}{30\,a^3\,f}\right)}{(e^{e\,1i+f\,x\,1i}-i)\,(e^{e\,1i+f\,x\,1i}+1i)^3}$$

$$-\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(-\frac{B\,1i}{a^3\,f}+\frac{A\,16i-B\,16i}{40\,a^3\,f}+\frac{A\,80i-B\,80i}{40\,a^3\,f}+\frac{(160\,A-120\,B)\,1i}{40\,a^3\,f}\right)}{(e^{e\,1i+f\,x\,1i}-i)\,(e^{e\,1i+f\,x\,1i}+1i)^4}$$

$$-\frac{B\,e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,8i}{3\,a^3\,f\,(e^{e\,1i+f\,x\,1i}-i)\,(e^{e\,1i+f\,x\,1i}+1i)^2}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^3,x)
```

output

```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B
*8i)*1i)/(10*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)
^5) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*
80i - B*120i)*1i)/(30*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*
1i) + 1i)^3) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*1i)/(a^3
*f) + (A*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*1i)/(40*a^3*f)))/((exp
(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (B*exp(e*1i + f*x*1i)
*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*8
i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2)
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

3.128.1 Optimal result 1180
3.128.2 Mathematica [C] (verified) 1181
3.128.3 Rubi [A] (verified) 1181
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3.128.5 Fricas [A] (verification not implemented) 1185
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3.128.1 Optimal result

Integrand size = 38, antiderivative size = 174

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} \\
&\quad - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} \\
&\quad - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}
\end{aligned}$$

output

```
-1/6*(A+B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x
+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^3/f+1/8*(A+B)*arctanh(1/2*cos(f*x+e)*c^
(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*(A+B)*sec(
f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^3/c/f
```

3.128.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(-A+B)-10(A+B)\left(\text{co}\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(
A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin
[e + f*x]])
```

3.128.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3154, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}}dx$$
$$\downarrow 3446$$
$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{2}c(A+B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$$\downarrow 3154$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}dx - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}dx - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$$\downarrow 3154$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right) - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right) - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-}{5f}}{a^3c^3}$$

$$\downarrow 3128$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(-\frac{c\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f} - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right) - \frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right) - }{a^3c^3}$$

$$\downarrow 219$$

3.128. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}dx$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\sec^5}{}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/f + ((A + B)*c*(
-1/3*(Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f + (c*((Sqrt[c]*ArcTanh[
(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(Sqrt[2]*f) -
(Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f))/2))/2)/(a^3*c^3)
```

3.128.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.128.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.15

method	result
default	$-\frac{(\sin(fx+e)-1)\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{5}{2}}c^2A-74c^{\frac{9}{2}}A-80Ac^{\frac{9}{2}}\sin(fx+e)-30Ac^{\frac{9}{2}}(\sin^2(fx+e))\right.}{120a^3c^{\frac{9}{2}}(1+\sin(fx+e))^2\cos(fx+}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/120/a^3*(sin(f*x+e)-1)/c^(9/2)/(1+sin(f*x+e))^2*(15*2^(1/2)*arctanh(1/2
*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A-
74*c^(9/2)*A-80*A*c^(9/2)*sin(f*x+e)-30*A*c^(9/2)*sin(f*x+e)^2+15*2^(1/2)*
arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(
5/2)*c^2*B-26*c^(9/2)*B-80*B*c^(9/2)*sin(f*x+e)-30*B*c^(9/2)*sin(f*x+e)^2)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{15\sqrt{2}\left((A+B)\cos(fx+e)^3 - 2(A+B)\cos(fx+e)\sin(fx+e) - 2(A+B)\cos(fx+e)\right)\sqrt{c}\log\left(-\frac{c}{2}\right.}{2}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
 + e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)
*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c
*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*co
s(f*x + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e)
 + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a
^3*c*f*cos(f*x + e))
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^3\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e
) + c)), x)
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(151) = 302$.

Time $= 0.46$ (sec) , antiderivative size $= 450$, normalized size of antiderivative $= 2.59$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\frac{15\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^3 c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\sqrt{2}\left(23A\sqrt{c}+17B\sqrt{c}+\frac{70A\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{70B\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right.\right.}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}$$
$$=$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/240*(15*sqrt(2)*(A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) + 4*sqrt(2)*(23*A*sqrt(c) + 17*B*sqrt(c) + 70*A*sqrt(c)*
(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
+ 70*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 140*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 80*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*A*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 +
 90*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)^3 + 45*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*c*((cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^3\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2
)), x)
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

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3.129.1 Optimal result

Integrand size = 38, antiderivative size = 224

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(7A+3B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}$$

output

```
1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*(A-B)*sec(f*x+e
)^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+1/32*(7*A+3*B)*arctanh(1/2*cos(f*x+e)
*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-1/12*(7*A+3
*B)*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+3*B)*sec(f*x+e)^3*
(c-c*sin(f*x+e))^(1/2)/a^3/c^2/f
```

3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.59

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+s}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A +
 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A
+ B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f
*(1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

3.129.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{10}c(7A+3B)\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(7A+3B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^4}dx - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\downarrow 3154$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\downarrow 3166$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) - \frac{(A-B)\sec^5(e+fx)(c-c\sin}{5f}}{a^3c^3}$$

$$\downarrow 3129$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) -}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c} + \frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right) -}{a^3c^3}$$

3.129. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

$$\downarrow 3128$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{}}{}}{a^3c^3}$$

$$\downarrow 219$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin(}}{3f}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2
)),x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/f + ((7*A + 3*B)
*c*(-1/3*(Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/f + (5*c*(-(Sec[e + f*x
]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f
*(c - c*Sin[e + f*x])^(3/2))))/2))/6))/10)/(a^3*c^3)
```

3.129.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.129.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.38

method	result
default	$-\frac{105A(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+45B(c(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{}}{2\sqrt{c}}\right.$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/480/c^(9/2)/a^3*(105*A*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*
(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+45*B*(c*(1+sin(f*x+e))
)^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(
f*x+e)*c-210*A*sin(f*x+e)^3*c^(7/2)-90*B*sin(f*x+e)^3*c^(7/2)+18*B*sin(f*x
+e)*c^(7/2)-350*A*sin(f*x+e)^2*c^(7/2)-150*B*sin(f*x+e)^2*c^(7/2)+42*A*sin
(f*x+e)*c^(7/2)+278*A*c^(7/2)-18*B*c^(7/2)-45*B*(c*(1+sin(f*x+e)))^(5/2)*2
^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-105*A*(c*(1
+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*c)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}\big((7A+3B)\cos(fx+e)^3\sin(fx+e)+(7A+3B$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*c
os(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x
+ e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (
c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e)
+ 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2
+ 3*(5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*
B)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3
*c^2*f*cos(f*x + e)^3)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(197) = 394$.

Time $= 0.46$ (sec) , antiderivative size $= 650$, normalized size of antiderivative $= 2.90$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
1/1920*(30*sqrt(2)*(7*A*sqrt(c) + 3*B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + B*sqrt(c)*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 14
*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1) - 6*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1))*(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*c^2*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) +
32*sqrt(2)*(29*A*sqrt(c) + 6*B*sqrt(c) + 100*A*sqrt(c)*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*B*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 17
0*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)^2 + 30*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 120*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
+ 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 30*B*sqrt(c)*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3
+ 45*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)^4)/(a^3*c^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos...
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^3\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2
)), x)
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

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3.130.1 Optimal result

Integrand size = 38, antiderivative size = 258

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{7(9A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} + \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}$$

output

```
7/128*(9*A+B)*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/240*(9*A+B)*sec(
f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/256*(9*A+B)*arctanh(1/2*cos(f*x+e)
*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-7/96*(9*A+B
)*sec(f*x+e)/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/30*(9*A+B)*sec(f*x+e)^3/a^
3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(1/
2)/a^3/c^3/f
```

3.130.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+s}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (10
5 + 105*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 +
 Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))
```

3.130.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 3446, 3042, 3334, 3042, 3166, 3042, 3160, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3334$$

$$\frac{\frac{1}{10}c(9A+B)\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(9A+B)\int \frac{1}{\cos(e+fx)^4\sqrt{c-c\sin(e+fx)}}dx - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\downarrow 3166$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\downarrow 3160$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(}}{5f}}{a^3c^3}$$

$$\downarrow 3166$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c} + \frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right) - \frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{(A-B)\sec^{5}}{}}{a^3c^3}$$

3.130. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^5}{}}{a^3c^3}$$

$$\downarrow 3129$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx}{3f\sqrt{c-c\sin(e-}}\right)}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx}{3f\sqrt{c-c\sin(e-}}\right)}{a^3c^3}$$

$$\downarrow 3128$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)}{a^3c^3}$$

$$\downarrow 219$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\arctanh\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{}{3f}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2
)),x]
```

3.130. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/f + ((9*A + B)*c*(
-1/3*Sec[e + f*x]^3/(f*Sqrt[c - c*Sin[e + f*x]]) + (7*c*(Sec[e + f*x]/(4*f
*(c - c*Sin[e + f*x])^(3/2)) + (5*(-(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*
x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2
))))/2))/(8*c)))/6))/10)/(a^3*c^3)
```

3.130.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3160
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Simp[(m + p + 1)/(a*(2*m + p + 1))   Int[
(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &
& IntegersQ[2*m, 2*p]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.130.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\left(-1890c^{\frac{9}{2}}A-210c^{\frac{9}{2}}B\right)(\cos^4(fx+e))+\left(1260c^{\frac{9}{2}}A+140c^{\frac{9}{2}}B\right)(\cos^2(fx+e))\sin(fx+e)+\left(-945\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/3840/c^(13/2)/a^3*((-1890*c^(9/2)*A-210*c^(9/2)*B)*cos(f*x+e)^4+(1260*c
^(9/2)*A+140*c^(9/2)*B)*cos(f*x+e)^2*sin(f*x+e)+(-945*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+252*c
^(9/2)*A-105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(
c+c*sin(f*x+e))^(5/2)*c^2*B+28*c^(9/2)*B)*cos(f*x+e)^2+(-1890*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2
*A+864*c^(9/2)*A-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B+96*c^(9/2)*B)*sin(f*x+e)+1890*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)
*c^2*A+96*c^(9/2)*A+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B+864*c^(9/2)*B)/(1+sin(f*x+e))^2/(si
n(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{105\sqrt{2}(9A+B)\sqrt{c}\cos(fx+e)^5\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{}}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^
2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x +
e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(c
os(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
105*(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A +
B)*cos(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin
(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.130.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(227) = 454$.

Time $= 0.56$ (sec) , antiderivative size $= 901$, normalized size of antiderivative $= 3.49$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
1/30720*(420*sqrt(2)*(9*A*sqrt(c) + B*sqrt(c))*log(-(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1))/(a^3*c^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(c) + B*sqrt(c) - 32*A*sqrt(c
)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
) - 16*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) + 378*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 42*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2)*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^2/(a^3*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 256*sqrt(2)*(54*A*sqrt(c) - 4*B*sqrt(c)
 + 195*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f
*x + 1/2*e) + 1) - 5*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-
1/4*pi + 1/2*f*x + 1/2*e) + 1) + 315*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 25*B*sqrt(c)*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 22
5*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1)^3 - 15*B*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3/(cos
(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 75*A*sqrt(c)*(cos(-1/4*pi + 1/2*f*x +
 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*sqrt(c)*(cos(
-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^...
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^3\,(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2
)), x)
```

3.131 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

3.131.1 Optimal result . . . 1205
3.131.2 Mathematica [A] (verified) . . . 1205
3.131.3 Rubi [A] (verified) . . . 1206
3.131.4 Maple [A] (verified) . . . 1207
3.131.5 Fricas [A] (verification not implemented) . . . 1208
3.131.6 Sympy [F(-1)] . . . 1208
3.131.7 Maxima [F] . . . 1209
3.131.8 Giac [A] (verification not implemented) . . . 1209
3.131.9 Mupad [B] (verification not implemented) . . . 1210

3.131.1 Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/4*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
5*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(-60A+23B)\sin(e+fx)+4\cos(2(e+fx))(-}{160f}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
-1/160*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x
]]*(4*(-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(
5*A - 6*B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x]
)))/f
```

3.131.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx}{c}$$

$$\downarrow 3217$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
-1/4*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

3.131.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.131.4 Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

method	result
default	$\frac{c^3\tan(fx+e)\left(4B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+5A\sin(fx+e)\left(\cos^2(fx+e)\right)+15B\left(\sin^3(fx+e)\right)-20A\left(\cos^2(fx+e)\right)-24B\left(\sin^2(fx+e\right.\right.}{20f}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,c^3\left(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)-8\tan(fx+e)+7\sec(fx+e)\right)}{4f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/20*c^3/f*tan(f*x+e)*(4*B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*sin(f*x+e)*cos(f*
x+e)^2+15*B*sin(f*x+e)^3-20*A*cos(f*x+e)^2-24*B*sin(f*x+e)^2-35*A*sin(f*x+
e)+10*B*sin(f*x+e)+40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2
)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.49

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{\left(5\,(A-3\,B)c^3\cos\left(fx+e\right)^4-40\,(A-B)c^3\cos\left(fx+e\right)^2+5\,(7\,A-5\,B)c^3+4\,\left(Bc^3\cos\left(fx+e\right)^4+(5\,A\right.}{20\,f\cos\left(fx\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*
(7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2
 - 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e))
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.131.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(7/2), x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{4\left(8\,Bc^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-5\,Ac^3\mathrm{sgn}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/5*(8*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*A*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^8 - 5*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

3.131.9 Mupad [B] (verification not implemented)

Time = 16.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{c^3\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(100\,B\,\cos(e+f\,x)-140\,A\,\cos(e+f\,x)-135\,A\,\cos$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
-(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*co
s(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5
*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*
f*x) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*
x) + 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.132 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

3.132.1 Optimal result 1211
3.132.2 Mathematica [A] (verified) 1211
3.132.3 Rubi [A] (verified) 1212
3.132.4 Maple [A] (verified) 1213
3.132.5 Fricas [A] (verification not implemented) 1214
3.132.6 Sympy [F(-1)] 1214
3.132.7 Maxima [F] 1214
3.132.8 Giac [A] (verification not implemented) 1215
3.132.9 Mupad [B] (verification not implemented) 1215

3.132.1 Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/3*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
4*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e+fx))+16(7A-2B)\sin}{96f}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
(c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B
*Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*
A + 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

3.132.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx}{c}$$

$$\downarrow 3217$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/3*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.132.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
default	$-\frac{c^2\tan(fx+e)\left(-3B\left(\sin^3(fx+e)\right)+4A\left(\cos^2(fx+e)\right)+8B\left(\sin^2(fx+e)\right)+12A\sin(fx+e)-6B\sin(fx+e)-16A\right)\sqrt{-c(\sin(fx+e)-1}}{12f}$
parts	$-\frac{A\,c^2\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-4\tan(fx+e)+3\sec(fx+e))}{3f}+\frac{B\sec(fx+e)(3}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/12*c^2/f*tan(f*x+e)*(-3*B*sin(f*x+e)^3+4*A*cos(f*x+e)^2+8*B*sin(f*x+e)^
2+12*A*sin(f*x+e)-6*B*sin(f*x+e)-16*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin
(f*x+e)))^(1/2)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(3Bc^2\cos(fx+e)^4+12(A-B)c^2\cos(fx+e)^2-3(4A-3B)c^2-4\left((A-2B)c^2\cos(fx\right.\right.}{12f\cos(fx+e}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*
B)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2), x)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{4\left(3\,Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8-2\,Ac^2\mathrm{sgn}(\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/3*(3*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*c^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^6 - 2*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

3.132.9 Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e$$
$$+fx))^{5/2}\,dx = \frac{c^2\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(48\,A\,\cos(e+f\,x)-36\,B\,\cos(e+f\,x)+}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(
e + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*
x) + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x)
- 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x)
+ 1))
```

3.133 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.133.1 Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/2*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
3*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(2(6A-B)\sin(e+fx)+\cos(2(e+fx$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*
A - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(1
2*f)
```

3.133.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx}{c}$$

$$\downarrow 3217$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

3.133.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.133.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c\tan(fx+e)\left(2B\left(\sin^2(fx+e)\right)+3A\sin(fx+e)-3B\sin(fx+e)-6A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{6f}$
parts	$\frac{Ac\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)+2\tan(fx+e)-\sec(fx+e))}{2f}+\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e))(2}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6*c/f*tan(f*x+e)*(2*B*sin(f*x+e)^2+3*A*sin(f*x+e)-3*B*sin(f*x+e)-6*A)*(
-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{\left(3\,(A-B)c\cos\left(fx+e\right)^2-3\,(A-B)c+2\left(Bc\cos\left(fx+e\right)^2+(3\,A-B)c\right)\sin\left(fx+e\right)\right)\sqrt{}}{6\,f\cos\left(fx+e\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3
*A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)/(f*cos(f*x + e))
```

3.133.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*
sin(e + f*x)), x)
```

3.133.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2), x)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(4\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-3\,Ac\mathrm{sgn}(\mathrm{co}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-2/3*(4*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 3*A*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^4 - 3*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

3.133.9 Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e$$
$$+fx))^{3/2}\,dx = \frac{c\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(3\,A\,\cos(e+f\,x)-3\,B\,\cos(e+f\,x)+3\,A}{12\,f\,(}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
(c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
 f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 1
2*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(
cos(2*e + 2*f*x) + 1))
```

3.134 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

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3.134.2 Mathematica [A] (verified) 1223
3.134.3 Rubi [A] (verified) 1224
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3.134.7 Maxima [F] 1226
3.134.8 Giac [A] (verification not implemented) 1227
3.134.9 Mupad [B] (verification not implemented) 1227

3.134.1 Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

output
```
1/2*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-a*(A+
B)*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}(A+B\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}{2Bf}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(A + B*Sin[e + f*x])^2*Sqrt[c - c
*Sin[e + f*x]])/(2*B*f)
```

3.134.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx}{c}$$

$$\downarrow 3217$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
-((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e +
f*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

3.134.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.134.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\tan(fx+e)(B\sin(fx+e)+2A)\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}}{2f}$	49
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}}{f}+\frac{B\sin(fx+e)\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}}{2f}$	81

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/f*tan(f*x+e)*(B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*
x+e)))^(1/2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= -\frac{\left(B\cos(fx+e)^2-2A\sin(fx+e)-B\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2f\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.134.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int \sqrt{a\left(\sin(e+fx)+1\right)}\sqrt{-c\left(\sin(e+fx)-1\right)}(A+B\sin(e+fx))\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin
(e + f*x)), x)
```

3.134.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c), x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\left(B\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-A\operatorname{sgn}\left(\cos\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-2*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(a)*sqrt(c)/f
```

3.134.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(B\,\cos(e+f\,x)+B\,\cos(3\,e+3\,f\,x)-4\,A\,\sin(2\,e+}{4\,f\,(\cos(2\,e+2\,f\,x)+1)}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*
x) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) +
1))
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.135.1 Optimal result 1229
3.135.2 Mathematica [C] (verified) 1229
3.135.3 Rubi [A] (verified) 1230
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3.135.7 Maxima [A] (verification not implemented) 1234
3.135.8 Giac [A] (verification not implemented) 1234
3.135.9 Mupad [F(-1)] 1235

3.135.1 Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

output

```
-a*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2
)
```

3.135.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left((A+B)\left(-ifx+2\log\left(i-e^{i(e+fx)}\right)\right)+B\,\mathrm{s}\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
-(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A +
B)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

3.135.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3450$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\downarrow 3216$$

$$\frac{ac(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\downarrow 3042$$

$$\frac{ac(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\downarrow 3146$$

$$-\frac{a(A+B)\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\int\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\downarrow 16$$

$$-\frac{B\int\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}-\frac{a(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3217$$

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
-((a*(A + B)*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x
]])/(c*f*Sqrt[a + a*Sin[e + f*x]])
```

3.135.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(92) = 184$.

Time = 2.53 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.00

method	result
parts	$\frac{A\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sqrt{a(1+\sin(fx+e))}\,(-\cos(fx+e)+\sin(fx+e)-1)}{f(\cos(fx+e)+\sin(fx+e)+1)\sqrt{-c(\sin(fx+e)-1)}} - \frac{B\left(2\ln(\csc(fx+e)-\cot$
default	$-\frac{\left(2A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-2A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin\right.}{}$

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
A/f*(2*ln(csc(f*x+e)-cot(f*x+e)-1)-ln(2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e))
)^(1/2)*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+
e)-1))^(1/2)-B/f*(2*ln(csc(f*x+e)-cot(f*x+e)-1)*cos(f*x+e)-2*ln(csc(f*x+e)
-cot(f*x+e)-1)*sin(f*x+e)-ln(2/(1+cos(f*x+e)))*cos(f*x+e)+ln(2/(1+cos(f*x+
e)))*sin(f*x+e)+cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)+2*ln(csc(f*x+e)-cot(f*x
+e)-1)-ln(2/(1+cos(f*x+e)))+sin(f*x+e)-1)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*
x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.135.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c*sin(f*x + e) - c), x)
```

3.135.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e +
f*x) - 1)), x)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}} + \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(c+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) + A\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
(B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*si
n(f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)
)) + A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(
a)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{2\sqrt{2}B\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{c}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\sqrt{2}\left(A\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log}{c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/2*sqrt(2)*(2*sqrt(2)*B*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))/(sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2
)*(A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sqrt(c)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)))*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(c*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

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3.136.2 Mathematica [C] (verified) 1236
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3.136.8 Giac [A] (verification not implemented) 1241
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3.136.1 Optimal result

Integrand size = 40, antiderivative size = 99

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+a*B*cos
(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(}{f\left(\cos\left(\frac{1}{2}(e+fx\right)\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B -
 I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e
 + f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*
Sin[e + f*x])^(3/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3450$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\downarrow 3216$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx - \frac{aB\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx - \frac{aB\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx + \frac{aB\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx + \frac{aB\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3217$$

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + (a*B*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

3.136.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.136.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

method	result
default	$\frac{\sec(fx+e)\left(-2B\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+B\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx+e)+B\sin(fx+e)+2B\ln(\csc(f}{cf\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\tan(fx+e)\sqrt{a(1+\sin(fx+e))}}{fc\sqrt{-c(\sin(fx+e)-1)}} - \frac{B\sec(fx+e)\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-\sin(fx+}{fc\sqrt{-c(\sin(fx+e)-1}}$

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/c/f*sec(f*x+e)*(-2*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+B*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)+B*sin(f*x+e)+2*B*ln(csc(f*x+e)-cot(f*x+
e)-1)-B*ln(2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))
^(1/2)
```

3.136.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

3.136.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
 - 1))**(3/2), x)
```

3.136.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(3/2), x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left(4\,B\sqrt{c}\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{A\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+B\sqrt{c}\operatorname{sgn}(c}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{2\,c^2 f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-1/2*(4*B*sqrt(c)*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + (A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*
sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2)*sqrt(a)/(c^2*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

3.137.1 Optimal result 1242
3.137.2 Mathematica [A] (verified) 1242
3.137.3 Rubi [A] (verified) 1243
3.137.4 Maple [A] (verified) 1244
3.137.5 Fricas [A] (verification not implemented) 1245
3.137.6 Sympy [F] 1245
3.137.7 Maxima [F] 1245
3.137.8 Giac [A] (verification not implemented) 1246
3.137.9 Mupad [F(-1)] 1246

3.137.1 Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}$$

output
```
1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-a*B
*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{a(1+\sin(e+fx))}(A-B+2B\sin(e+fx))\sqrt{c-}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

3.137.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3450$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\downarrow 3217$$

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(3/2))
```

3.137.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.137.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\tan(fx+e)(A\sin(fx+e)-B\sin(fx+e)-2A)\sqrt{a(1+\sin(fx+e))}}{2c^2 f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}}$	71
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)+2\tan(fx+e)-\sec(fx+e))}{2f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}+\frac{B\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)-\sec(fx+e))}{2f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}$	128

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/c^2/f*tan(f*x+e)*(A*sin(f*x+e)-B*sin(f*x+e)-2*A)*(a*(1+sin(f*x+e)))^(1
/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(2\,B\sin(fx+e)+A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2\left(c^3 f\cos(fx+e)^3+2\,c^3 f\cos(fx+e)\sin(fx+e)-2\,c^3 f\cos(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3
*f*cos(f*x + e))
```

3.137.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\left(\sin(e+fx)+1\right)}(A+B\sin(e+fx))}{\left(-c\left(\sin(e+fx)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
 - 1))**(5/2), x)
```

3.137.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(5/2), x)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(4\,B\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\right.\right.}{8\,c^3 f\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/8*(4*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^3*f*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

3.138.1 Optimal result 1247
3.138.2 Mathematica [A] (verified) 1247
3.138.3 Rubi [A] (verified) 1248
3.138.4 Maple [A] (verified) 1249
3.138.5 Fricas [A] (verification not implemented) 1250
3.138.6 Sympy [F(-1)] 1250
3.138.7 Maxima [F] 1250
3.138.8 Giac [A] (verification not implemented) 1251
3.138.9 Mupad [B] (verification not implemented) 1251

3.138.1 Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}$$

output
```
1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)-1/2
*a*B*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{a(1+\sin(e+fx))}(2A-B+3B\sin(e+fx))\sqrt{c}}{6c^4 f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
(Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

3.138.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3450$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx - \frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\downarrow 3217$$

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(5/2))
```

3.138.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.138.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

method	result
default	$\frac{\tan(fx+e)\left(2A\left(\cos^2(fx+e)\right)+B\left(\sin^2(fx+e)\right)+6A\sin(fx+e)-3B\sin(fx+e)-8A\right)\sqrt{a(1+\sin(fx+e))}}{6c^3 f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-4\tan(fx+e)+3\sec(fx+e))}{3f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}\,c^3} - \frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+$

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/6/c^3/f*tan(f*x+e)*(2*A*cos(f*x+e)^2+B*sin(f*x+e)^2+6*A*sin(f*x+e)-3*B*s
in(f*x+e)-8*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*
(sin(f*x+e)-1))^(1/2)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{(3\,B\sin(fx+e)+2\,A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{6\left(3\,c^4 f\cos(fx+e)^3-4\,c^4 f\cos(fx+e)-\left(c^4 f\cos(fx+e)^3-4\,c^4 f\cos(fx+e)\right)\sin(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x
 + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(7/2), x)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(3\,B\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{24\,c^4 f\,\mathrm{sgn}\left(\sin\left(-\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
1/24*(3*B*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^2 - A*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(c)*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^4*f*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{2\,A\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}-B\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}+3\,B\sin(e}{\frac{9\,c^4 f\cos(3\,e+3\,fx)}{2}+\frac{21\,c^4 f\sin(2\,e+2\,fx)}{2}-\frac{3\,c^4 f\sin(4\,e+4\,fx)}{4}-\frac{21\,c^4 f\,\mathrm{co}}{2}}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
-(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin
(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2
+ (21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f
*cos(e + f*x))/2)
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

3.139.1 Optimal result 1252
3.139.2 Mathematica [A] (verified) 1253
3.139.3 Rubi [A] (verified) 1253
3.139.4 Maple [A] (verified) 1255
3.139.5 Fricas [A] (verification not implemented) 1256
3.139.6 Sympy [F(-1)] 1256
3.139.7 Maxima [F] 1257
3.139.8 Giac [A] (verification not implemented) 1257
3.139.9 Mupad [B] (verification not implemented) 1258

3.139.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

output

```
-1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/30*a^2
*(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/15*a
*(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.139.2 Mathematica [A] (verified)

Time = 4.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.40

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(15(16A-11B)\cos(2(e+fx))+30(2A$$

$$960f\ (c$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*
Sin[e + f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e +
 f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] +
60*A*Sin[3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24
*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)
```

3.139.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3452$$

$$\frac{1}{3}(3A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A-B)\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$$\downarrow 3219$$

$$\frac{1}{3}(3A-B)\left(\frac{2}{5}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A-B)\left(\frac{2}{5}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$$\downarrow 3217$$

$$\frac{1}{3}(3A-B)\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

input

```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output

```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + ((3*A - B)*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*S
qrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(7/2))/(5*f)))/3
```

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.139.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a\,c^3\tan(fx+e)\left(5B\left(\sin^5(fx+e)\right)+6A\left(\cos^4(fx+e)\right)-12\left(\sin^4(fx+e)\right)B+15A\sin(fx+e)\left(\cos^2(fx+e)\right)-12A\left(\cos^2(fx+e)\right)+20B(s\right.}{30f}$
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c^3a\left(2\left(\cos^3(fx+e)\right)\sin(fx+e)-5\left(\cos^3(fx+e)\right)-4\cos(fx+e)\sin(fx+e)-8\tan(fx+e)+\right.}{10f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/30*a*c^3/f*tan(f*x+e)*(5*B*sin(f*x+e)^5+6*A*cos(f*x+e)^4-12*sin(f*x+e)^
4*B+15*A*sin(f*x+e)*cos(f*x+e)^2-12*A*cos(f*x+e)^2+20*B*sin(f*x+e)^2+15*A*
sin(f*x+e)-15*B*sin(f*x+e)-24*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e
)))^(1/2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int (a + a\sin(e+fx))^{3/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{(5\,Bac^3\cos(fx+e)^6 + 15\,(A-B)ac^3\cos(fx+e)^4 - 5\,(3\,A-2\,B)ac^3 - 2\,(3\,(A-$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A
- 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos
(f*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^{3/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{7/2}\,dx = \frac{8\left(20\,Bac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
8/15*(20*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 12*A*a*c^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^10 - 36*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 15*A*a*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 15*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^8)*sqrt(a)*sqrt(c)/f
```

3.139.9 Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{e^{-e\,6i-f\,x\,6i}\sqrt{c-c\sin(e+f\,x)}\left(\frac{B\,a\,c^3\,e^{e\,6i+f\,x\,6i}\cos(6\,e+6\,f\,x)\sqrt{a+a\sin(e+f\,x)}}{96\,f} - \frac{a\,c^3\,e^{e\,6i+f\,x\,6i}}{}\right.$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*
6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i
+ f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f)
+ (a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x)
)^(1/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)
*(a + a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e +
3*f*x)*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*
6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)
/(40*f)))/(2*cos(e + f*x))
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.140.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{20f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

output

```
-1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/30*a^2
*(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/20*a
*(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.140.2 Mathematica [A] (verified)

Time = 4.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$

$$-c\sin(e+fx))^{5/2}\,dx = \frac{c^2(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(4(100A-11B)\sin}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*
B*Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f
*x])))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3)
```

3.140.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3452$$

$$\frac{1}{5}(5A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A-B)\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$$\downarrow 3219$$

$$\frac{1}{5}(5A-B)\left(\frac{1}{2}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A-B)\left(\frac{1}{2}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$$\downarrow 3217$$

$$\frac{1}{5}(5A-B)\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

input

```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output

```
-1/5*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + ((5*A - B)*(-1/6*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sq
rt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*
Sin[e + f*x])^(5/2))/(4*f)))/5
```

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.140.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a\,c^2\tan(fx+e)\left(12B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+15A\sin(fx+e)\left(\cos^2(fx+e)\right)+15B\left(\sin^3(fx+e)\right)-20A\left(\cos^2(fx+e)\right)+8B(\sin^2($
parts	$\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c^2a\left(3\left(\cos^3(fx+e)\right)+4\cos(fx+e)\sin(fx+e)+8\tan(fx+e)-3\sec(fx+e)\right)}{12f}+\frac{B\sqrt{-c(\sin($

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/60*a*c^2/f*tan(f*x+e)*(12*B*sin(f*x+e)^2*cos(f*x+e)^2+15*A*sin(f*x+e)*c
os(f*x+e)^2+15*B*sin(f*x+e)^3-20*A*cos(f*x+e)^2+8*B*sin(f*x+e)^2+15*A*sin(
f*x+e)-30*B*sin(f*x+e)-40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^
(1/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{(15\,(A-B)ac^2\cos(fx+e)^4 - 15\,(A-B)ac^2 + 4\,(3\,Bac^2\cos(fx+e)^4 + (5\,A-B}{60\,f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*co
s(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.140.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{5/2}\,dx = \frac{4\left(24\,Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
4/15*(24*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a*c^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^8 - 45*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 20*A*a*c^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 20*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^6)*sqrt(a)*sqrt(c)/f
```

3.140.9 Mupad [B] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{5/2}\,dx = \frac{a\,c^2\,\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(60\,A\,\cos(e+f\,x)-60\,B\,\cos(}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*co
s(e + f*x) - 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*
f*x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 400*A*sin(2*e + 2*f
*x) + 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x
) + 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.141.1 Optimal result

Integrand size = 40, antiderivative size = 134

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

output

```
-1/4*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/f-1/3*a^2*
A*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*A*cos(f
*x+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.141.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(9B+1}{96f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c
- c*Sin[e + f*x]]*(9*B + 12*B*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)] - 72
*A*Sin[e + f*x] - 8*A*Sin[3*(e + f*x)]))/(96*f)
```

3.141.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3452$$

$$A\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$$\downarrow 3042$$

$$A\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$$\downarrow 3219$$

$$A\left(\frac{2}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$$\downarrow 3042$$

$$A\left(\frac{2}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$$\downarrow 3217$$

$$A\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + A*(-1/3*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*
Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))/(3*f))
```

3.141.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.141.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

method	result
default	$\frac{ac\tan(fx+e)\left(3B\left(\cos^2(fx+e)\right)\sin(fx+e)+4A\left(\cos^2(fx+e)\right)+3B\sin(fx+e)+8A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{12f}$
parts	$\frac{A\tan(fx+e)ca\left(\cos^2(fx+e)+2\right)\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}{3f}+\frac{B\sin(fx+e)\tan(fx+e)ca\left(\cos^2(fx+e)+1\right)\sqrt{-c(\sin(fx$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/12*a*c/f*tan(f*x+e)*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*si
n(f*x+e)+8*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{\left(3\,Bac\cos\left(fx+e\right)^4-3\,Bac-4\left(Aac\cos\left(fx+e\right)^2+2\,Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin(}}{12\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*
c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos
(f*x + e))
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(116) = 232$.

Time $= 0.50$ (sec) , antiderivative size $= 237$, normalized size of antiderivative $= 1.77$

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = \frac{4\left(3\,Bac\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
4/3*(3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*a*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^6 - 6*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 3*A*a*c*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^4 + 3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(
c)/f
```

3.141.9 Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{a\,c\,\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(12\,B\,\cos(e+f\,x)+15\,B\,\cos(3\,e+3\,f\,x)+3\,B\,\cos($$
$$96\,f\,(\cos(2\,e+2\,f\,x)+1)$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos
(e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e + 
2*f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

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3.142.8 Giac [A] (verification not implemented) 1277
3.142.9 Mupad [B] (verification not implemented) 1278

3.142.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{3/2}(A + B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-2(6A+B)\sin(e+fx)+\cos(2(e+fx))(3(A+$$
$$12f$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
-1/12*(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*
(-2*(6*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x
]))/f
```

3.142.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3217$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

3.142.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.142.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

method	result
default	$\frac{a\tan(fx+e)\left(2B\left(\sin^2(fx+e)\right)+3A\sin(fx+e)+3B\sin(fx+e)+6A\right)\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}}{6f}$
parts	$-\frac{Aa\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-2\tan(fx+e)-\sec(fx+e))}{2f}$ $-\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx+e)$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/6*a/f*tan(f*x+e)*(2*B*sin(f*x+e)^2+3*A*sin(f*x+e)+3*B*sin(f*x+e)+6*A)*(-
c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\left(3\,(A+B)a\cos(fx+e)^2-3\,(A+B)a+2\left(Ba\cos(fx+e)^2-(3\,A+B)a\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e}}{6\,f\cos(fx+e)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (
3*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e))
```

3.142.6 Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A$$
$$+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-c\,(\sin(e+fx)-1)}(A$$
$$+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*
sin(e + f*x)), x)
```

3.142.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A$$
$$+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}\,d$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=$$
$$-\frac{2\left(4\,Ba\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Aa\cos\left(\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-2/3*(4*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*A*a*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 3*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

3.142.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{a\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(3\,A\,\cos(e+f\,x)+3\,B\,\cos(e+f\,x)+3\,A\,\cos(3\,e+3}{12\,f\,(\cos(2\,e+2\,}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e
+ f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) -
12*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*
(cos(2*e + 2*f*x) + 1))
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.143.1 Optimal result . . . 1279
3.143.2 Mathematica [A] (verified) . . . 1279
3.143.3 Rubi [A] (verified) . . . 1280
3.143.4 Maple [B] (verified) . . . 1283
3.143.5 Fricas [F] . . . 1283
3.143.6 Sympy [F] . . . 1284
3.143.7 Maxima [F] . . . 1284
3.143.8 Giac [A] (verification not implemented) . . . 1284
3.143.9 Mupad [F(-1)] . . . 1285

3.143.1 Optimal result

Integrand size = 40, antiderivative size = 145

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-2*a^2*(A
+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)-a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(-B\cos(2(e+fx))+16(A+B)\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(
-(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*
Sqrt[c - c*Sin[e + f*x]])
```

3.143.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3452, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3452$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3216$$

$$(A+B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$(A+B)\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$(A+B)\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/2*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))
```

3.143.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.143.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(131) = 262$.

Time = 2.66 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.92

method	result
default	$-\frac{a\left(-B(\cos^3(fx+e))+B(\cos^2(fx+e))\sin(fx+e)+2A(\cos^2(fx+e))+2A\sin(fx+e)\cos(fx+e)-4A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+$
parts	$-\frac{A\left(\cos(fx+e)\sin(fx+e)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sin(fx+e)-4\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)+\cos^2(fx+e)-2\ln\left(\frac{2}{1+\cos(fx}\right.$ $f(\cos(fx+e)+$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2*a/f*(-B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2+2*A*s
in(f*x+e)*cos(f*x+e)-4*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*A*cos(f*x+e)*ln
(csc(f*x+e)-cot(f*x+e)-1)+4*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-8*A*sin(f*x+
e)*ln(csc(f*x+e)-cot(f*x+e)-1)+3*B*cos(f*x+e)^2+4*B*cos(f*x+e)*sin(f*x+e)-
4*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+8*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e
)-1)+4*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-8*B*sin(f*x+e)*ln(csc(f*x+e)-cot(
f*x+e)-1)+2*A*sin(f*x+e)-4*A*ln(2/(1+cos(f*x+e)))+8*A*ln(csc(f*x+e)-cot(f*
x+e)-1)+cos(f*x+e)*B+3*B*sin(f*x+e)-4*B*ln(2/(1+cos(f*x+e)))+8*B*ln(csc(f*
x+e)-cot(f*x+e)-1)-2*A-3*B)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+sin(f*x+e
)+1)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.143.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

3.143.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e
 + f*x) - 1)), x)
```

3.143.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Aa\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Ba\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}f}{c\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
sqrt(2)*sqrt(a)*(sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1/4*pi + 1/2*
f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (sqrt(2)*B*a
*c^(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a*c^(3/2)*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) + sqrt(2)*B*a*c^(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c
^2)/f
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

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3.144.1 Optimal result

Integrand size = 40, antiderivative size = 158

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(
A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*
x+e))^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 8.56 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(4A+3B+B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{2cf\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

input

```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
4*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*
Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

3.144.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((A + 3*B)*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/(2*c)
```

3.144.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.144.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.50

method	result
default	$-\frac{a\sec(fx+e)\left(2A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+6B\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+$
parts	$-\frac{A\sec(fx+e)\left(2\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-2\ln(\csc(fx+e)-\cot(fx+e)-1)+\ln\left(\frac{1+\cos}{fc\sqrt{-c(\sin(fx+e)-1)}}\right.\right.$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-a/c/f*sec(f*x+e)*(2*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-A*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))+6*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-3*B*sin(f
*x+e)*ln(2/(1+cos(f*x+e)))+B*sin(f*x+e)^2-2*A*ln(csc(f*x+e)-cot(f*x+e)-1)+
A*ln(2/(1+cos(f*x+e)))-2*A*sin(f*x+e)-6*B*ln(csc(f*x+e)-cot(f*x+e)-1)+3*B*
ln(2/(1+cos(f*x+e)))-3*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x
+e)-1))^(1/2)
```

3.144.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*si
n(f*x + e) - 2*c^2), x)
```

3.144.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(-c\,(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f
*x) - 1))**(3/2), x)
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(142) = 284.

Time = 0.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.32

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{B\left(\frac{6\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{2\left(\frac{3\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(
f*x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(
3/2) + 4*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))
/f
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}Ba\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{c^{\frac{3}{2}}\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}+\frac{\sqrt{2}(Aa\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+3\,Ba\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}\right)\sqrt{a}}{2f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))/(c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqr
t(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 +
 8)/(c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*a*sqrt(c)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))))*sqrt(a)/f
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

3.145.1 Optimal result . . . 1293
3.145.2 Mathematica [A] (verified) . . . 1293
3.145.3 Rubi [A] (verified) . . . 1294
3.145.4 Maple [A] (verified) . . . 1297
3.145.5 Fricas [F] . . . 1297
3.145.6 Sympy [F] . . . 1298
3.145.7 Maxima [F] . . . 1298
3.145.8 Giac [A] (verification not implemented) . . . 1298
3.145.9 Mupad [F(-1)] . . . 1299

3.145.1 Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+
e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 9.00 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f$$

input

```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos
[2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[
(e + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

3.145.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - (B*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e +
 f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c
```

3.145.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.145.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.45

method	result
default	$-\frac{a\sec(fx+e)\left(-B(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+2B(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+2B(\sin^2(fx+e))-2B\sin(fx+$
parts	$-\frac{A\tan(fx+e)\sqrt{a(1+\sin(fx+e))}\,a}{f(\sin(fx+e)-1)\sqrt{-c(\sin(fx+e)-1)}\,c^2}+\frac{B\sec(fx+e)\left((\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-a/c^2/f*sec(f*x+e)*(-B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)^2
*ln(csc(f*x+e)-cot(f*x+e)-1)+2*B*sin(f*x+e)^2-2*B*sin(f*x+e)*ln(2/(1+cos(f
*x+e)))+4*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+A*sin(f*x+e)-B*sin(f*x+
e)+2*B*ln(2/(1+cos(f*x+e)))-4*B*ln(csc(f*x+e)-cot(f*x+e)-1))*(a*(1+sin(f*x
+e)))^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.145.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

3.145.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f
*x) - 1))**(5/2), x)
```

3.145.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(5/2), x)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{4\sqrt{2}Ba\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx-\right.\right.\right.}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/8*sqrt(2)*sqrt(a)*(4*sqrt(2)*B*a*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))) + sqrt(2)*(A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 5*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*a*sqrt(c)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

3.146.1 Optimal result 1300
3.146.2 Mathematica [A] (verified) 1300
3.146.3 Rubi [A] (verified) 1301
3.146.4 Maple [A] (verified) 1302
3.146.5 Fricas [A] (verification not implemented) 1303
3.146.6 Sympy [F(-1)] 1303
3.146.7 Maxima [F] 1303
3.146.8 Giac [B] (verification not implemented) 1304
3.146.9 Mupad [F(-1)] 1304

3.146.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

output
```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*
(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

3.146.2 Mathematica [A] (verified)

Time = 9.78 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(A+4B-3B\cos(2(e+fx))+3(A-B)\sin(}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
-1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
A + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]
])
```

3.146.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

$$\downarrow 3042$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

$$\downarrow 3221$$

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c
- c*Sin[e + f*x])^(5/2))
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.146.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

method	result
default	$\frac{a\tan(fx+e)\left(A\left(\cos^2(fx+e)\right)-B\left(\sin^2(fx+e)\right)+3A\sin(fx+e)-3B\sin(fx+e)-7A\right)\sqrt{a(1+\sin(fx+e))}}{6c^3 f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a(\cos(fx+e)\sin(fx+e)-3\cos(fx+e)-7\tan(fx+e)+3\sec(fx+e))}{6f\left(\cos^2(fx+e)+2\sin(fx+e)-2\right)\sqrt{-c(\sin(fx+e)-1)}\,c^3}+\frac{B\sec(fx+e)(\cos(fx+e)-1)(1+\cos(fx-$ $6f(\cos^2(fx+e)+2\sin(fx+e$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/6*a/c^3/f*tan(f*x+e)*(A*cos(f*x+e)^2-B*sin(f*x+e)^2+3*A*sin(f*x+e)-3*B*s
in(f*x+e)-7*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*
(sin(f*x+e)-1))^(1/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(6\,Ba\cos(fx+e)^2 - 3\,(A-B)a\sin(fx+e) - (A+7$$
$$6\left(3\,c^4 f\cos(fx+e)^3 - 4\,c^4 f\cos(fx+e) - \left(c^4 f\cos($$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c
^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x
+ e))
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(7/2), x)
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(84) = 168$.

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{\left(12\,Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-3\,Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-1/24*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 - 3*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - 9*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 2*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 2*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt
(a)/(c^4*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^6)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

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3.147.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}$$

output

```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*
(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*
(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)
```

3.147.2 Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input

```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output

```
(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A +
 3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

3.147.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c -
 c*Sin[e + f*x])^(7/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*
f*(c - c*Sin[e + f*x])^(5/2))))/(4*c)
```

3.147.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.147.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.87

method	result
default	$\frac{a\tan(fx+e)\left(A\sin(fx+e)\left(\cos^2(fx+e)\right)-4A\left(\cos^2(fx+e)\right)-7A\sin(fx+e)+3B\sin(fx+e)+10A\right)\sqrt{a(1+\sin(fx+e))}}{6c^4f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-3\left(\cos^2(fx+e)\right)-4\sin(fx+e)+4\right)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a\left(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)-10\tan(fx+e)+7\sec(fx+e)\right)}{6f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-3\left(\cos^2(fx+e)\right)-4\sin(fx+e)+4\right)\sqrt{-c(\sin(fx+e)-1)}\,c^4}-\frac{B}{2f\left(\left(\cos^2(fx+e)\right)\sin$

input

```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/6*a/c^4/f*tan(f*x+e)*(A*sin(f*x+e)*cos(f*x+e)^2-4*A*cos(f*x+e)^2-7*A*sin
(f*x+e)+3*B*sin(f*x+e)+10*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*
x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$

$$-\frac{\left(3\,Ba\cos(fx+e)^2-2\,Aa\sin(fx+e)-(A+3\,B)a\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+}}{6\left(c^5f\cos(fx+e)^5-8\,c^5f\cos(fx+e)^3+8\,c^5f\cos(fx+e)+4\left(c^5f\cos(fx+e)^3-2\,c^5f\cos(fx+e)\right)\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*co
s(f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*co
s(f*x + e))*sin(f*x + e))
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

3.147.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{\left(12\,Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-4\,Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
-1/96*(12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^4 - 4*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 - 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 3*A*a*sqrt(c)*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 3*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqr
t(a)/(c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^8)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{c-c\,\sin(e+f\,x)}\left(\frac{8\,a\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,(2\,A+3\,B)\,\sqrt{a+a\,\sin(}}{3\,c^5\,f}\right.}{84\,\cos(e+f\,x)\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(3\,e+3\,f\,x)}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((8*a*exp(e*5i + f*x*5i)*(2*A + 3*B)*(a + a*si
n(e + f*x))^(1/2))/(3*c^5*f) + (32*A*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a
+ a*sin(e + f*x))^(1/2))/(3*c^5*f) - (8*B*a*exp(e*5i + f*x*5i)*cos(2*e + 2
*f*x)*(a + a*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*
x*5i) - 54*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(
5*e + 5*f*x) - 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*
5i)*sin(4*e + 4*f*x))
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

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3.148.1 Optimal result

Integrand size = 40, antiderivative size = 154

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}$$

output

```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)-1/1
20*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^
(1/2)+1/40*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+
e))^(9/2)
```

3.148.2 Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(9(A+B)-10B\cos(2(e+fx))+5(3A+B)}{60c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
-1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e
+ f*x]])
```

3.148.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3218, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\downarrow 3451$$

$$\frac{(3A-7B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-7B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3218$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3217$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((3*A - 7*B)*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*
(c - c*Sin[e + f*x])^(9/2)) - (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(7/2))))/(10*c)
```

3.148.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.148.4 Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.22

method	result
default	$\frac{a\tan(fx+e)\left(9A\left(\cos^4(fx+e)\right)+B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+45A\sin(fx+e)\left(\cos^2(fx+e)\right)+5B\left(\sin^3(fx+e)\right)-108A\left(\cos^2(fx+e)\right)\right.}{60c^5 f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a\left(3\left(\cos^3(fx+e)\right)\sin(fx+e)-15\left(\cos^3(fx+e)\right)-36\cos(fx+e)\sin(fx+e)+60\cos(fx+e)+53\tan(fx+e)-45\sec(\right.}{20f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

input

```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output

```
1/60*a/c^5/f*tan(f*x+e)*(9*A*cos(f*x+e)^4+B*sin(f*x+e)^2*cos(f*x+e)^2+45*A
*sin(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-108*A*cos(f*x+e)^2-11*B*sin(f*x+
e)^2-135*A*sin(f*x+e)+30*B*sin(f*x+e)+159*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos
(f*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(si
n(f*x+e)-1))^(1/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$

$$-\frac{\left(20\,Ba\cos(fx+e)^2-5\,(3\,A+B)a\sin(fx+e)-(9\,A+19\,B)a\right)\sqrt{a\sin(fx+e)+a}}{60\left(5\,c^6 f\cos(fx+e)^5-20\,c^6 f\cos(fx+e)^3+16\,c^6 f\cos(fx+e)-\left(c^6 f\cos(fx+e)^5-12\,c^6 f\cos(fx+\right.\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
-1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a
)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)
^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)
^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$

$$-\frac{\left(40\,Ba\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Aa\sqrt{c}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
-1/960*(40*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^4 - 15*A*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin
(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 45*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 12*A*a*sqrt(c)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 12*B*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
*sqrt(a)/(c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^10)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 21.01 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,(}{}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,2}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(A + B)*(a + a*sin(e +
f*x))^(1/2)*48i)/(5*c^6*f) - (B*a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a +
 a*sin(e + f*x))^(1/2)*32i)/(3*c^6*f) + (16*a*exp(e*6i + f*x*6i)*sin(e + f
*x)*(A*3i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f)))/(cos(e + f*x)*ex
p(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6
i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330
i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e +
 6*f*x)*2i)
```

3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

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3.149.9 Mupad [B] (verification not implemented) 1323

3.149.1 Optimal result

Integrand size = 40, antiderivative size = 198

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{105f}$$
$$-\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

output

```
-1/42*a*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f
-1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/105*a^
3*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/105
*a^2*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.149.2 Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}(525(A-B)\cos(2(e+fx))+210(A-$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c
*Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)
] + 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] -
525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A
*Sin[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5)
```

3.149.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$\frac{1}{7}(7A-B)\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A-B)\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$$\downarrow 3219$$

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7}}{6f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7}}{6f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right.$$

$$\downarrow 3219$$

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))}{5f}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))}{5f}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right.$$

$$\downarrow 3217$$

$$\frac{1}{7}(7A - B)\left(\frac{2}{3}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right) - \frac{a\c$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/7*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + ((7*A - B)*(-1/6*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(7/2))/f + (2*a*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(7/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5*f)))/3))/7
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.149.4 Maple [A] (verified)

Time = 73.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a^2c^3\tan(fx+e)\left(30B\left(\cos^4(fx+e)\right)\left(\sin^2(fx+e)\right)+35A\left(\cos^4(fx+e)\right)\sin(fx+e)+35B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)-42A\left(\cos^4(fx+$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^2c^3\left(5\left(\cos^5(fx+e)\right)+6\left(\cos^3(fx+e)\right)\sin(fx+e)+8\cos(fx+e)\sin(fx+e)+16\tan(fx+e)-\right.}{30f}$

input

```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output

```
-1/210*a^2*c^3/f*tan(f*x+e)*(30*B*cos(f*x+e)^4*sin(f*x+e)^2+35*A*cos(f*x+e
)^4*sin(f*x+e)+35*B*cos(f*x+e)^2*sin(f*x+e)^3-42*A*cos(f*x+e)^4+24*B*sin(f
*x+e)^2*cos(f*x+e)^2+35*A*sin(f*x+e)*cos(f*x+e)^2+70*B*sin(f*x+e)^3-56*A*c
os(f*x+e)^2+16*B*sin(f*x+e)^2+35*A*sin(f*x+e)-105*B*sin(f*x+e)-112*A)*(-c*
(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c$$

$$-c\sin(e+fx))^{7/2}\,dx = \frac{\left(35\,(A-B)a^2c^3\cos(fx+e)^6-35\,(A-B)a^2c^3+2\left(15\,Ba^2c^3\cos(fx+e)^6+3\,(7\,A\right.\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^
2*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^
2*c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.149.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{5}{2}}(-c\sin(fx + e) + c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(174) = 348$.

Time $= 0.49$ (sec) , antiderivative size $= 355$, normalized size of antiderivative $= 1.79$

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{16\left(120\,Ba^2c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{14} - 70\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-16/105*(120*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^2*c^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^12 - 350*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^1
2 + 168*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 336*B*a^2*c^3*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^10 - 105*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 1
05*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

3.149.9 Mupad [B] (verification not implemented)

Time $= 18.72$ (sec) , antiderivative size $= 383$, normalized size of antiderivative $= 1.93$

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c$$

$$-c\sin(e+fx))^{7/2}\,dx = \frac{\mathrm{e}^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\sqrt{c - c\sin(e+f\,x)}\left(-\frac{a^2\,c^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\sin(e+f\,x)}\,5\mathrm{i}}{32\,f} - \right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output

```
(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*
7i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^
2*c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x
))^(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i -
 B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)
*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5
*a^2*c^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1
/2))/(32*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a +
 a*sin(e + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7
*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.150.1 Optimal result

Integrand size = 40, antiderivative size = 180

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f}$$
$$-\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

output

```
-1/5*a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/6*B*
cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*A*cos(
f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*A*cos(f*x+e
)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.150.2 Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(50B+75B\cos(2(e+fx))+30B\cos(4(e+fx)}{960f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
-1/960*(a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e +
 f*x]]*(50*B + 75*B*Cos[2*(e + f*x)] + 30*B*Cos[4*(e + f*x)] + 5*B*Cos[6*(
e + f*x)] - 600*A*Sin[e + f*x] - 100*A*Sin[3*(e + f*x)] - 12*A*Sin[5*(e +
f*x)]))/f
```

3.150.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$A\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{5/2}dx -$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$
$$\downarrow 3042$$

$$A\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\downarrow 3219$$

$$A\left(\frac{4}{5}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/}}{5f}\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\right)$$

$$\downarrow 3042$$

$$A\left(\frac{4}{5}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/}}{5f}\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\right)$$

$$\downarrow 3219$$

$$A\left(\frac{4}{5}a\left(\frac{1}{2}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/}}{4f}\right)\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\right)$$

$$\downarrow 3042$$

$$A\left(\frac{4}{5}a\left(\frac{1}{2}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/}}{4f}\right)\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\right)$$

$$\downarrow 3217$$

$$A\left(\frac{4}{5}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{a\cos}{}\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

3.150. $\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

output
```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + A*(-1/5*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(5/2))/f + (4*a*(-1/6*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f
*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(5/2))/(4*f)))/5)
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.150.4 Maple [A] (verified)

Time = 72.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

method	result
default	$\frac{a^2c^2\tan(fx+e)(5B(\cos^4(fx+e))\sin(fx+e)+6A(\cos^4(fx+e))+5B(\cos^2(fx+e))\sin(fx+e)+8A(\cos^2(fx+e))+5B\sin(fx+e)+16A}{30f}$
parts	$\frac{A\tan(fx+e)a^2c^2(3(\cos^4(fx+e))+4(\cos^2(fx+e))+8)\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}{15f} - \frac{B\sec(fx+e)a^2c^2(\cos^4(fx+e)+c}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/30*a^2*c^2/f*tan(f*x+e)*(5*B*cos(f*x+e)^4*sin(f*x+e)+6*A*cos(f*x+e)^4+5*
B*cos(f*x+e)^2*sin(f*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(sin(f
*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{\left(5\,Ba^2c^2\cos(fx+e)^6-5\,Ba^2c^2-2\left(3\,Aa^2c^2\cos(fx+e)^4+4\,Aa^2c^2\cos(fx+e)^2+8\,Aa^2c^2\right)\sin(fx+\right.}{30\,f\cos(fx+e)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
 e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.150.7 Maxima [F]

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(156) = 312$.

Time $= 0.44$ (sec) , antiderivative size $= 355$, normalized size of antiderivative $= 1.97$

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{16\left(10\,Ba^2c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12} - 6\,Ac\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
-16/15*(10*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 6*A*a^2*c^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^10 - 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 +
15*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 30*B*a^2*c^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^8 - 10*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 10*B*a^2*c
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

3.150.9 Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^2c^2\sqrt{a\,(\sin(e+fx)+1)}\sqrt{-c\,(\sin(e+fx)-1)}\,(75B\cos(e+fx)+105B\cos(3e+3fx)+35B}{960f\,(\cos}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
-(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B
*cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7
*e + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6
*e + 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))
```

3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

3.151.1 Optimal result 1332
3.151.2 Mathematica [A] (verified) 1333
3.151.3 Rubi [A] (verified) 1333
3.151.4 Maple [A] (verified) 1335
3.151.5 Fricas [A] (verification not implemented) 1336
3.151.6 Sympy [F(-1)] 1336
3.151.7 Maxima [F] 1337
3.151.8 Giac [B] (verification not implemented) 1337
3.151.9 Mupad [B] (verification not implemented) 1338

3.151.1 Optimal result

Integrand size = 40, antiderivative size = 142

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

output

```
-1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(5*
A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/20*(
5*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

3.151.2 Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.16

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}(4(100A+11B)\sin(e+fx)+4\cos(2(e+$$

$$480f\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)^3(c$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
-1/480*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[
e + f*x]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B)
 + 4*(5*A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e
 + f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5)
```

3.151.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3452$$

$$\frac{1}{5}(5A+B)\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{3/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A+B)\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$$\downarrow 3219$$

$$\frac{1}{5}(5A+B)\left(\frac{1}{2}c\int(\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A+B)\left(\frac{1}{2}c\int(\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$$\downarrow 3217$$

$$\frac{1}{5}(5A+B)\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

input

```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output

```
-1/5*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + ((5*A + B)*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[
c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c -
 c*Sin[e + f*x]])/(4*f)))/5
```

3.151.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.151.4 Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

method	result
default	$\frac{a^2 c \tan(fx+e)(12B(\sin^2(fx+e))(\cos^2(fx+e))+15A\sin(fx+e)(\cos^2(fx+e))-15B(\sin^3(fx+e))+20A(\cos^2(fx+e))+8B(\sin^2(fx+e)}{60f}$
parts	$\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c\,a^2(-3(\cos^3(fx+e))+4\cos(fx+e)\sin(fx+e)+8\tan(fx+e)+3\sec(fx+e))}{12f} - \frac{B\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c\,a^2(-3(\cos^3(fx+e))+4\cos(fx+e)\sin(fx+e)+8\tan(fx+e)+3\sec(fx+e))}{12f}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/60*a^2*c/f*tan(f*x+e)*(12*B*sin(f*x+e)^2*cos(f*x+e)^2+15*A*sin(f*x+e)*co
s(f*x+e)^2-15*B*sin(f*x+e)^3+20*A*cos(f*x+e)^2+8*B*sin(f*x+e)^2+15*A*sin(f
*x+e)+30*B*sin(f*x+e)+40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(
1/2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{(15\,(A+B)a^2c\cos(fx+e)^4 - 15\,(A+B)a^2c + 4\,(3\,Ba^2c\cos(fx+e)^4 - (5\,A+B)a^2c\cos(fx+e)^2 - 2}{60\,f\cos(fx+e)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*c
os(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(124) = 248$.

Time $= 0.44$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 2.40$

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{4\left(24\,Ba^2c\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-15\,Aa^2\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-4/15*(24*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a^2*c*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^8 - 75*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 40*A*a^2
*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 80*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^6 - 30*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 30*B*a^2*c*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

3.151.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{a^2 c \sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(60\,A\,\cos(e+f\,x)+60\,B\,\cos(e+f\,x)+75\,A\,\cos(3$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*c
os(e + f*x) + 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5
*f*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*
f*x) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*
x) + 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

3.152.1 Optimal result 1339
3.152.2 Mathematica [A] (verified) 1339
3.152.3 Rubi [A] (verified) 1340
3.152.4 Maple [A] (verified) 1341
3.152.5 Fricas [A] (verification not implemented) 1342
3.152.6 Sympy [F(-1)] 1342
3.152.7 Maxima [F] 1342
3.152.8 Giac [A] (verification not implemented) 1343
3.152.9 Mupad [B] (verification not implemented) 1343

3.152.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/3*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int (a+a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B
*Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A
 + 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

3.152.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A-B)\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx + \frac{B\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx + \frac{B\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3217$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

3.152.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.152.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

method	result
default	$-\frac{a^2\tan(fx+e)(-3B(\sin^3(fx+e))+4A(\cos^2(fx+e))-8B(\sin^2(fx+e))-12A\sin(fx+e)-6B\sin(fx+e)-16A)\sqrt{-c(\sin(fx+e)-1}}{12f}$
parts	$-\frac{A\,a^2\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)\sin(fx+e)+3\cos(fx+e)-4\tan(fx+e)-3\sec(fx+e))}{3f}+\frac{B\sec(fx+e)(3}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/12*a^2/f*tan(f*x+e)*(-3*B*sin(f*x+e)^3+4*A*cos(f*x+e)^2-8*B*sin(f*x+e)^
2-12*A*sin(f*x+e)-6*B*sin(f*x+e)-16*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin
(f*x+e)))^(1/2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{\left(3\,Ba^2\cos(fx+e)^4 - 12\,(A+B)a^2\cos(fx+e)^2 + 3\,(4\,A+3\,B)a\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*
B)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sin(fx+e)+c}\,a$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$
$$-\frac{4\left(3\,Ba^2\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 2\,Aa^2\cos$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/3*(3*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*cos(-1/4*pi + 1/2*f
*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - 2*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

3.152.9 Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.55

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$
$$-\frac{a^2\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(48\,A\cos(e+f\,x) + 36\,B\cos(e+f\,x) + 48\,A\cos(3\,e$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output

```
-(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos
(e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f
*x) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x)
 - 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x)
 + 1))
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.153.1 Optimal result . . . 1345
3.153.2 Mathematica [A] (verified) . . . 1346
3.153.3 Rubi [A] (verified) . . . 1346
3.153.4 Maple [B] (verified) . . . 1350
3.153.5 Fricas [F] . . . 1350
3.153.6 Sympy [F(-1)] . . . 1351
3.153.7 Maxima [F] . . . 1351
3.153.8 Giac [A] (verification not implemented) . . . 1351
3.153.9 Mupad [F(-1)] . . . 1352

3.153.1 Optimal result

Integrand size = 40, antiderivative size = 193

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

output

```
-1/2*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/
3*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*(A+B)
*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)-2*a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 11.68 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-3(A+3B)\cos(2(e+fx))+96A\log\left(\cos\right.\right.}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*
(-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e
 + f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*
Sqrt[c - c*Sin[e + f*x]])
```

3.153.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$
$$\downarrow 3452$$
$$(A+B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$
$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3216$$

$$(A+B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+\right.$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$(A+B)\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)}{2f\sqrt{c}}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$(A+B)\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e}{2f\sqrt{c-c\sin(}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*Sqrt[c
- c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))
```

3.153.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(173) = 346$.

Time = 3.76 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.55

method	result
default	$-\frac{a^2\left(15A\sin(fx+e)-15A-17B-2B\left(\cos^3(fx+e)\right)\sin(fx+e)+3A\cos(fx+e)+17B\sin(fx+e)+48A\cos(fx+e)\ln(\csc(fx+e)-\cot(\right.}{}$
parts	$\frac{A\left(\cos^3(fx+e)-\left(\cos^2(fx+e)\right)\sin(fx+e)-5\left(\cos^2(fx+e)\right)-16\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx\right.}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6*a^2/f*(15*A*sin(f*x+e)-15*A-17*B+7*B*cos(f*x+e)^2*sin(f*x+e)+3*A*cos(
f*x+e)+17*B*sin(f*x+e)+19*B*cos(f*x+e)^2+15*A*cos(f*x+e)^2+48*A*cos(f*x+e)
*ln(csc(f*x+e)-cot(f*x+e)-1)+3*A*sin(f*x+e)*cos(f*x+e)^2+18*A*sin(f*x+e)*c
os(f*x+e)+26*B*cos(f*x+e)*sin(f*x+e)-24*A*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+
24*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*B*cos(f*x+e)^4-24*B*ln(2/(1+cos(f*x
+e)))+9*cos(f*x+e)*B+48*A*ln(csc(f*x+e)-cot(f*x+e)-1)+48*B*ln(csc(f*x+e)-c
ot(f*x+e)-1)-3*A*cos(f*x+e)^3-48*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+
48*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-48*B*sin(f*x+e)*ln(csc(f*x+e)-
cot(f*x+e)-1)-24*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+24*B*sin(f*x+e)*ln(2/(1
+cos(f*x+e)))-9*B*cos(f*x+e)^3-24*A*ln(2/(1+cos(f*x+e)))-2*B*cos(f*x+e)^3*
sin(f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*
x+e)-1))^(1/2)
```

3.153.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c*sin(f*x + e) - c), x)
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.153.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^2\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^2\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.\right.\right.}{\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx\right.\right.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + sqrt(2)*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-
cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) + sqrt(2)*(4*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 3*A*a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^(5/2)*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a^2*c^(5/2)*cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2
*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)))/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

3.154.1 Optimal result 1353
3.154.2 Mathematica [A] (verified) 1354
3.154.3 Rubi [A] (verified) 1354
3.154.4 Maple [A] (verified) 1357
3.154.5 Fricas [F] 1358
3.154.6 Sympy [F(-1)] 1358
3.154.7 Maxima [F] 1359
3.154.8 Giac [F(-2)] 1359
3.154.9 Mupad [F(-1)] 1359

3.154.1 Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a
*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^
3*(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin
(f*x+e))^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 11.77 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(28A+16B+2(2A+7B)\cos(2(e+fx))+\right.$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8
*A + 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e +
 f*x] + B*Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-
1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

3.154.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$
$$\downarrow 3451$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$
$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((A + 2*B)*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f
*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c
```

3.154.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.154.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

method	result
default	$-\frac{a^2\sec(fx+e)\left(B(\sin^3(fx+e))+2(\sin^2(fx+e))A-8A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+16A\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(8\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-(\cos^2(fx+e))-8\ln(\csc(fx+e)-\cot(fx-\right.}{fc\sqrt{-c(\sin(fx+e)-1)}}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2*a^2/c/f*sec(f*x+e)*(B*sin(f*x+e)^3+2*sin(f*x+e)^2*A-8*A*sin(f*x+e)*ln
(2/(1+cos(f*x+e)))+16*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+7*B*sin(f*x
+e)^2-16*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+32*B*sin(f*x+e)*ln(csc(f*x+e)-c
ot(f*x+e)-1)-10*A*sin(f*x+e)+8*A*ln(2/(1+cos(f*x+e)))-16*A*ln(csc(f*x+e)-c
ot(f*x+e)-1)-16*B*sin(f*x+e)+16*B*ln(2/(1+cos(f*x+e)))-32*B*ln(csc(f*x+e)-
cot(f*x+e)-1))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.154.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.154.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(3/2), x)
```

3.154.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument Value
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

3.155.1 Optimal result 1360
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3.155.1 Optimal result

Integrand size = 40, antiderivative size = 212

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}$$
$$-\frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}}$$
$$-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a
*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*
(A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx))$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A
 + B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

3.155.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{4c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{4c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{4c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{4c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - ((A + 5*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c -
 c*Sin[e + f*x])^(3/2)) - (2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x
]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c))/(4*c)
```

3.155.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.155.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.74

method	result
default	$-\frac{a^2 \sec(fx+e)\left(2A(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B(\sin^3(fx+e))+10B(\cos^2(fx$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sin(fx+e)-4\ln(cs}{f(\sin(fx+e)-1)\sqrt{-}}$

input

```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output

```
-a^2/c^2/f*sec(f*x+e)*(2*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-A*cos(
f*x+e)^2*ln(2/(1+cos(f*x+e)))-B*sin(f*x+e)^3+10*B*cos(f*x+e)^2*ln(csc(f*x+
e)-cot(f*x+e)-1)-5*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*sin(f*x+e)^2*A+4*
A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)
))+8*B*sin(f*x+e)^2+20*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-10*B*sin(f
*x+e)*ln(2/(1+cos(f*x+e)))-4*A*ln(csc(f*x+e)-cot(f*x+e)-1)+2*A*ln(2/(1+cos
(f*x+e)))-5*B*sin(f*x+e)-20*B*ln(csc(f*x+e)-cot(f*x+e)-1)+10*B*ln(2/(1+cos
(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/
2)
```

3.155.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*
sin(f*x + e)), x)
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(190) = 380$.

Time = 0.32 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.39

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$\left(\frac{8\,a^{\frac{5}{2}}\sqrt{c}\sin(fx+e)^2}{\left(c^3-\frac{4\,c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6\,c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4\,c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2}-\frac{2\,a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}+\frac{a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e}{(\cos(fx+e)}\right.}{c^{\frac{5}{2}}}$$

$$-$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x +
e) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x
+ e) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2)
+ a^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*
a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e
)/(cos(f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14
*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5
/2) - 4*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*
c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{4\sqrt{2}Ba^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{2\sqrt{2}}{}\right.$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/4*sqrt(2)*(4*sqrt(2)*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))/(c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*
sqrt(2)*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sqrt(
c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^2 + 32)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*a^2*
sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^2*sqrt(c)*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - 4*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 2*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi +
1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

3.156.1 Optimal result 1369
3.156.2 Mathematica [A] (verified) 1369
3.156.3 Rubi [A] (verified) 1370
3.156.4 Maple [A] (verified) 1373
3.156.5 Fricas [F] 1374
3.156.6 Sympy [F(-1)] 1374
3.156.7 Maxima [F] 1375
3.156.8 Giac [A] (verification not implemented) 1375
3.156.9 Mupad [F(-1)] 1376

3.156.1 Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output

```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e
)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(4(A+B)-6(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.$$

input

```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

3.156.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) - (B*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin
[e + f*x])^(5/2)) - (a*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c -
c*Sin[e + f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c))/c
```

3.156.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.156.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.63

method	result
default	$\frac{a^2\sec(fx+e)\left(-6B\sin(fx+e)(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+3B\sin(fx+e)(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx$
parts	$\frac{A\tan(fx+e)a^2(\cos^2(fx+e)-4)\sqrt{a(1+\sin(fx+e))}}{3f(\cos^2(fx+e)+2\sin(fx+e)-2)\sqrt{-c(\sin(fx+e)-1)}\,c^3}-\frac{B\sec(fx+e)\left(6(\cos^2(fx+e))\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/3*a^2/c^3/f*sec(f*x+e)*(-6*B*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f
*x+e)-1)+3*B*sin(f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)*cos
(f*x+e)^2-7*B*sin(f*x+e)^3+18*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-9
*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+6*B*sin(f*x+e)^2+24*B*sin(f*x+e)*ln(c
sc(f*x+e)-cot(f*x+e)-1)-12*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-4*A*sin(f*x+e
)-3*B*sin(f*x+e)-24*B*ln(csc(f*x+e)-cot(f*x+e)-1)+12*B*ln(2/(1+cos(f*x+e))
))*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-
1))^(1/2)
```

3.156.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*
cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.156.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(7/2), x)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{6\sqrt{2}Ba^2\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(3\,(Aa^2\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\,Ba^2\sqrt{c}\operatorname{sgn}\right.\right.}{\left.\left.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-1/12*sqrt(2)*sqrt(a)*(6*sqrt(2)*B*a^2*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) - sqrt(2)*(3*(A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 
1/2*e)) + 5*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^4 + A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 10*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*(A*a^2*sqrt(c)*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - 1)^3*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

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3.157.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

output
```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*
(A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

3.157.2 Mathematica [A] (verified)

Time = 12.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
 - 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[
3*(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e
 + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

3.157.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A-7B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3042$$

$$\frac{(A-7B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\downarrow 3221$$

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c
- c*Sin[e + f*x])^(7/2))
```

3.157.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.157.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

method	result
default	$\frac{a^2\tan(fx+e)(A\sin(fx+e)(\cos^2(fx+e))+B(\sin^3(fx+e))-4A(\cos^2(fx+e))+2B(\sin^2(fx+e))-4A\sin(fx+e)+3B\sin(fx+e)+10A}{6c^4f((\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)+4)\sqrt{-c(\sin(fx+e)-1)}}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2(\cos^3(fx+e)+4\cos(fx+e)\sin(fx+e)-5\cos(fx+e)-10\tan(fx+e)+4\sec(fx+e))}{6f((\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)+4)\sqrt{-c(\sin(fx+e)-1)}\,c^4}+\frac{B\sec(fx+e)(\cos(f$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/6*a^2/c^4/f*tan(f*x+e)*(A*sin(f*x+e)*cos(f*x+e)^2+B*sin(f*x+e)^3-4*A*cos
(f*x+e)^2+2*B*sin(f*x+e)^2-4*A*sin(f*x+e)+3*B*sin(f*x+e)+10*A)*(a*(1+sin(f
*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*
(sin(f*x+e)-1))^(1/2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.72

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{\left(3\,(A-B)a^2\cos\left(fx+e\right)^2-4\,(A-B)a^2+2\left(3\,Ba^2\cos\left(fx+e\right)^2-(A+5\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin(}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x +
e)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(
f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

3.157.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}}{(-c\sin\left(fx+e\right)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(84) = 168$.

Time $= 0.40$ (sec) , antiderivative size $= 268$, normalized size of antiderivative $= 2.79$

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx =$$

$$-\frac{\left(24\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 6\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^4 \mathrm{sg}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
-1/48*(24*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 6*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 42*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*A*a^2*sqrt(c)*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 28*B*a^2*sqrt
(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*sqrt(c)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
 - 1)^4*c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(9/2), x)
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

3.158.1 Optimal result . . . 1383
3.158.2 Mathematica [A] (verified) . . . 1383
3.158.3 Rubi [A] (verified) . . . 1384
3.158.4 Maple [A] (verified) . . . 1386
3.158.5 Fricas [A] (verification not implemented) . . . 1387
3.158.6 Sympy [F(-1)] . . . 1387
3.158.7 Maxima [F] . . . 1387
3.158.8 Giac [B] (verification not implemented) . . . 1388
3.158.9 Mupad [B] (verification not implemented) . . . 1388

3.158.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}$$

output

```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/4
0*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/2
40*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

3.158.2 Mathematica [A] (verified)

Time = 13.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}}{120c^5f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input

```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36
*A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 1
5*B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-
1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

3.158.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A-4B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{(A-4B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3222$$

$$\frac{(A-4B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{(A-4B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}+$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+$$
$$\frac{(A-4B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((A - 4*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c
 - c*Sin[e + f*x])^(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*
c*f*(c - c*Sin[e + f*x])^(7/2))))/(5*c)
```

3.158.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.158.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^2\tan(fx+e)(4A(\cos^4(fx+e))+B(\sin^2(fx+e))(\cos^2(fx+e))+20A\sin(fx+e)(\cos^2(fx+e))+5B(\sin^3(fx+e))-48A(\cos^2(fx+e))}{30c^5f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2(2(\cos^3(fx+e))\sin(fx+e)-10(\cos^3(fx+e))-24\cos(fx+e)\sin(fx+e)+35\cos(fx+e)+37\tan(fx+e)-25\sec}{15f(\cos^4(fx+e)+4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-8\sin(fx+e)+8)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/30*a^2/c^5/f*tan(f*x+e)*(4*A*cos(f*x+e)^4+B*sin(f*x+e)^2*cos(f*x+e)^2+20
*A*sin(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-48*A*cos(f*x+e)^2+4*B*sin(f*x+
e)^2-50*A*sin(f*x+e)+15*B*sin(f*x+e)+74*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f
*x+e)^4+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(sin(
f*x+e)-1))^(1/2)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{\left(5\,(2\,A+B)a^2\cos(fx+e)^2-2\,(7\,A+2\,B)a^2+5\left(3\,Ba^2\cos(fx+e)^2-2\,(A+2\,B)a^2\right)\sin(fx+e)\right)\sqrt{}}{30\left(5\,c^6f\cos(fx+e)^5-20\,c^6f\cos(fx+e)^3+16\,c^6f\cos(fx+e)-\left(c^6f\cos(fx+e)^5-12\,c^6f\cos(fx+\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
-1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos
(f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 +
16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 +
16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(128) = 256$.

Time $= 0.44$ (sec) , antiderivative size $= 268$, normalized size of antiderivative $= 1.84$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(30\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
1/240*(30*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 10*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*a^2*sqrt(c)*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*sqr
t(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^2*sqrt(c)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^
2 - 1)^5*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.158.9 Mupad [B] (verification not implemented)

Time $= 21.07$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 2.34$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\left(\frac{16\,a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,(A\,6\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}}{5\,c^6\,f}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output

```
((c - c*sin(e + f*x))^(1/2)*((16*a^2*exp(e*6i + f*x*6i)*(A*6i + B*1i)*(a +
 a*sin(e + f*x))^(1/2))/(5*c^6*f) - (16*a^2*exp(e*6i + f*x*6i)*cos(2*e + 2
*f*x)*(A*2i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f) + (a^2*exp(e*6i
+ f*x*6i)*sin(e + f*x)*(8*A + 13*B)*(a + a*sin(e + f*x))^(1/2)*8i)/(3*c^6*
f) - (B*a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)
*8i)/(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*
cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i
 + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i
 - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

3.159.1 Optimal result . . . 1390
3.159.2 Mathematica [A] (verified) . . . 1390
3.159.3 Rubi [A] (verified) . . . 1391
3.159.4 Maple [A] (verified) . . . 1393
3.159.5 Fricas [A] (verification not implemented) . . . 1394
3.159.6 Sympy [F(-1)] . . . 1394
3.159.7 Maxima [F] . . . 1395
3.159.8 Giac [A] (verification not implemented) . . . 1395
3.159.9 Mupad [B] (verification not implemented) . . . 1396

3.159.1 Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}$$

output

```
1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/4
0*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/
160*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
+1/960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7
/2)
```

3.159.2 Mathematica [A] (verified)

Time = 14.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}}{120c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(13/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*
A + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B
*Sin[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

3.159.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{11/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{11/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(13/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x
])^(13/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(
c - c*Sin[e + f*x])^(11/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(
8*f*(c - c*Sin[e + f*x])^(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)
)/(48*c*f*(c - c*Sin[e + f*x])^(7/2)))/(5*c)))/(4*c)
```

3.159.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.159.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11

method	result
default	$\frac{a^2\tan(fx+e)\left(7A\left(\cos^4(fx+e)\right)\sin(fx+e)-B\left(\sin^5(fx+e)\right)-42A\left(\cos^4(fx+e)\right)+6\left(\sin^4(fx+e)\right)B-119A\sin(fx+e)\left(\cos^2(fx+e)\right)-\right.}{60c^6f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(c}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^2\left(7\left(\cos^5(fx+e)\right)+42\left(\cos^3(fx+e)\right)\sin(fx+e)-126\left(\cos^3(fx+e)\right)-224\cos(fx+e)\sin(fx+e)+321\cos(fx+\right.}{60f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(\cos^2(fx+e))+16\sin(fx+e)-16)\sqrt{}}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/60*a^2/c^6/f*tan(f*x+e)*(7*A*cos(f*x+e)^4*sin(f*x+e)-B*sin(f*x+e)^5-42*A
*cos(f*x+e)^4+6*sin(f*x+e)^4*B-119*A*sin(f*x+e)*cos(f*x+e)^2-15*B*sin(f*x+
e)^3+224*A*cos(f*x+e)^2+202*A*sin(f*x+e)-30*B*sin(f*x+e)-242*A)*(a*(1+sin(
f*x+e)))^(1/2)/(cos(f*x+e)^4*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin
(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)-16)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(15\,(A+B)a^2\cos(fx+e)^2 - 2\,(11\,A+7\,B)}{60\,(c^7 f\cos(fx+e)^7 - 18\,c^7 f\cos(fx+e)^5 + 48\,c^7 f\cos}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="fricas")
```

output
```
1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos
(f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 4
8*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5
 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2
),x)
```

output
```
Timed out
```

3.159.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(13/2), x)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(40\,Ba^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+15\,Aa^2\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 s}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="giac")
```

output
```
-1/960*(40*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 15*A*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 45*B*a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*a^2*sqrt(c)*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a^2*sq
rt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + A*a^2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*sqrt(c)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)
^2 - 1)^6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.159.9 Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.82

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\left(\frac{a^2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,(A\,29\mathrm{i}+B\,13\mathrm{i})\,\sqrt{}}{15\,c^7\,f}\right.}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(13/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a +
a*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e +
2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp
(e*7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^
7*f) + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^
(1/2))/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e +
f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(
e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x)
 - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e
 + 6*f*x))
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

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3.160.2 Mathematica [A] (verified) 1398
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3.160.7 Maxima [F] 1403
3.160.8 Giac [B] (verification not implemented) 1403
3.160.9 Mupad [B] (verification not implemented) 1404

3.160.1 Optimal result

Integrand size = 40, antiderivative size = 250

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx =$$
$$-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{126f}$$
$$-\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}$$
$$-\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

output

```
-1/84*a^2*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)
/f-1/72*a*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)
/f-1/9*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/f-1/315*
a^4*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/1
26*a^3*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.160.2 Mathematica [A] (verified)

Time = 11.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{9/2}\,dx = \frac{a^3c^4(-1+\sin(e+fx))^4(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(9/2),x]
```

output
```
(a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*
x])]*Sqrt[c - c*Sin[e + f*x]]*(17640*(A - B)*Cos[2*(e + f*x)] + 8820*(A -
B)*Cos[4*(e + f*x)] + 2520*A*Cos[6*(e + f*x)] - 2520*B*Cos[6*(e + f*x)] +
315*A*Cos[8*(e + f*x)] - 315*B*Cos[8*(e + f*x)] + 176400*A*Sin[e + f*x] -
17640*B*Sin[e + f*x] + 35280*A*Sin[3*(e + f*x)] + 7056*A*Sin[5*(e + f*x)]
+ 2016*B*Sin[5*(e + f*x)] + 720*A*Sin[7*(e + f*x)] + 900*B*Sin[7*(e + f*x)
] + 140*B*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

3.160.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$

$$\frac{1}{9}(9A-B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{9/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$$\downarrow 3042$$

$$\frac{1}{9}(9A-B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{9/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$$\downarrow 3219$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9}}{8f}\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right)$$

$$\downarrow 3042$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9}}{8f}\right.$$
$$\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right)$$

$$\downarrow 3219$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+\right.\right.$$
$$\left.\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right)$$

$$\downarrow 3042$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+\right.\right.$$
$$\left.\left.-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right)$$

$$\downarrow 3219$$

$$\frac{1}{9}(9A - B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(\frac{1}{3}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+\right.\right.\right.$$

$$\left.\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right.\right.\right.$$

$$\downarrow 3042$$

$$\frac{1}{9}(9A - B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(\frac{1}{3}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+\right.\right.\right.$$

$$\left.\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right.\right.\right.$$

$$\downarrow 3217$$

$$\frac{1}{9}(9A - B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15f\sqrt{a\sin(e+fx)+a}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{6f}\right)\right.\right.$$

$$\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}\right.\right)$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
9/2),x]
```

output
```
-1/9*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2)
)/f + ((9*A - B)*(-1/8*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*S
in[e + f*x])^(9/2))/f + (3*a*(-1/7*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2)*(c - c*Sin[e + f*x])^(9/2))/f + (4*a*(-1/15*(a^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(9/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/(6*f)))/7))/4))/9
```

3.160.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.160.4 Maple [A] (verified)

Time = 74.96 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

method	result
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^3c^4\left(35\left(\cos^7(fx+e)\right)+40\left(\cos^5(fx+e)\right)\sin(fx+e)+48\left(\cos^3(fx+e)\right)\sin(fx+e)+64\cos(f$
default	$-\frac{a^3c^4\tan(fx+e)\left(280B\left(\sin^2(fx+e)\right)\left(\cos^6(fx+e)\right)+315A\sin(fx+e)\left(\cos^6(fx+e)\right)+315B\left(\sin^3(fx+e)\right)\left(\cos^4(fx+e)\right)-360A\left(\cos^6\right.\right.$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/280*A/f*(a*(1+sin(f*x+e)))^(1/2)*(-c*(sin(f*x+e)-1))^(1/2)*a^3*c^4*(35*c
os(f*x+e)^7+40*cos(f*x+e)^5*sin(f*x+e)+48*cos(f*x+e)^3*sin(f*x+e)+64*cos(f
*x+e)*sin(f*x+e)+128*tan(f*x+e)-35*sec(f*x+e))+1/2520*B/f*sec(f*x+e)*(280*
cos(f*x+e)^6*sin(f*x+e)-315*cos(f*x+e)^6+240*cos(f*x+e)^4*sin(f*x+e)-315*c
os(f*x+e)^4+192*cos(f*x+e)^2*sin(f*x+e)-315*cos(f*x+e)^2+128*sin(f*x+e)-31
5)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*c^4*a^3*(cos(f*x+e)^
2-1)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{9/2}\,dx = \frac{(315\,(A-B)a^3c^4\cos(fx+e)^8 - 315\,(A-B)a^3c^4 + 8\,(35\,Ba^3c^4\cos(fx+e)^8 + 5\,($$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B
*a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)
*a^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B
)*a^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)/(f*cos(f*x + e))
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

3.160.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{9/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(9/2), x)
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(220) = 440$.

Time $= 0.55$ (sec) , antiderivative size $= 453$, normalized size of antiderivative $= 1.81$

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{9/2}\,dx = \frac{32\left(560\,Ba^3c^4\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
32/315*(560*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^18 - 315*A*a^3*c^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^16 - 2205*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
16 + 1080*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 + 3240*B*a^3*c^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^14 - 1260*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^1
2 - 2100*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 504*A*a^3*c^4*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^10 + 504*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)*
sqrt(a)*sqrt(c)/f
```

3.160.9 Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \frac{\mathrm{e}^{-e\,9\mathrm{i}-f\,x\,9\mathrm{i}}\sqrt{c-c\sin(e+f\,x)}\left(-\frac{a^3\,c^4\,\mathrm{e}^{e\,9\mathrm{i}+f\,x\,9\mathrm{i}}\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}-B\,1\mathrm{i})\,\sqrt{a+a\sin(e+f\,x)}\,7\mathrm{i}}{64\,f}-\right.$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
9/2),x)
```

output

```
(exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^4*exp(e*9i + f*x*
9i)*sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^
3*c^4*exp(e*9i + f*x*9i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x
))^(1/2)*7i)/(128*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos(6*e + 6*f*x)*(A*1i
 - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(64*f) - (a^3*c^4*exp(e*9i + f*x*9i
)*cos(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(512*f) -
(a^3*c^4*exp(e*9i + f*x*9i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e +
f*x))^(1/2)*7i)/(64*f) + (a^3*c^4*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(4*A
 + 5*B)*(a + a*sin(e + f*x))^(1/2))/(896*f) + (7*A*a^3*c^4*exp(e*9i + f*x*
9i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e
*9i + f*x*9i)*sin(e + f*x)*(10*A - B)*(a + a*sin(e + f*x))^(1/2))/(64*f) +
 (B*a^3*c^4*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(1152*f)))/(2*cos(e + f*x))
```

3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

3.161.1 Optimal result 1406
3.161.2 Mathematica [A] (verified) 1407
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3.161.1 Optimal result

Integrand size = 40, antiderivative size = 226

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f}$$
$$-\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f}$$
$$-\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

output

```
-1/7*a^2*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*
a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/8*B*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*A*cos(f*x+e
)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*A*cos(f*x+e)*(c
-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

3.161.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(1225B+1960B\cos(2(e+fx))+980B\cos(4(e$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/35840*(a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
 + f*x]]*(1225*B + 1960*B*Cos[2*(e + f*x)] + 980*B*Cos[4*(e + f*x)] + 280*
B*Cos[6*(e + f*x)] + 35*B*Cos[8*(e + f*x)] - 19600*A*Sin[e + f*x] - 3920*A
*Sin[3*(e + f*x)] - 784*A*Sin[5*(e + f*x)] - 80*A*Sin[7*(e + f*x)]))/f
```

3.161.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$A\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{7/2}dx -$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$
$$\downarrow 3042$$

$$A\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\downarrow 3219$$

$$A\left(\frac{6}{7}a\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/}}{7f}\right.$$
$$\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right)$$

$$\downarrow 3042$$

$$A\left(\frac{6}{7}a\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/}}{7f}\right.$$
$$\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right)$$

$$\downarrow 3219$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+f}\right.\right.$$
$$\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right)\right)$$

$$\downarrow 3042$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+f}\right.\right.$$
$$\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right)\right)$$

$$\downarrow 3219$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(\frac{2}{5}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+f}\right.\right.\right.$$
$$\left.\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right)\right)\right)$$

$$\downarrow 3042$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(\frac{2}{5}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+f}{5f}\right.\right.\right.$$
$$\left.\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right.\right.\right.$$

$$\downarrow 3217$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\right.\right.$$
$$\left.\left.\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\right.\right)$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/8*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + A*(-1/7*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(7/2))/f + (6*a*(-1/6*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c -
c*Sin[e + f*x])^(7/2))/f + (2*a*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(7/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5*f)))/3))/7)
```

3.161.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.161.4 Maple [A] (verified)

Time = 74.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

method	result
default	$\frac{a^3c^3\tan(fx+e)(35B\sin(fx+e)(\cos^6(fx+e))+40A(\cos^6(fx+e))+35B(\cos^4(fx+e))\sin(fx+e)+48A(\cos^4(fx+e))+35B(\cos^2(fx+e)}{280f}$
parts	$\frac{A\tan(fx+e)a^3c^3(5(\cos^6(fx+e))+6(\cos^4(fx+e))+8(\cos^2(fx+e))+16)\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}{35f} - \frac{B\sec(fx+e)a^3}{}$

input

```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/280*a^3*c^3/f*tan(f*x+e)*(35*B*sin(f*x+e)*cos(f*x+e)^6+40*A*cos(f*x+e)^6
+35*B*cos(f*x+e)^4*sin(f*x+e)+48*A*cos(f*x+e)^4+35*B*cos(f*x+e)^2*sin(f*x+
e)+64*A*cos(f*x+e)^2+35*B*sin(f*x+e)+128*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(
1+sin(f*x+e)))^(1/2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.59

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{(35\,Ba^3c^3\cos(fx+e)^8 - 35\,Ba^3c^3 - 8\,(5\,Aa^3c^3\cos(fx+e)^6 + 6\,Aa^3c^3\cos(fx+e)^4 + 8\,Aa^3c^3\cos(fx+$$
$$280\,f\cos(fx+e)$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*
x + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*
a^3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.161.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(196) = 392$.

Time $= 0.54$ (sec) , antiderivative size $= 453$, normalized size of antiderivative $= 2.00$

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = \frac{32\left(35\,Ba^3c^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi +\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
32/35*(35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^16 - 20*A*a^3*c^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^14 - 140*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 +
 70*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 210*B*a^3*c^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^12 - 84*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 140*B
*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 35*A*a^3*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^8 + 35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c
)/f
```

3.161.9 Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{e^{-e\,8i-f\,x\,8i}\sqrt{c-c\sin(e+f\,x)}\left(\frac{35\,A\,a^3\,c^3\,e^{e\,8i+f\,x\,8i}\sin(e+f\,x)\sqrt{a+a\sin(e+f\,x)}}{32\,f} - \frac{7\,B\,a^3\,c^3\,e^{e\,8i+}}{}\right.$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
(exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i +
 f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*ex
p(e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*
B*a^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/
(128*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*
x))^(1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*
sin(e + f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3
*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)
*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8
i + f*x*8i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(
e + f*x))
```

3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

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3.162.1 Optimal result

Integrand size = 40, antiderivative size = 192

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{105f} + \frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

output

```
1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f-
1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f+1/105*(7*
A+B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/105*
(7*A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

3.162.2 Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{5/2}\,dx = \frac{a^3c^2(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*
x])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)
*Cos[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A
*Sin[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(
e + f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e
+ f*x)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7)
```

3.162.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$\frac{1}{7}(7A+B)\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{5/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{5/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$$\downarrow 3219$$

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx + \frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3}}{6f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx + \frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3}}{6f} - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\right.$$

$$\downarrow 3219$$

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx + \frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx + \frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\right.$$

$$\downarrow 3217$$

$$\frac{1}{7}(7A + B)\left(\frac{2}{3}c\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)+\frac{c\cos(}{}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/7*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + ((7*A + B)*((c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e
+ f*x])^(3/2))/(6*f) + (2*c*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))
/(10*f*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f)))/3))/7
```

3.162.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.162.4 Maple [A] (verified)

Time = 75.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

method	result
default	$\frac{a^3c^2\tan(fx+e)\left(30B\left(\cos^4(fx+e)\right)\left(\sin^2(fx+e)\right)+35A\left(\cos^4(fx+e)\right)\sin(fx+e)-35B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)+42A\left(\cos^4(fx+e)\right.\right.}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^3c^2\left(-5\left(\cos^5(fx+e)\right)+6\left(\cos^3(fx+e)\right)\sin(fx+e)+8\cos(fx+e)\sin(fx+e)+16\tan(fx+e)\right.}{30f}$

input

```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/210*a^3*c^2/f*tan(f*x+e)*(30*B*cos(f*x+e)^4*sin(f*x+e)^2+35*A*cos(f*x+e)
^4*sin(f*x+e)-35*B*cos(f*x+e)^2*sin(f*x+e)^3+42*A*cos(f*x+e)^4+24*B*sin(f*
x+e)^2*cos(f*x+e)^2+35*A*sin(f*x+e)*cos(f*x+e)^2-70*B*sin(f*x+e)^3+56*A*co
s(f*x+e)^2+16*B*sin(f*x+e)^2+35*A*sin(f*x+e)+105*B*sin(f*x+e)+112*A)*(-c*(
sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{\left(35\,(A+B)a^3c^2\cos(fx+e)^6-35\,(A+B)a^3c^2+2\left(15\,Ba^3c^2\cos(fx+e)^6-3\,(7\,A+B)a^3c^2\cos(fx+\right.\right.}{210\,f\,c}$$

input

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a
^3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a
^3*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.162.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{7}{2}}(-c\sin(fx + e) + c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(168) = 336$.

Time $= 0.56$ (sec) , antiderivative size $= 453$, normalized size of antiderivative $= 2.36$

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{16\left(120\,Ba^3c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
16/105*(120*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^3*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^12 - 490*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12
 + 252*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 756*B*a^3*c^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^10 - 315*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 52
5*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 140*A*a^3*c^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^6 + 140*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sq
rt(c)/f
```

3.162.9 Mupad [B] (verification not implemented)

Time $= 17.66$ (sec) , antiderivative size $= 383$, normalized size of antiderivative $= 1.99$

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \frac{\mathrm{e}^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\sqrt{c - c\,\sin(e+f\,x)}\left(\frac{a^3\,c^2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\cos(2\,e+2\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,5\mathrm{i}}{32\,f} + \frac{a^{3}}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*
7i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) +
 (a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e +
 f*x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*
1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x
*7i)*sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a
^3*c^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))
^(1/2))/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a +
 a*sin(e + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7
*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.163 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

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3.163.1 Optimal result

Integrand size = 40, antiderivative size = 142

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

output

```
-1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(3*
A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/15*(
3*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

3.163.2 Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{a^3c(-1+\sin(e+fx))(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-15(16A+11B)\cos(2$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
-1/960*(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e +
 f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*
(2*A + B)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 2
40*B*Sin[e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*S
in[5*(e + f*x)] - 24*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

3.163.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$\frac{1}{3}(3A+B)\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$
$$\downarrow 3042$$

$$\frac{1}{3}(3A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$$\downarrow 3219$$

$$\frac{1}{3}(3A+B)\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A+B)\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$$\downarrow 3217$$

$$\frac{1}{3}(3A+B)\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

input

```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output

```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + ((3*A + B)*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(5*f)))/3
```

3.163.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.163.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a^3 c \tan(fx+e)(5B(\sin^5(fx+e))+6A(\cos^4(fx+e))+12(\sin^4(fx+e))B-15A\sin(fx+e)(\cos^2(fx+e))-12A(\cos^2(fx+e))-20B(s}{30f}$
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,\sqrt{a(1+\sin(fx+e))}\,c\,a^3(2(\cos^3(fx+e))\sin(fx+e)+5(\cos^3(fx+e))-4\cos(fx+e)\sin(fx+e)-8\tan(fx+e)-}{10f}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/30*a^3*c/f*tan(f*x+e)*(5*B*sin(f*x+e)^5+6*A*cos(f*x+e)^4+12*sin(f*x+e)^
4*B-15*A*sin(f*x+e)*cos(f*x+e)^2-12*A*cos(f*x+e)^2-20*B*sin(f*x+e)^2-15*A*
sin(f*x+e)-15*B*sin(f*x+e)-24*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e
)))^(1/2)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{(5\,Ba^3c\cos(fx+e)^6 - 15\,(A+B)a^3c\cos(fx+e)^4 + 5\,(3\,A+2\,B)a^3c - 2\,(3\,(A+$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A
+ 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos
(f*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.74

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c$$
$$-c\sin(e+fx))^{3/2}\,dx = \frac{8\left(20\,Ba^3c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
8/15*(20*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 36*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^
3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)))*sqrt(a)*sqrt(c)/f
```

3.163.9 Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.26

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{e^{-e\,6i-f\,x\,6i}\sqrt{c-c\sin(e+f\,x)}\left(\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\cos(4\,e+4\,f\,x)\,(2\,A+B)\,\sqrt{a+a\sin(e+f\,x)}}{16\,f}-\frac{B\,a^3\,c\,e^{e\,6i+f\,x\,6i}\cos(6\,e+6\,f\,x)\,\sqrt{a+}}{96\,f}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6
i)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*
c*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) +
 (a^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))
^(1/2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B
)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e +
 3*f*x)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e
*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i
)/(40*f)))/(2*cos(e + f*x))
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

3.164.1 Optimal result 1429
3.164.2 Mathematica [A] (verified) 1429
3.164.3 Rubi [A] (verified) 1430
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3.164.7 Maxima [F] 1433
3.164.8 Giac [A] (verification not implemented) 1433
3.164.9 Mupad [B] (verification not implemented) 1434

3.164.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{7/2}(A + B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.26

$$\int (a+a\sin(e+fx))^{7/2}(A + B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(60A+23$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
(a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(
60*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x
]) - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(16
0*f)
```

3.164.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A-B)\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{9/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{9/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3217$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

3.164.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.164.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^3\tan(fx+e)\left(4B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+5A\sin(fx+e)\left(\cos^2(fx+e)\right)-15B\left(\sin^3(fx+e)\right)+20A\left(\cos^2(fx+e)\right)-24B\left(\sin^2(fx\right.}{20f}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}\,a^3\left(\cos^3(fx+e)-4\cos(fx+e)\sin(fx+e)-8\cos(fx+e)+8\tan(fx+e)+7\sec(fx+e)\right)}{4f}+\frac{B}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/20*a^3/f*tan(f*x+e)*(4*B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*sin(f*x+e)*cos(f
*x+e)^2-15*B*sin(f*x+e)^3+20*A*cos(f*x+e)^2-24*B*sin(f*x+e)^2-35*A*sin(f*x
+e)-10*B*sin(f*x+e)-40*A)*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/
2)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int (a+a\sin(e+fx))^{7/2}(A$$
$$+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{\left(5\,(A+3\,B)a^3\cos(fx+e)^4-40\,(A+B)a^3\cos(fx+e)^2+5\,(7\,A$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(
7*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2
+ 2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e))
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.164.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A$$
$$+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}\sqrt{-c\sin(fx+e)+c}\,d$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=$$
$$-\frac{4\left(8\,Ba^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,Aa^3\,\mathrm{co}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/5*(8*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^3*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) - 5*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

3.164.9 Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.80

$$\int (a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$

$$-\frac{a^3\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(140\,A\,\cos(e+f\,x)+100\,B\,\cos(e+f\,x)+135\,A\,\cos$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*co
s(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5
*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*
f*x) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*
x) - 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.165.1 Optimal result 1435
3.165.2 Mathematica [A] (verified) 1436
3.165.3 Rubi [A] (verified) 1436
3.165.4 Maple [B] (verified) 1440
3.165.5 Fricas [F] 1441
3.165.6 Sympy [F(-1)] 1441
3.165.7 Maxima [F] 1441
3.165.8 Giac [B] (verification not implemented) 1442
3.165.9 Mupad [F(-1)] 1442

3.165.1 Optimal result

Integrand size = 40, antiderivative size = 239

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

output

```
-a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*
a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-1/4*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*(A+B)*cos(
f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*
a^3*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 12.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\left(-12(8A+15B)\cos(2(e+\right.}{96f\left(\cos\left(\frac{1}{2}\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt
[a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e +
 f*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A
+ 36*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

3.165.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3452$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\left(2a\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3219$$

$$(A+B)\left(2a\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\left(2a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3216$$

$$\frac{(A+B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$\frac{(A+B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+}{2f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$\frac{(A+B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a}{2f\sqrt{c-c}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*(-1/3*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*Sqrt[c
- c*Sin[e + f*x]]) + 2*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))
/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e
+ f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))))
```

3.165.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(215) = 430.

Time = 3.30 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.30

method	result
default	$\frac{a^3\left(-64A\sin(fx+e)+64A+67B+16B\left(\cos^3(fx+e)\right)\sin(fx+e)-24A\cos(fx+e)-67B\sin(fx+e)-192A\cos(fx+e)\ln(\csc(fx+e)-c\right.}{}$
parts	$\frac{A\left(\cos^4(fx+e)+\left(\cos^3(fx+e)\right)\sin(fx+e)+6\left(\cos^3(fx+e)\right)-5\left(\cos^2(fx+e)\right)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx+e)-24\ln\left(\frac{}{1+\cos}\right.\right.}{}$

input

```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/12*a^3/f*(-64*A*sin(f*x+e)+64*A+67*B-32*B*cos(f*x+e)^2*sin(f*x+e)-24*A*c
os(f*x+e)-67*B*sin(f*x+e)-80*B*cos(f*x+e)^2-68*A*cos(f*x+e)^2-192*A*cos(f*
x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-20*A*sin(f*x+e)*cos(f*x+e)^2-88*A*sin(f*x
+e)*cos(f*x+e)-112*B*cos(f*x+e)*sin(f*x+e)+96*A*cos(f*x+e)*ln(2/(1+cos(f*x
+e)))-96*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+13*B*cos(f*x+e)^4+96*B*ln(2/(1+
cos(f*x+e)))+3*B*cos(f*x+e)^4*sin(f*x+e)-45*cos(f*x+e)*B+4*A*cos(f*x+e)^4-
192*A*ln(csc(f*x+e)-cot(f*x+e)-1)-192*B*ln(csc(f*x+e)-cot(f*x+e)-1)+24*A*c
os(f*x+e)^3+192*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-192*B*cos(f*x+e)*
ln(csc(f*x+e)-cot(f*x+e)-1)+192*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+9
6*B*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-96*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+4
8*B*cos(f*x+e)^3+96*A*ln(2/(1+cos(f*x+e)))-3*B*cos(f*x+e)^5+4*A*cos(f*x+e)
^3*sin(f*x+e)+16*B*cos(f*x+e)^3*sin(f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(cos(
f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.165.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

3.165.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(215) = 430$.

Time = 0.78 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^3\sqrt{c}\operatorname{sgn}(\cos(-\right.\right.}{c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
2/3*sqrt(2)*sqrt(a)*(6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-
cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) + (3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*
A*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(2)*B*a^3*c^(7/2)*cos
(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*A*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*
x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 3*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sq
rt(2)*A*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*B*a^3*c^(7/
2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^4)/f
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

3.166.1 Optimal result 1444
3.166.2 Mathematica [A] (verified) 1445
3.166.3 Rubi [A] (verified) 1445
3.166.4 Maple [A] (verified) 1449
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3.166.6 Sympy [F(-1)] 1450
3.166.7 Maxima [F] 1450
3.166.8 Giac [A] (verification not implemented) 1450
3.166.9 Mupad [F(-1)] 1451

3.166.1 Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}$$
$$+\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a
^2*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+
1/6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/
2)+4*a^4*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/
(c-c*sin(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c
/f/(c-c*sin(f*x+e))^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 12.86 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.08

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+f}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-13
2*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A
*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e
+ f*x)]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x]
)*Sqrt[c - c*Sin[e + f*x]])
```

3.166.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a}{3f\sqrt{c-}}\right.}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a}{3f\sqrt{c-}}\right.}{2c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos}{2c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a}{2f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\c}{2c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((3*A + 5*B)*(-1/3*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c
- c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
 (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))
)/(2*c)
```

3.166.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.166.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

method	result
default	$-\frac{a^3\sec(fx+e)\left(-2B(\sin^2(fx+e))(\cos^2(fx+e))+3(\sin^3(fx+e))A+13B(\sin^3(fx+e))+27(\sin^2(fx+e))A+144A\sin(fx+e)\ln(\csc$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\sin(fx+e)-48\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+9(\cos^2(fx+e)\right.}{2fc\sqrt{-c(\sin(fx+e)-1)}}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6*a^3/c/f*sec(f*x+e)*(-2*B*sin(f*x+e)^2*cos(f*x+e)^2+3*sin(f*x+e)^3*A+1
3*B*sin(f*x+e)^3+27*sin(f*x+e)^2*A+144*A*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+
e)-1)-72*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+59*B*sin(f*x+e)^2+240*B*sin(f*x
+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-120*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-78*A
*sin(f*x+e)-144*A*ln(csc(f*x+e)-cot(f*x+e)-1)+72*A*ln(2/(1+cos(f*x+e)))-12
0*B*sin(f*x+e)-240*B*ln(csc(f*x+e)-cot(f*x+e)-1)+120*B*ln(2/(1+cos(f*x+e))
))*(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.166.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2
*sin(f*x + e) - 2*c^2), x)
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.166.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(3/2), x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{6\left(3\sqrt{2}Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5\sqrt{2}Ba^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{6\left(\sqrt{2}A\right.}{}\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-1/3*sqrt(2)*sqrt(a)*(6*(3*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 5*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*
log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))) - 6*(sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/((cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^2 - 1)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt
(2)*(4*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 3*A*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a^3*c^(9/2)*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c^(9/2)*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*B*a^3*c^(9/2)
*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(c^
6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

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3.167.1 Optimal result

Integrand size = 40, antiderivative size = 263

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}$$
$$- \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}}$$
$$- \frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$- \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}}$$
$$- \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(5/2)-1/2*a
*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)-3/4*
a^2*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
-6*a^4*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c
-c*sin(f*x+e))^(1/2)-3*a^3*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f
/(c-c*sin(f*x+e))^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 12.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
 + B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(
e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 -
4*(A + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

3.167.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{3a\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{3a\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+f}{2f\sqrt{c-c\sin(e+}}\right.}{c}\right.}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{3a\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+f}{2f\sqrt{c-c\sin(e+}}\right.}{c}\right.}{2c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{3a\left(2a\left(-\frac{2a^2\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+}{2}\right.}{c}\right.}{2c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{3a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right) - \frac{a\cos(e+fx)(a\sin(e+f}{2f\sqrt{c-c\sin(e+}}\right.}{c}\right.}{2c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

3.167. $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - ((A + 3*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c -
 c*Sin[e + f*x])^(3/2)) - (3*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*
Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c))
/(2*c)
```

3.167.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.167.4 Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.54

method	result
default	$-\frac{a^3\sec(fx+e)\left(B(\sin^2(fx+e))(\cos^2(fx+e))+24A(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-12A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right.\right.$
parts	$-\frac{A\sec(fx+e)\left((\cos^2(fx+e))\sin(fx+e)+12(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)-6(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+24\ln(\right.$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2*a^3/c^2/f*sec(f*x+e)*(B*sin(f*x+e)^2*cos(f*x+e)^2+24*A*cos(f*x+e)^2*l
n(csc(f*x+e)-cot(f*x+e)-1)-12*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-2*sin(f*
x+e)^3*A+72*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-36*B*cos(f*x+e)^2*l
n(2/(1+cos(f*x+e)))-10*B*sin(f*x+e)^3+20*sin(f*x+e)^2*A+48*A*sin(f*x+e)*ln
(csc(f*x+e)-cot(f*x+e)-1)-24*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+54*B*sin(f*
x+e)^2+144*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-72*B*sin(f*x+e)*ln(2/(
1+cos(f*x+e)))-10*A*sin(f*x+e)-48*A*ln(csc(f*x+e)-cot(f*x+e)-1)+24*A*ln(2/
(1+cos(f*x+e)))-36*B*sin(f*x+e)-144*B*ln(csc(f*x+e)-cot(f*x+e)-1)+72*B*ln(
2/(1+cos(f*x+e))))*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(-c*(sin(f*x+e)
-1))^(1/2)
```

3.167.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c
^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.167.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(5/2), x)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{6\sqrt{2}(Aa^3\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+3Ba^3\sqrt{c}\operatorname{sgn}(\cos(-\frac{1}{4}\pi}{c^3\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx}\right.$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(a)*(6*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-cos(-1
/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
+ 2*(sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*A*a^3*
c^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*B*a^3*c^(7/2)*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)))/c^6 + (5*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 9*sqrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) - 2*(3*sqrt(2)*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*s
qrt(2)*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^3*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

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3.168.1 Optimal result

Integrand size = 40, antiderivative size = 264

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}$$
$$- \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}}$$
$$+ \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}}$$
$$+ \frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+ \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*
a*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4
*a^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2
)+a^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/
f/(c-c*sin(f*x+e))^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 12.49 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx))$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A
 + B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

3.168.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{(A+7B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}\right)}{6c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}\right)}{6c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}\right)}{6c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}\right)}{6c}$$

$$\downarrow 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx - \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+}}{f\sqrt{c-c\sin(e+fx}}\right.}{c}\right.}{2c}\right.}{6c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e+}}{f\sqrt{c-c\sin(e+fx}}\right.}{c}\right.}{2c}\right.}{6c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+\right.}{c}\right.}{2c}\right.$$

$$\frac{}{6c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} -$$

$$(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a\sin(e}\right.}{c}\right.}{2c}\right.$$

$$\frac{}{6c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) - ((A + 7*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c
 - c*Sin[e + f*x])^(5/2)) - (3*a*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/
2))/(f*(c - c*Sin[e + f*x])^(3/2)) - (2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*
Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c))/
(2*c)))/(6*c)
```

3.168.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(236) = 472.

Time = 4.48 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.06

method	result
default	$\frac{a^3 \sec(fx+e)\left(3A\sin(fx+e)(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-6A\sin(fx+e)(\cos^2(fx+e))\ln(\csc(fx+e)-\cot(fx+e)-1)+21B\sin(fx\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(6(\cos^2(fx+e))\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-3(\cos^2(fx+e))\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-18(\cos^2(fx-\right.}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/3*a^3/c^3/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))
-6*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+21*B*sin(f*x+e)*c
os(f*x+e)^2*ln(2/(1+cos(f*x+e)))-42*B*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e
)-cot(f*x+e)-1)-3*B*sin(f*x+e)^2*cos(f*x+e)^2-9*A*cos(f*x+e)^2*ln(2/(1+cos
(f*x+e)))+18*A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-8*sin(f*x+e)^3*A-6
3*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+126*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot
(f*x+e)-1)-41*B*sin(f*x+e)^3-12*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+24*A*sin
(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+6*sin(f*x+e)^2*A-84*B*sin(f*x+e)*ln(2/
(1+cos(f*x+e)))+168*B*sin(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+54*B*sin(f*x+
e)^2+12*A*ln(2/(1+cos(f*x+e)))-24*A*ln(csc(f*x+e)-cot(f*x+e)-1)-6*A*sin(f*
x+e)+84*B*ln(2/(1+cos(f*x+e)))-168*B*ln(csc(f*x+e)-cot(f*x+e)-1)-21*B*sin(
f*x+e))*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/(-c*(sin(f*
x+e)-1))^(1/2)
```

3.168.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*
cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(236) = 472$.

Time $= 0.36$ (sec) , antiderivative size $= 749$, normalized size of antiderivative $= 2.84$

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
-1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*
a^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/
2)*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2
)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^
(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(
7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7
/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x
 + e)/(cos(f*x + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 22*a^(7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2
)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 15*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*si...
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{12\sqrt{2}Ba^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\sqrt{2}\left(Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+7Ba^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx\right.\right.\right.}{c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-1/12*sqrt(2)*(12*sqrt(2)*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))/(c^(7/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) +
 6*sqrt(2)*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a^3*sq
rt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-192*cos(-1/4*pi + 1/2*f*x
+ 1/2*e)^2 + 192)/(c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(11*
A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1
/4*pi + 1/2*f*x + 1/2*e)^4 - 3*(9*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 31*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*c^4*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(a)/f
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

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3.169.1 Optimal result

Integrand size = 40, antiderivative size = 247

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} - \frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output

```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*
cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)
*ln(1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 12.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\left(6(A+B)-4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output
```
((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
 + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^7*(c - c*Sin[e + f*x])^(9/2))
```

3.169.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{7/2}}dx}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{7/2}}dx}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}\right)}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}\right)}{c}$$

$$\downarrow 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}\right)}{c}$$

$$\downarrow 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) - (B*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin
[e + f*x])^(7/2)) - (a*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(
c - c*Sin[e + f*x])^(5/2)) - (a*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(f*(c - c*Sin[e + f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]
])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c))/c))/c
```

3.169.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.169.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.66

method	result
default	$-\frac{a^3\sec(fx+e)\left(-3B\left(\cos^4(fx+e)\right)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+6B\left(\cos^4(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)+8B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+\right.\right.$
parts	$-\frac{A\tan(fx+e)a^3\left(\cos^2(fx+e)-2\right)\sqrt{a(1+\sin(fx+e))}}{f((\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)+4)\sqrt{-c(\sin(fx+e)-1)}\,c^4}+\frac{B\sec(fx+e)\left(3\left(\cos^4(fx+e)\right)\ln\left(\frac{2}{1+\cos(f\right.\right.}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/3*a^3/c^4/f*sec(f*x+e)*(-3*B*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))+6*B*cos(
f*x+e)^4*ln(csc(f*x+e)-cot(f*x+e)-1)+8*B*sin(f*x+e)^2*cos(f*x+e)^2-12*B*si
n(f*x+e)*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+24*B*sin(f*x+e)*cos(f*x+e)^2*ln
(csc(f*x+e)-cot(f*x+e)-1)+3*A*sin(f*x+e)*cos(f*x+e)^2+11*B*sin(f*x+e)^3+24
*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-48*B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f
*x+e)-1)-20*B*sin(f*x+e)^2+24*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*B*sin(f
*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-6*A*sin(f*x+e)+3*B*sin(f*x+e)-24*B*ln(2/
(1+cos(f*x+e)))+48*B*ln(csc(f*x+e)-cot(f*x+e)-1))*(a*(1+sin(f*x+e)))^(1/2)
/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/(-c*(sin(f*x+e)-1
))^(1/2)
```

3.169.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c
^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 +
 16*c^5)*sin(f*x + e)), x)
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

3.169.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{24\sqrt{2}Ba^3\log\left(-2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.\right.\right.}{c^{\frac{9}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(a)*(24*sqrt(2)*B*a^3*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^2 + 2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c^(9/2)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))) - sqrt(2)*(12*(A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 7*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*p
i + 1/2*f*x + 1/2*e)^6 - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - 47*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 18*(A*a^3*sqrt
(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 4*(3*A*a^3*sqrt(c
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 41*B*a^3*sqrt(c)*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((cos(-1/4*pi + 1/2*
f*x + 1/2*e)^2 - 1)^4*c^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(9/2), x)
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

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3.170.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

output

```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/8
0*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

3.170.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 434 vs. 2(96) = 192.

Time = 17.07 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.52

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin$$
$$+\frac{(-3A-5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{2(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
(8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11
/2)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e
 + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c -
c*Sin[e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^7*(c - c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^7*(c - c*Sin[e + f*x])^(11/2))
```

3.170.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A-9B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{(A-9B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\downarrow 3221$$

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}} + \frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(
c - c*Sin[e + f*x])^(9/2))
```

3.170.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(84) = 168$.

Time = 4.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.98

method	result
default	$\frac{a^3 \tan(fx+e)\left(A\left(\cos^4(fx+e)\right)-B\left(\sin^2(fx+e)\right)\left(\cos^2(fx+e)\right)+5A\sin(fx+e)\left(\cos^2(fx+e)\right)+5B\left(\sin^3(fx+e)\right)-17A\left(\cos^2(fx+e)\right)+\right.}{10c^5 f\left(\cos^4(fx+e)+4\left(\cos^2(fx+e)\right)\sin(fx+e)-8\left(\cos^2(fx+e)\right)-8\sin(fx+e)+8\right.}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3\left(\left(\cos^3(fx+e)\right)\sin(fx+e)-5\left(\cos^3(fx+e)\right)-17\cos(fx+e)\sin(fx+e)+15\cos(fx+e)+26\tan(fx+e)-10\sec(f\right.}{10f\left(\cos^4(fx+e)+4\left(\cos^2(fx+e)\right)\sin(fx+e)-8\left(\cos^2(fx+e)\right)-8\sin(fx+e)+8\right)\sqrt{-c(\sin(fx+e)-1)}\,c^5}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/10*a^3/c^5/f*tan(f*x+e)*(A*cos(f*x+e)^4-B*sin(f*x+e)^2*cos(f*x+e)^2+5*A*
sin(f*x+e)*cos(f*x+e)^2+5*B*sin(f*x+e)^3-17*A*cos(f*x+e)^2+6*B*sin(f*x+e)^
2-10*A*sin(f*x+e)+5*B*sin(f*x+e)+26*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e
)^4+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-8*sin(f*x+e)+8)/(-c*(sin(f*x+
e)-1))^(1/2)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time $= 0.30$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 2.07$

$$\int \frac{(a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{11/2}}\,dx = \frac{(10\,Ba^3\cos(fx+e)^4 - 5\,(A+7\,B)a^3\cos(fx+e)^2 + 2}{10\,\left(5\,c^6 f\cos(fx+e)^5 - 20\,c^6 f\cos(fx+e\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A +
13*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
- 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

3.170.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(84) = 168$.

Time $= 0.42$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 3.55$

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(40\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
1/80*(40*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 10*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 90*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^3*sqrt(c)*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*B*a^3*sqr
t(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 5*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) - 45*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 9*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((cos(
-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^5*c^6*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)))
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{11/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(11/2), x)
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

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3.171.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

output

```
1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/6
0*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/
480*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

3.171.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. $2(146) = 292$.

Time = 17.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.03

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin$$
$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{3(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(13/2),x]
```

output
```
(4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13
/2)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[
e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(
a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

3.171.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3042$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3222$$

$$\frac{(A-5B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3042$$

$$\frac{(A-5B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-5B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(13/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x
])^(13/2)) + ((A - 5*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(
c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8
0*c*f*(c - c*Sin[e + f*x])^(9/2))))/(6*c)
```

3.171.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.171.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.41

method	result
default	$\frac{a^3 \tan(fx+e)(3A(\cos^4(fx+e))\sin(fx+e)-18A(\cos^4(fx+e))-51A\sin(fx+e)(\cos^2(fx+e))-15B(\sin^3(fx+e))+106A(\cos^2(fx+e)}{30c^6 f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(\cos^2(fx+e}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3(3(\cos^5(fx+e))+18(\cos^3(fx+e))\sin(fx+e)-54(\cos^3(fx+e))-106\cos(fx+e)\sin(fx+e)+129\cos(fx+e)}{30f((\cos^4(fx+e))\sin(fx+e)-5(\cos^4(fx+e))-12(\cos^2(fx+e))\sin(fx+e)+20(\cos^2(fx+e))+16\sin(fx+e)-16)\sqrt{-}}$

input

```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,meth
od=_RETURNVERBOSE)
```

output

```
1/30*a^3/c^6/f*tan(f*x+e)*(3*A*cos(f*x+e)^4*sin(f*x+e)-18*A*cos(f*x+e)^4-5
1*A*sin(f*x+e)*cos(f*x+e)^2-15*B*sin(f*x+e)^3+106*A*cos(f*x+e)^2-10*B*sin(
f*x+e)^2+78*A*sin(f*x+e)-15*B*sin(f*x+e)-118*A)*(a*(1+sin(f*x+e)))^(1/2)/(
cos(f*x+e)^4*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)+20*cos(f
*x+e)^2+16*sin(f*x+e)-16)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.47

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(15\,Ba^3\cos(fx+e)^4-15\,(A+3\,B)a^3\cos(fx+e)^2+6\,(3\,A+5\,B)a^3-2\left(5\,(A+B)a^3\cos(fx+e)^2-\right.\right.}{30\left(c^7 f\cos(fx+e)^7-18\,c^7 f\cos(fx+e)^5+48\,c^7 f\cos(fx+e)^3-32\,c^7 f\cos(fx+e)+2\left(3\,c^7 f\cos\right.\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="fricas")
```

output
```
-1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A
+ 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e
)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x
 + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos
(f*x + e))*sin(f*x + e))
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2
),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(13/2), x)
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(128) = 256$.

Time = 0.55 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.34

$$\int \frac{(a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(60\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 20\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 s\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="giac")
```

output
```
-1/480*(60*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 20*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 100*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*
x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^3*sqrt(c)*cos(-1
/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 75*B*a^3*
sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 6*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) - 30*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 5*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((c
os(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^6*c^7*f*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)))
```

3.171.9 Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

$$\int \frac{(a + a\sin(e+fx))^{7/2}(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\sqrt{c - c\sin(e+fx)}\left(\frac{56\,a^3\,e^{e\,7i+f\,x\,7i}\,(4A+5B)\,\sqrt{a+a\sin(e+fx)}}{5\,c^7\,f} + \frac{a^3\,e^{e\,7i+f\,x\,7i}\sin(3e+3fx)\,(A\,1i+B\,1i)\,\sqrt{a+a\sin(e+fx)}\,32i}{3\,c^7\,f} - \right.}{-858\cos(e+fx)\,e^{e\,7i+f\,x\,7i} + 858\,e^{e\,7i+f\,x\,7i}\cos(3e+3fx) - 130\,e^{e\,7i+f\,x\,7i}\cos(5e+5fx) +}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(13/2),x)
```

output

```
-((c - c*sin(e + f*x))^(1/2)*((56*a^3*exp(e*7i + f*x*7i)*(4*A + 5*B)*(a +
a*sin(e + f*x))^(1/2))/(5*c^7*f) + (a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x
)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*c^7*f) - (32*a^3*exp(e*
7i + f*x*7i)*cos(2*e + 2*f*x)*(A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(c^7*f
) + (8*B*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2
))/(c^7*f) - (a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(A*13i + B*5i)*(a + a*si
n(e + f*x))^(1/2)*32i)/(5*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x
) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e +
 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*
sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i +
 f*x*7i)*sin(6*e + 6*f*x))
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

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3.172.1 Optimal result

Integrand size = 40, antiderivative size = 202

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}$$

output

```
1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/1
68*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2
)+1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e)
)^(11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*si
n(f*x+e))^(9/2)
```

3.172.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. $2(202) = 404$.

Time = 17.20 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.19

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin$$
$$-\frac{2(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{6(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(15/2),x]
```

output
```
(8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15
/2)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[
e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(
a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

3.172.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\downarrow 3451$$

$$\frac{(3A-11B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-11B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3222$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3222$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\Bigg)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\Bigg)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{(3A-11B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}\right)}{14c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(15/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x
])^(15/2)) + ((3*A - 11*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*
f*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
)/(10*f*(c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^
(7/2))/(80*c*f*(c - c*Sin[e + f*x])^(9/2)))/(6*c)))/(14*c)
```

3.172.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.172.4 Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.40

method	result
default	$\frac{a^3\tan(fx+e)\left(39A\left(\cos^6(fx+e)\right)+3B\left(\cos^4(fx+e)\right)\left(\sin^2(fx+e)\right)+273A\left(\cos^4(fx+e)\right)\sin(fx+e)+21B\left(\cos^2(fx+e)\right)\left(\sin^3(fx+e)\right)-\right.}{420c^7 f\left(\cos^6(fx+e)+6\left(\cos^4(fx+e)\right.\right.}$
parts	$\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3\left(13\left(\cos^5(fx+e)\right)\sin(fx+e)-91\left(\cos^5(fx+e)\right)-312\left(\cos^3(fx+e)\right)\sin(fx+e)+728\left(\cos^3(fx+e)\right)+1075\cos(fx\right.}{140f\left(\cos^6(fx+e)+6\left(\cos^4(fx+e)\right)\sin(fx+e)-18\left(\cos^4(fx+e)\right)-32\left(\cos^2(fx+e)\right)\sin(fx+e)+48\left(\cos^2(fx+e)\right)\right.}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/420*a^3/c^7/f*tan(f*x+e)*(39*A*cos(f*x+e)^6+3*B*cos(f*x+e)^4*sin(f*x+e)^
2+273*A*cos(f*x+e)^4*sin(f*x+e)+21*B*cos(f*x+e)^2*sin(f*x+e)^3-936*A*cos(f
*x+e)^4-69*B*sin(f*x+e)^2*cos(f*x+e)^2-1911*A*sin(f*x+e)*cos(f*x+e)^2-266*
B*sin(f*x+e)^3+3225*A*cos(f*x+e)^2-4*B*sin(f*x+e)^2+2268*A*sin(f*x+e)-210*
B*sin(f*x+e)-2748*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^6+6*cos(f*x+e)^4
*sin(f*x+e)-18*cos(f*x+e)^4-32*cos(f*x+e)^2*sin(f*x+e)+48*cos(f*x+e)^2+32*
sin(f*x+e)-32)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx =$$
$$-\frac{\left(140\,Ba^3\cos\left(fx+e\right)^4-7\left(27\,A+61\,B\right)a^3\cos\left(fx+e\right)^2+4\left(57\,A+71\,B\right)a^3-7\left(5\left(3\,A+5\,B\right)a^3\cos\right.\right.}{420\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(f\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="fricas")
```

output
```
-1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*
(57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^
3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8
*f*cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 6
4*c^8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 8
0*c^8*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2
),x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(15/2), x)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.69

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{\left(280\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="giac")
```

output
```
1/6720*(280*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 105*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 385*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 63*A*a^3*sqrt(c)*cos(
-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 231*B*a
^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 21*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) - 77*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 11*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt
(a)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^7*c^8*f*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)))
```

3.172.9 Mupad [B] (verification not implemented)

Time = 25.38 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.09

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(15/2),x)
```

output

```
-((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*
((B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a +
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(
3*c^8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*
11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1
i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 4
1*B)*16i)/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*
f) + (a^3*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i +
 f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))
^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)
*32i)/(35*c^8*f)))/(exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e
*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2
002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*143
0i + 2002*exp(e*10i + f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e...
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

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3.173.1 Optimal result

Integrand size = 40, antiderivative size = 246

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}$$

output

```
1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/5
6*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/
224*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2
)+1/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^
(11/2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x
+e))^(9/2)
```

3.173.2 Mathematica [A] (verified)

Time = 17.21 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin($$
$$f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+$$
$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(17/2),x]
```

output
```
((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
 - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 +
 Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*S
in[e + f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
9*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^7*(c - c*Sin[e + f*x])^(17/2))
```

3.173.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{3\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{3\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3222$$

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3042$$

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}+$$

$$\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{3\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}\right)}{14c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(17/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x
])^(17/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(
c - c*Sin[e + f*x])^(15/2)) + (3*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
)/(12*f*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])
^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^(9/2)))/(6*c)))/(14*c)))/(4*c)
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.173.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.26

method	result
default	$\frac{a^3\tan(fx+e)(12A\sin(fx+e)(\cos^6(fx+e))-B(\cos^2(fx+e))(\sin^5(fx+e))-96A(\cos^6(fx+e))+8B(\cos^2(fx+e))(\sin^4(fx+e))-372}{140c^8f((\cos^6(fx+e))\sin(fx+e)-7(\cos^6(fx+e)}$
parts	$-\frac{A\sqrt{a(1+\sin(fx+e))}\,a^3(3(\cos^7(fx+e))+24(\cos^5(fx+e))\sin(fx+e)-96(\cos^5(fx+e))-240(\cos^3(fx+e))\sin(fx+e)+480(\cos^3(f}{35f((\cos^6(fx+e))\sin(fx+e)-7(\cos^6(fx+e))-24(\cos^4(fx+e))\sin(fx+e)+56(\cos^4(fx+e))+80(\cos^2(fx+e))\sin(}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/140*a^3/c^8/f*tan(f*x+e)*(12*A*sin(f*x+e)*cos(f*x+e)^6-B*cos(f*x+e)^2*si
n(f*x+e)^5-96*A*cos(f*x+e)^6+8*B*cos(f*x+e)^2*sin(f*x+e)^4-372*A*cos(f*x+e
)^4*sin(f*x+e)+29*B*sin(f*x+e)^5+960*A*cos(f*x+e)^4-64*sin(f*x+e)^4*B+1548
*A*sin(f*x+e)*cos(f*x+e)^2+105*B*sin(f*x+e)^3-2332*A*cos(f*x+e)^2-1468*A*s
in(f*x+e)+70*B*sin(f*x+e)+1608*A)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^6*s
in(f*x+e)-7*cos(f*x+e)^6-24*cos(f*x+e)^4*sin(f*x+e)+56*cos(f*x+e)^4+80*cos
(f*x+e)^2*sin(f*x+e)-112*cos(f*x+e)^2-64*sin(f*x+e)+64)/(-c*(sin(f*x+e)-1)
)^(1/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(35\,Ba^3\cos(fx+e)^4 - 56\,(A+2\,B)}{140\,(c^9 f\cos(fx+e)^9 - 32\,c^9 f\cos(fx+e)^7 + 160\,c^9 f\,c}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="fricas")
```

output
```
1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
 + 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*
x + e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*
cos(f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9
*f*cos(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f
*x + e))
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2
),x)
```

output
```
Timed out
```

3.173.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="maxima")
```

output
```
Timed out
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx =$$
$$-\frac{\left(140\,Ba^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+56\,Aa^3\sqrt{c}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="giac")
```

output
```
-1/8960*(140*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 56*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 168*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 28*A*a^3*sqrt(c)*cos(
-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 84*B*a^
3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 8*A*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) - 24*B*a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 3*B*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(
(cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^8*c^9*f*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)))
```

3.173.9 Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.42

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(17/2),x)
```

output
```
((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f)
- (64*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f
*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(
1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(
5*c^9*f) - (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/
2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*ex
p(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(
a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*
13i + B*10i))/(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f
)))/(exp(e*1i + f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*
544i + 1700*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i
+ f*x*6i) - exp(e*7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(
e*10i + f*x*10i) + exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12...
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.174.1 Optimal result 1513
3.174.2 Mathematica [A] (verified) 1513
3.174.3 Rubi [A] (verified) 1514
3.174.4 Maple [B] (verified) 1517
3.174.5 Fricas [F] 1518
3.174.6 Sympy [F(-1)] 1518
3.174.7 Maxima [F] 1519
3.174.8 Giac [A] (verification not implemented) 1519
3.174.9 Mupad [F(-1)] 1520

3.174.1 Optimal result

Integrand size = 40, antiderivative size = 197

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

output

```
1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3
*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+4*(A-B)*c^3*
cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
)+2*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 11.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\left(3(A-3B)\cos(2(e+fx))\right.}{12f\left(\cos\left(\frac{1}{2}(\right.\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin
[e + f*x]],x]
```

output
```
-1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqr
t[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
+ (36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

3.174.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^{5/2}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^{5/2}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3452$$

$$(A-B)\int \frac{(c - c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(c - c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3219$$

$$(A-B)\left(2c\int \frac{(c - c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx + \frac{c\cos(e+fx)(c - c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right) - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\left(2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3219$$

$$(A-B)\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3216$$

$$(A-B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e}{2f\sqrt{a\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e}{2f\sqrt{a\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3146$$

$$(A-B)\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-}{2f\sqrt{a\sin(e}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 16$$

$$(A - B)\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin($$
$$\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/3*(B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[e + f*x
]]) + (A - B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a
*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))
```

3.174.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(177) = 354$.

Time = 4.17 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.51

method	result
default	$\frac{c^2\left(15A\sin(fx+e)+15A-17B+2B\left(\cos^3(fx+e)\right)\sin(fx+e)-3A\cos(fx+e)-17B\sin(fx+e)+18A\sin(fx+e)\cos(fx+e)-26B\cos(f\right.}{}$
parts	$\frac{A\left(\left(\cos^2(fx+e)\right)\sin(fx+e)+\cos^3(fx+e)+6\cos(fx+e)\sin(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-16\ln(-\cot(fx+e)+\csc(fx+e)+\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/6*c^2/f*(15*A*sin(f*x+e)+15*A-17*B-7*B*cos(f*x+e)^2*sin(f*x+e)-3*A*cos(f
*x+e)-17*B*sin(f*x+e)+19*B*cos(f*x+e)^2-15*A*cos(f*x+e)^2+3*A*sin(f*x+e)*c
os(f*x+e)^2+18*A*sin(f*x+e)*cos(f*x+e)-26*B*cos(f*x+e)*sin(f*x+e)+24*A*cos
(f*x+e)*ln(2/(1+cos(f*x+e)))+24*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*B*cos(
f*x+e)^4-24*B*ln(2/(1+cos(f*x+e)))+9*cos(f*x+e)*B+3*A*cos(f*x+e)^3-24*B*co
s(f*x+e)*ln(2/(1+cos(f*x+e)))-24*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-48*A*co
s(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*cos(f*x+e)*ln(-cot(f*x+e)+csc(f
*x+e)+1)-48*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*sin(f*x+e)*ln(-
cot(f*x+e)+csc(f*x+e)+1)-9*B*cos(f*x+e)^3+24*A*ln(2/(1+cos(f*x+e)))+2*B*co
s(f*x+e)^3*sin(f*x+e)-48*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+48*B*ln(-cot(f*x+e
)+csc(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(
1+sin(f*x+e)))^(1/2)
```

3.174.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
 e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(
f*x + e) + a), x)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
+ e) + a), x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.53

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}(A\sqrt{a}c^2\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-B\sqrt{a}c^2\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\log\left(-2\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+2\right)}{a\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}\left(4Ba^{\frac{5}{2}}c^2\mathrm{sg}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-1/3*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-2*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 + 2)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) -
 sqrt(2)*(4*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^6 - 3*A*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 3*B*a^(5/2)*c^2*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 6*A*a^(5/2)*c^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 6*B*a^(5/2)*
c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/
(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.175.1 Optimal result 1521
3.175.2 Mathematica [A] (verified) 1521
3.175.3 Rubi [A] (verified) 1522
3.175.4 Maple [B] (verified) 1525
3.175.5 Fricas [F] 1525
3.175.6 Sympy [F] 1526
3.175.7 Maxima [F] 1526
3.175.8 Giac [A] (verification not implemented) 1526
3.175.9 Mupad [F(-1)] 1527

3.175.1 Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/2*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+2*(A-B)*
c^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)+(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 6.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\left(B\cos(2(e+fx))-4\left(4(-A\right.\right.}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1-}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin
[e + f*x]],x]
```

output
```
-1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
 c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

3.175.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3452, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c - c\sin(e+fx))^{3/2}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3452$$

$$(A-B)\int \frac{(c - c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(c - c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3219$$

$$(A-B)\left(2c\int \frac{\sqrt{c - c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx + \frac{c\cos(e+fx)\sqrt{c - c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right) - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\left(2c\int \frac{\sqrt{c - c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx + \frac{c\cos(e+fx)\sqrt{c - c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right) - \frac{B\cos(e+fx)(c - c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3216$$

$$(A-B)\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3146$$

$$(A-B)\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 16$$

$$(A-B)\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/2*(B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e + f*x
]]) + (A - B)*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e
 + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

3.175.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(132) = 264$.

Time = 4.08 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.92

method	result
default	$\frac{c\left(B\left(\cos^3(fx+e)\right)+B\left(\cos^2(fx+e)\right)\sin(fx+e)+2A\left(\cos^2(fx+e)\right)-2A\sin(fx+e)\cos(fx+e)+8A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx}{}$
parts	$\frac{A\left(2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos(fx+e)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-4\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-4\ln(-\cot(fx+e)+\right.}{f(-\cos(fx-}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2*c/f*(B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2-2*A*sin
(f*x+e)*cos(f*x+e)+8*A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*cos(f*x
+e)*ln(2/(1+cos(f*x+e)))+8*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*s
in(f*x+e)*ln(2/(1+cos(f*x+e)))-3*B*cos(f*x+e)^2+4*B*cos(f*x+e)*sin(f*x+e)-
8*B*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*B*cos(f*x+e)*ln(2/(1+cos(f*x
+e)))-8*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*B*sin(f*x+e)*ln(2/(1+c
os(f*x+e)))-2*A*sin(f*x+e)+8*A*ln(-cot(f*x+e)+csc(f*x+e)+1)-4*A*ln(2/(1+co
s(f*x+e)))-cos(f*x+e)*B+3*B*sin(f*x+e)-8*B*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*
B*ln(2/(1+cos(f*x+e)))-2*A+3*B)*(-c*(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)-sin(
f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)
```

3.175.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c
*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

3.175.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{\sqrt{a\,(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e
 + f*x) + 1)), x)
```

3.175.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
+ e) + a), x)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{\sqrt{2}(A\sqrt{ac}\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-B\sqrt{ac}\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\log\left(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1\right)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}Ba^{\frac{3}{2}}c\operatorname{sgn}(\cos(-\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-sqrt(2)*sqrt(c)*(sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
 - B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-1/4*pi + 1/2
*f*x + 1/2*e)^2 + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (sqrt(2)*B*
a^(3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(3/2)*c*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^2 + sqrt(2)*B*a^(3/2)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/
a^2)/f
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.176.1 Optimal result 1528
3.176.2 Mathematica [C] (verified) 1528
3.176.3 Rubi [A] (verified) 1529
3.176.4 Maple [B] (verified) 1531
3.176.5 Fricas [F] 1532
3.176.6 Sympy [F] 1532
3.176.7 Maxima [A] (verification not implemented) 1533
3.176.8 Giac [A] (verification not implemented) 1533
3.176.9 Mupad [F(-1)] 1534

3.176.1 Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
(A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

3.176.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\left(-ifx+2\log\left(i+e^{i(e+fx)}\right)\right)+B\sin(e+fx)\right)\sqrt{c-c\sin(}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
 + f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I
*(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

3.176.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3450$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx + \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c - c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx + \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c - c\sin(e+fx)}dx}{a}$$

$$\downarrow 3216$$

$$\frac{ac(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c - c\sin(e+fx)}} + \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c - c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{ac(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c - c\sin(e+fx)}} + \frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c - c\sin(e+fx)}dx}{a}$$

$$\downarrow 3146$$

$$\frac{c(A-B)\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a} d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}}{a}$$

$$\downarrow 16$$

$$\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a} + \frac{c(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3217$$

$$\frac{c(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
f*Sqrt[a + a*Sin[e + f*x]])
```

3.176.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(88) = 176$.

Time = 3.39 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.19

method	result
parts	$-\frac{A\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)+\sin(fx+e)+1)}{f(-\cos(fx+e)+\sin(fx+e)-1)\sqrt{a(1+\sin(fx+e))}}+\frac{B\left(2\cos(fx+e)\ln($
default	$\frac{\left(A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+A\cos(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2A\cos(fx+e)\ln(-\right.$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-A/f*(2*ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)
-1))^(1/2)*(cos(f*x+e)+sin(f*x+e)+1)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+sin(
f*x+e)))^(1/2)+B/f*(2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*ln(-cot(f*
x+e)+csc(f*x+e)+1)*sin(f*x+e)-ln(2/(1+cos(f*x+e)))*cos(f*x+e)-ln(2/(1+cos(
f*x+e)))*sin(f*x+e)+cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+2*ln(-cot(f*x+e)+cs
c(f*x+e)+1)-ln(2/(1+cos(f*x+e)))-sin(f*x+e)-1)*(-c*(sin(f*x+e)-1))^(1/2)/(
-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)
```

3.176.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e
) + a), x)
```

3.176.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e +
f*x) + 1)), x)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.83

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}} - \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(a+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) - A\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
(B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*si
n(f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)
)) - A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(
c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{2\sqrt{2}B\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-B\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right.}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/2*sqrt(2)*(2*sqrt(2)*B*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^2/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2
)*(A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)))*log(-4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 4)/(a*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

3.177.1 Optimal result 1535
3.177.2 Mathematica [A] (verified) 1535
3.177.3 Rubi [A] (verified) 1536
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3.177.8 Giac [F(-2)] 1539
3.177.9 Mupad [F(-1)] 1540

3.177.1 Optimal result

Integrand size = 40, antiderivative size = 113

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)\cos(e+fx)\log(1+\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)+1/2*(A-B)*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2
)/(c-c*sin(f*x+e))^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\left(B\log(\cos(e+fx)) + A\left(\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]]),x]
```

output
```
-((Cos[e + f*x]*(B*Log[Cos[e + f*x]] + A*(Log[1 - Tan[(e + f*x)/2]] - Log[
1 + Tan[(e + f*x)/2]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e +
f*x]]))
```

3.177.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3042, 3448, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3448$$

$$\frac{(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{2a} + \frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{2c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{2a} + \frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{2c}$$

$$\downarrow 3216$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{c(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} + \frac{c(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$\frac{(A-B)\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$\frac{(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]),x]
```

output
```
((A - B)*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) - ((A + B)*Cos[e + f*x]*Log[c - c*Sin[e + f*x
]])/(2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

3.177.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3448

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp
[(A*b + a*B)/(2*a*b)   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]
], x], x] + Simp[(B*c + A*d)/(2*c*d)   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a
+ b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.177.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(A\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\ln(\csc(fx+e)-\cot(fx+e)-1)-B\ln(-\cot(fx+e)+\csc(fx+e)+1)+B\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B\ln\right.}{f\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}}$
parts	$\frac{A\cos(fx+e)(\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln(\csc(fx+e)-\cot(fx+e)-1))}{f\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}+\frac{B\left(-\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln(\csc(fx+e)\right.}{f\sqrt{a(1+\sin(fx+e)}}$

input

```
int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/f*(A*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*ln(csc(f*x+e)-cot(f*x+e)-1)-B*ln(-co
t(f*x+e)+csc(f*x+e)+1)+B*ln(2/(1+cos(f*x+e)))-B*ln(csc(f*x+e)-cot(f*x+e)-1
))*cos(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

3.177.5 Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a*c*cos(f*x + e)^2), x)
```

3.177.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A + B\sin(e+fx)}{\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
 f*x) - 1))), x)
```

3.177.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)), x)
```

3.177.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{\sqrt{a+a\,\sin(e+f\,x)}\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

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3.178.1 Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*(
A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(A+B+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)),x]
```

output
```
((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*
x])]*(c - c*Sin[e + f*x])^(3/2))
```

3.178.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3220$$

$$\frac{(A-B)\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 4257$$

$$\frac{(A-B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2)),x]
```

output
```
((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

3.178.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.178.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(91) = 182.

Time = 3.12 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.51

method	result
default	$\frac{A\sin(fx+e)\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\ln(\csc(fx+e)-\cot(fx+e)-1)\sin(fx+e)\cos(fx+e)-A(\cos^2(fx+e))\ln($
parts	$\frac{A((\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-\cos(fx+e)\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-(\cos^2(fx+e))\ln(\csc(fx+$

input
```
int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/c/f*(A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*ln(csc(f*x
+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(
f*x+e)+1)+A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*sin(f*x+e)*cos(f*x+
e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*c
os(f*x+e)+B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*cos(f*x+e)^2*ln(cs
c(f*x+e)-cot(f*x+e)-1)-A*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^2-A*cos(f*x+e)
*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-B*c
os(f*x+e)*sin(f*x+e)+B*cos(f*x+e)^2+B*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)
+1)-B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-A*sin(f*x+e)-B*sin(f*x+e)-A-B
)/(-cos(f*x+e)+sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^
(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.27

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \left[-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx\right.$$

$$\left.-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right.}{2\left(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(fx+e)\right.}\right]$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
[-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c
)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*
x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*s
in(f*x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))
 + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*co
s(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

3.178.6 Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin(e+fx)}{\sqrt{a\left(\sin(e+fx)+1\right)}\left(-c\left(\sin(e+fx)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x
) - 1))**(3/2)), x)
```

3.178.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2)), x)
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.98

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A\sqrt{a}-B\sqrt{a})\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\,(A\sqrt{a}}{ac^{\frac{3}{2}}\operatorname{sgn}(\cos(-}$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/4*((A*sqrt(a) - B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a
*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) - 2*(A*sqrt(a) - B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))/(a*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))) + (A*sqrt(a) + B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
- 1)*a*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))))/f
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{\sqrt{a+a\,\sin(e+f\,x)}\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

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3.179.1 Optimal result

Integrand size = 40, antiderivative size = 153

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{4c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(
A-B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B
)*arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.45

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(A+B+(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right.$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(5/2)),x]
```

output
```
((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x]
)^(5/2))
```

3.179.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c} + \frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c} + \frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{(A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c} + \frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c} + \frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3220$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 4257$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2)),x]
```

output
```
((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(5/2)) + ((A - B)*(Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/(2*c)
```

3.179.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.179.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(135) = 270.

Time = 3.64 (sec) , antiderivative size = 773, normalized size of antiderivative = 5.05

method	result	size
default	Expression too large to display	773
parts	Expression too large to display	815

input
```
int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/4/c^2/f*(-A*sin(f*x+e)-A-B-B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(
f*x+e)+1)+2*A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*A*ln(cs
c(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)-2*B*sin(f*x+e)*cos(f*x+e)*ln(
-cot(f*x+e)+csc(f*x+e)+1)+2*B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f
*x+e)+2*A*cos(f*x+e)-B*sin(f*x+e)+B*cos(f*x+e)^2+A*cos(f*x+e)^2+2*A*cos(f*
x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*sin(f*x+e)*cos(f*x+e)^2-3*A*sin(f*x+e
)*cos(f*x+e)-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x
+e)+csc(f*x+e)+1)-B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-A*cos(f*x+e)^
3*ln(csc(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e)^3*ln(csc(f*x+e)-cot(f*x+e)-1)+A
*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*cos(f*x+e)^3+B*sin(f*x+e)*co
s(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*
x+e)-1)-2*A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*B*cos(f*x+e)*ln(-cot
(f*x+e)+csc(f*x+e)+1)-A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1
)+B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)^3*ln(-cot(f*x+e
)+csc(f*x+e)+1)-B*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^2
*ln(-cot(f*x+e)+csc(f*x+e)+1))/(cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)-cos(f*x
+e)+2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.77

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{\left((A-B)\cos(fx+e)^3+2\,(A-B)\cos(fx+e)\sin(\right.\right.$$

$$\left.\left.-\frac{\left((A-B)\cos(fx+e)^3+2\,(A-B)\cos(fx+e)\sin(fx+e)-2\,(A-B)\cos(fx+e)\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-}\right.}{4\left(ac^3f\cos(fx+e)^3+2\,ac^3f\cos(fx+\right.}\right]$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
[-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(
A - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
e) + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos
(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x +
 e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt
(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)/(a*c*cos(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*
c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

3.179.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A + B\sin(e+fx)}{\sqrt{a\left(\sin(e+fx)+1\right)}\left(-c\left(\sin(e+fx)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x
) - 1))**(5/2)), x)
```

3.179.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2)), x)
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{2\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}}{16\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-1/16*(2*(A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1
)/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))) - 4*(A*sqrt(a) - B*sqrt(a))*log(abs(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))) + (2*(A*sqrt(a) - B*sqrt(a))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 + A*sqrt(a) + B*sqrt(a))/(a*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4))/
f
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.180.1 Optimal result 1554
3.180.2 Mathematica [A] (verified) 1555
3.180.3 Rubi [A] (verified) 1555
3.180.4 Maple [A] (verified) 1559
3.180.5 Fricas [F] 1559
3.180.6 Sympy [F(-1)] 1560
3.180.7 Maxima [F] 1560
3.180.8 Giac [A] (verification not implemented) 1561
3.180.9 Mupad [F(-1)] 1561

3.180.1 Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*
(3*A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)
-1/6*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1
/2)-4*(3*A-5*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)
/(c-c*sin(f*x+e))^(1/2)-2*(3*A-5*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/
a/f/(a+a*sin(f*x+e))^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 12.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(132A-45B+2(27A-59B)\cos(2(e+fx))-\right.$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 57
6*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*
(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]
))^(3/2))
```

3.180.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\downarrow 3451$$
$$-\frac{(3A-5B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}\,dx}{2a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-5B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(3A-5B)\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-5B)\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(3A-5B)\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-5B)\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-}{3f\sqrt{a\sin}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-}{3f\sqrt{a\sin}}\right.}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}}^{2a}$$

$$\downarrow 3216$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+c\cos}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}}^{2a}$$

$$\downarrow 3042$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+c\cos}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}}^{2a}$$

$$\downarrow 3146$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}}^{2a}$$

$$\downarrow 16$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+c\cos}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}}^{2a}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((3*A - 5*B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(
3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*
Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))))/(2
*a)
```

3.180.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.180.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

method	result
default	$-\frac{c^3\sec(fx+e)\left(-2B(\sin^2(fx+e))(\cos^2(fx+e))+3(\sin^3(fx+e))A-13B(\sin^3(fx+e))-27(\sin^2(fx+e))A+144A\sin(fx+e)\ln(-c\right.$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\sin(fx+e)-48\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-9(\cos^2(fx+\right.}{2f\sqrt{a(1+\sin(fx+e))\,a}}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output

```
-1/6*c^3/a/f*sec(f*x+e)*(-2*B*sin(f*x+e)^2*cos(f*x+e)^2+3*sin(f*x+e)^3*A-1
3*B*sin(f*x+e)^3-27*sin(f*x+e)^2*A+144*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x
+e)+1)-72*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+59*B*sin(f*x+e)^2-240*B*sin(f*
x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+120*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-78
*A*sin(f*x+e)+144*A*ln(-cot(f*x+e)+csc(f*x+e)+1)-72*A*ln(2/(1+cos(f*x+e)))
+120*B*sin(f*x+e)-240*B*ln(-cot(f*x+e)+csc(f*x+e)+1)+120*B*ln(2/(1+cos(f*x
+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

3.180.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*
sin(f*x + e) - 2*a^2), x)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
 + a)^(3/2), x)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.45

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{6\left(3\sqrt{2}A\sqrt{a}c^3\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-5\sqrt{2}B\sqrt{a}c^3\mathrm{sgn}(\sin(-\right.\right.}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
1/3*sqrt(2)*sqrt(c)*(6*(3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 5*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*l
og(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))) - 6*(sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/((sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(
2)*(4*B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^6 - 3*A*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^4 + 9*B*a^(9/2)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 12*A*a^(9/2)*c^3*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 24*B*a^(9/2)*c^3*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^6
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.181.1 Optimal result . . . 1562
3.181.2 Mathematica [A] (verified) . . . 1563
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3.181.1 Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*
(A-2*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-4*(
A-2*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)-2*(A-2*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*si
n(f*x+e))^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(28A-16B+2(2A-7B)\cos(2(e+fx))+64\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
28*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*
A + 31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e +
f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(3/2))
```

3.181.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\downarrow 3451$$
$$-\frac{(A-2B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$
$$\downarrow 3042$$

$$-\frac{(A-2B)\int\frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-2B)\left(2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-2B)\left(2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-2B)\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-2B)\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3216$$

$$-\frac{(A-2B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-2B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3146$$

$$-\frac{(A-2B)\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 16$$

$$-\frac{(A-2B)\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((A - 2*B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*
x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*
x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/a
```

3.181.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.181.4 Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

method	result
default	$\frac{c^2\sec(fx+e)\left(B(\sin^3(fx+e))+2(\sin^2(fx+e))A-16A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+8A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)-\right.$
parts	$-\frac{A\sec(fx+e)\left(8\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+\cos^2(fx+e)+8\ln(-\cot(fx+e)+\csc(f\right.}{fa\sqrt{a(1+\sin(fx+e))}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2*c^2/a/f*sec(f*x+e)*(B*sin(f*x+e)^3+2*sin(f*x+e)^2*A-16*A*sin(f*x+e)*ln
(-cot(f*x+e)+csc(f*x+e)+1)+8*A*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-7*B*sin(f*x
+e)^2+32*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-16*B*sin(f*x+e)*ln(2/(1
+cos(f*x+e)))+10*A*sin(f*x+e)-16*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+8*A*ln(2/(
1+cos(f*x+e)))-16*B*sin(f*x+e)+32*B*ln(-cot(f*x+e)+csc(f*x+e)+1)-16*B*ln(2
/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e)))^(1/2)
```

3.181.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
 + a)^(3/2), x)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(B\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+A\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}(\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-2*(B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + A*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) - 2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*l
og(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/co
s(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.182.1 Optimal result 1570
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3.182.1 Optimal result

Integrand size = 40, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-(A-3
*B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f
*x+e))^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 11.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(4A-3B-B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{2f\left(\cos\left(\right.\right.\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*
A - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
] - 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.182.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$
$$\downarrow 3451$$
$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$
$$\downarrow 3042$$

$$-\frac{(A-3B)\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-3B)\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3216$$

$$-\frac{(A-3B)\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3146$$

$$-\frac{(A-3B)\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 16$$

$$-\frac{(A-3B)\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((A - 3*B)*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[
c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/(2*a)
```

3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.182.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.50

method	result
default	$-\frac{c\sec(fx+e)\left(2A\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+B\left(\sin^2(fx+e)\right)-6B\sin(fx+e)\ln(-\right.}{}$
parts	$-\frac{A\sec(fx+e)\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+2\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\right.\right.}{fa\sqrt{a(1+\sin(fx+e))}}$

input

```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output

```
-c/a/f*sec(f*x+e)*(2*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*sin(f*x+e
)*ln(2/(1+cos(f*x+e)))+B*sin(f*x+e)^2-6*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*
x+e)+1)+3*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))-2*A*sin(f*x+e)+2*A*ln(-cot(f*x
+e)+csc(f*x+e)+1)-A*ln(2/(1+cos(f*x+e)))+3*B*sin(f*x+e)-6*B*ln(-cot(f*x+e)
+csc(f*x+e)+1)+3*B*ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+s
in(f*x+e)))^(1/2)
```

3.182.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*s
in(f*x + e) - 2*a^2), x)
```

3.182.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f
*x) + 1))**(3/2), x)
```

3.182.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(143) = 286$.

Time $= 0.32$ (sec) , antiderivative size $= 367$, normalized size of antiderivative $= 2.31$

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{B\left(\frac{6c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{3c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{2\left(\frac{3c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(
f*x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(
3/2) - 4*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))
/f
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.42

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left(\frac{2\sqrt{2}Bc\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\sqrt{a}c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right.}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqr
t(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-8*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 +
 8)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a)*c*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))))*sqrt(c)/f
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.183.1 Optimal result 1578
3.183.2 Mathematica [C] (verified) 1578
3.183.3 Rubi [A] (verified) 1579
3.183.4 Maple [A] (verified) 1581
3.183.5 Fricas [F] 1582
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3.183.7 Maxima [F] 1582
3.183.8 Giac [A] (verification not implemented) 1583
3.183.9 Mupad [F(-1)] 1583

3.183.1 Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+B*c*co
s(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
)
```

3.183.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B -
I*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e +
 f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + S
in[e + f*x]))^(3/2))
```

3.183.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\downarrow 3450$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\downarrow 3216$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{Bc\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{Bc\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{Bc\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 16$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx + \frac{Bc\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3217$$

$$\frac{Bc\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(
3/2),x]
```

output
```
-(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
+ f*x]])) + (B*c*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

3.183.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.183.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sec(fx+e)\left(2B\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-B\sin(fx+e)\ln\left(\frac{2}{1+\cos(fx+e)}\right)+A\sin(fx+e)-B\sin(fx+e)+2B\ln(-\cot$ $af\sqrt{a(1+\sin(fx+e))}$
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}}{fa\sqrt{a(1+\sin(fx+e))}}+\frac{B\sec(fx+e)\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+2\ln\right.}{fa\sqrt{a(1+\sin(fx$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/a/f*sec(f*x+e)*(2*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))+A*sin(f*x+e)-B*sin(f*x+e)+2*B*ln(-cot(f*x+e)+csc(f*x
+e)+1)-B*ln(2/(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+sin(f*x+e))
)^(1/2)
```

3.183.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

3.183.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{-c\left(\sin(e+fx)-1\right)}(A+B\sin(e+fx))}{\left(a\left(\sin(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
 + 1))**(3/2), x)
```

3.183.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
 a)^(3/2), x)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(4\,B\sqrt{a}\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-\frac{A\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-B\sqrt{a}\operatorname{sgn}(s}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}}{2\,a^2 f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-1/2*(4*B*sqrt(a)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) - (A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*
sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2)*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

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3.184.2 Mathematica [A] (verified) 1584
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3.184.1 Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*
(A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+s\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

3.184.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3220$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 4257$$

$$\frac{(A+B)\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

3.184.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.184.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(91) = 182.

Time = 2.68 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.47

method	result
default	$\frac{A(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+A\sin(fx+e)\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A(\cos^2(fx+e))\ln(\csc(f$
parts	$\frac{A((\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+\cos(fx+e)\sin(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-(\cos^2(fx+e))\ln(\csc(fx+$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/a/f*(A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*sin(f*x+e)*cos(f*x+
e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)
-A*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2*ln(-co
t(f*x+e)+csc(f*x+e)+1)+B*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1
)-B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*ln(csc(f*x+e)-cot(f*x+e)-1)
*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x
+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+A*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)+B*c
os(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+
e)-1)-B*cos(f*x+e)^2-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)-B*sin(f*x+e)-A+B
)/(cos(f*x+e)+sin(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(
1/2)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.19

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+\right.$$

$$\left.-\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right.}{2\left(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx+e)\right.}\right]$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
[1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x
 + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*si
n(f*x + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))
+ sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos
(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

3.184.6 Sympy [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{A + B\sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}\sqrt{-c(\sin(e + fx) - 1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(
e + f*x) - 1))), x)
```

3.184.7 Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{B\sin(fx + e) + A}{(a\sin(fx + e) + a)^{\frac{3}{2}}\sqrt{-c\sin(fx + e) + c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c)), x)
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(91) = 182$.

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\log\left(-512\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+512\right)}{a^{\frac{3}{2}}\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2}{a^{\frac{3}{2}}\sqrt{c}\operatorname{sgn}(\text{co}}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/4*((A + B)*log(-512*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 512)/(a^(3/2)*sqr
t(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) - 2*(A + B)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(3/2)*sqrt(c)*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) +
 (A*sqrt(a) - B*sqrt(a))/(a^2*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

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3.185.3 Rubi [A] (verified) 1591
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3.185.8 Giac [F(-2)] 1595
3.185.9 Mupad [F(-1)] 1596

3.185.1 Optimal result

Integrand size = 40, antiderivative size = 150

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{A\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output

```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*
A*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*A*arcta
nh(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

3.185.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\cos(e+fx)\left(2B - A\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right) + A\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right) + A\cos(2(e+fx))\left(-\log\right.\right.}{4cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)),x]
```

output
```
-1/4*(Cos[e + f*x]*(2*B - A*Log[1 - Tan[(e + f*x)/2]] + A*Log[1 + Tan[(e +
 f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[1 - Tan[(e + f*x)/2]] + Log[1 + Tan[(
e + f*x)/2]]) + 2*A*Sin[e + f*x]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e
+ f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

3.185.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx$$
$$\downarrow 3451$$
$$\frac{A\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$
$$\downarrow 3042$$
$$\frac{A\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3222$$

$$\frac{A\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3220$$

$$\frac{A\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 4257$$

$$\frac{A\left(\frac{\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)) + (A*(Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/a
```

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.185.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

method	result
default	$\frac{A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+B\tan(fx+e)\sin(fx+e)+\tan(fx+e)A}{2acf\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}$
parts	$\frac{A(-\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)+\tan(fx+e))}{2fa\sqrt{a(1+\sin(fx+e))}\,c\sqrt{-c(\sin(fx+e)-1)}}+\frac{B\sin(fx+e)}{2f\sqrt{a(1+\sin(fx+e))}\sqrt{}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/a/c/f/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)*(A*cos(f*x+e)
*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+B*t
an(f*x+e)*sin(f*x+e)+tan(f*x+e)*A)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{ac}A\cos(fx+e)^3\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\sqrt{}}{}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
[1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x
+ e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(
f*x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*
A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*
c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

3.185.6 Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e +
f*x) - 1))**(3/2)), x)
```

3.185.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2)), x)
```

3.185.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

3.186.1 Optimal result 1597
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3.186.1 Optimal result

Integrand size = 40, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx &= \\ -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} &\\ +\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} &\\ +\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} &\\ +\frac{(3A-B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8ac^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} &\end{aligned}$$

output

```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+1/8*
(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(
3*A-B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*
(3*A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.41

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right) +}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*
x])^(5/2))
```

3.186.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3451$$

$$\frac{(3A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3220$$

$$\frac{(3A-B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A-B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2c}-\frac{2a}{(A-B)\cos(e+fx)}$$
$$\frac{}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 4257$$

$$\frac{(3A-B)\left(\frac{\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2c}-\frac{2a}{(A-B)\cos(e+fx)}$$
$$\frac{}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)) + ((3*A - B)*(Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/(2*c)))/(2*a)
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.186.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sec(fx+e)\left(3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)-3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1\right.}{}$
parts	$\frac{A\sec(fx+e)\left(3\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)+2\right.}{8fc^2a(\sin(fx+e)-1)\sqrt{-c}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/8/a/c^2/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x
+e)-1)-3*A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-B*sin(f*x+
e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+B*sin(f*x+e)*cos(f*x+e)^2*ln(-
cot(f*x+e)+csc(f*x+e)+1)-2*sin(f*x+e)^3*A-3*A*cos(f*x+e)^2*ln(csc(f*x+e)-c
ot(f*x+e)-1)+3*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*B*sin(f*x+e)^
3+B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*cos(f*x+e)^2*ln(-cot(f*x+e)
+csc(f*x+e)+1)-sin(f*x+e)^2*A+3*B*sin(f*x+e)^2+5*A*sin(f*x+e)+B*sin(f*x+e)
)/(sin(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.01

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{((3\,A-B)\cos(fx+e)^3\sin(fx+e) - (3\,A-B)\,c\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
[-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)
*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)
^3) + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*
sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*
sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x +
 e))) + ((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*
sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)]
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2)), x)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.29

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{2\,(3\,A\sqrt{a}-B\sqrt{a})\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2c^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} - \frac{4\,(3$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/32*(2*(3*A*sqrt(a) - B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 +
1)/(a^2*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))) - 4*(3*A*sqrt(a) - B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)))/(a^2*c^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a) - B*sqrt(a))*cos(-1/4*pi + 1/2*f
*x + 1/2*e)^4 - 3*(3*A*sqrt(a) - B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 2*A*sqrt(a) - 2*B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*
a^2*c^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.187.1 Optimal result 1605
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3.187.1 Optimal result

Integrand size = 40, antiderivative size = 323

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/
2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)+(3
*A-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
+1/3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e)
)^(1/2)+8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))
^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^
(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 14.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{9}}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2)*(-96*(A
 - B) + 192*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 3*(A - 7*
B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 192*(3*A - 7
*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^4 - 3*(28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e
 + f*x] - B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[3*(e + f*x)]))/(12
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.187.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3451$$

$$-\frac{(3A-7B)\int \frac{(c-c\sin(e+fx))^{9/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\int \frac{(c-c\sin(e+fx))^{9/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3218$$

$$-\frac{(3A-7B)\left(-\frac{4c\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\left(-\frac{4c\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3219$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3219$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e}{f(a\sin(e+fx)+a}$$

$$-\frac{}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e}{f(a\sin(e+fx)+a}\right)}{4a}}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3219$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}}{4a}}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}}{4a}}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3216$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c}{3f\sqrt{a\sin(}}\right)}{a}}{4a}}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3042$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c}{3f\sqrt{a\sin(}}\right)}{a}}{4a}}{\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

3.187. $\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}dx$

$$\downarrow 3146$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e}{}}{a}\right.}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 16$$

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin}{}}{a}\right.}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*A - 7*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/
(f*(a + a*Sin[e + f*x])^(3/2))) - (4*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((c*Cos[e + f*x]*(c - c*Sin
[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x
]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])))))/a))/(4*a)
```

3.187.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.187.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.38

method	result
default	$-\frac{c^4 \sec(fx+e)\left(2B(\cos^2(fx+e))(\sin^3(fx+e))+3A(\cos^2(fx+e))(\sin^2(fx+e))-17B(\sin^2(fx+e))(\cos^2(fx+e))-144A(\cos^2(fx+$
parts	$\frac{A\sec(fx+e)\left(\cos^4(fx+e)+12(\cos^2(fx+e))\sin(fx+e)-96(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+48(\cos^2(fx+e))\ln\left(\frac{}{1+co}\right.\right.$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6*c^4/a^2/f*sec(f*x+e)*(2*B*cos(f*x+e)^2*sin(f*x+e)^3+3*A*cos(f*x+e)^2*
sin(f*x+e)^2-17*B*sin(f*x+e)^2*cos(f*x+e)^2-144*A*cos(f*x+e)^2*ln(2/(1+cos
(f*x+e)))+288*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+36*sin(f*x+e)^3*
A+336*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-672*B*cos(f*x+e)^2*ln(-cot(f*x+e
)+csc(f*x+e)+1)-106*B*sin(f*x+e)^3+222*sin(f*x+e)^2*A+288*A*sin(f*x+e)*ln(
2/(1+cos(f*x+e)))-576*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-490*B*sin(
f*x+e)^2-672*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+1344*B*sin(f*x+e)*ln(-cot(f
*x+e)+csc(f*x+e)+1)+138*A*sin(f*x+e)+288*A*ln(2/(1+cos(f*x+e)))-576*A*ln(-
cot(f*x+e)+csc(f*x+e)+1)-336*B*sin(f*x+e)-672*B*ln(2/(1+cos(f*x+e)))+1344*
B*ln(-cot(f*x+e)+csc(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(
a*(1+sin(f*x+e)))^(1/2)
```

3.187.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2
 + 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2
 - 8*(A - B)*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e)), x)
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.187.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{12\left(3\sqrt{2}A\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\sqrt{2}B\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{3\sqrt{2}\left(7\right.}{}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
-1/3*sqrt(2)*sqrt(c)*(12*(3*sqrt(2)*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - 7*sqrt(2)*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))) + 3*sqrt(2)*(7*A*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) - 11*B*sqrt(a)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*(2*A*sqrt(a)
*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c^4*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)
*(4*B*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^6 - 3*A*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^4 + 15*B*a^(13/2)*c^4*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 18*A*a^(13/2)*c^4*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 54*B*a^(13/2)*
c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/
(a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{9/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.188.1 Optimal result 1614
3.188.2 Mathematica [A] (verified) 1615
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3.188.1 Optimal result

Integrand size = 40, antiderivative size = 263

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)
-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3/4*
(A-3*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
+6*(A-3*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c
-c*sin(f*x+e))^(1/2)+3*(A-3*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
/(a+a*sin(f*x+e))^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 12.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{7/}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A
 - B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e
 + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4
*(A - 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.188.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3451$$

$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{2a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3218$$

$$-\frac{(A-3B)\left(-\frac{3c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(-\frac{3c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{\frac{2a}{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3216$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-cs}{f(a\sin(e+fx}\right)}{\frac{2a}{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3042$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-cs}{f(a\sin(e+fx}\right)}{\frac{2a}{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3146$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+f}{f(a}\right)}{\frac{2a}{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 16$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin}{f(a\sin(e+fx)-}\right)}{\frac{2a}{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}}$$

3.188. $\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}dx$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((A - 3*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f
*(a + a*Sin[e + f*x])^(3/2))) - (3*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a +
a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (
c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/a
))/(2*a)
```

3.188.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.188.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.55

method	result
default	$-\frac{c^3\sec(fx+e)\left(-B(\sin^2(fx+e))(\cos^2(fx+e))+2(\sin^3(fx+e))A-12A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+24A(\cos^2(fx+e))\ln(-\cos$
parts	$\frac{A\sec(fx+e)\left(6(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-12(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+(\cos^2(fx+e))\sin(fx+e)-12\ln\left($

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2*c^3/a^2/f*sec(f*x+e)*(-B*sin(f*x+e)^2*cos(f*x+e)^2+2*sin(f*x+e)^3*A-1
2*A*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+24*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc
(f*x+e)+1)-10*B*sin(f*x+e)^3+36*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))-72*B*c
os(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+20*sin(f*x+e)^2*A+24*A*sin(f*x+e)
*ln(2/(1+cos(f*x+e)))-48*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-54*B*si
n(f*x+e)^2-72*B*sin(f*x+e)*ln(2/(1+cos(f*x+e)))+144*B*sin(f*x+e)*ln(-cot(f
*x+e)+csc(f*x+e)+1)+10*A*sin(f*x+e)+24*A*ln(2/(1+cos(f*x+e)))-48*A*ln(-cot
(f*x+e)+csc(f*x+e)+1)-36*B*sin(f*x+e)-72*B*ln(2/(1+cos(f*x+e)))+144*B*ln(-
cot(f*x+e)+csc(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+s
in(f*x+e)))^(1/2)
```

3.188.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^
3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{c}\left(\frac{6\sqrt{2}(A\sqrt{a}c^3\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-3B\sqrt{a}c^3\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\log\left(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1\right)}{a^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{2\left(\sqrt{2}Ba^{\frac{7}{2}}c^3\mathrm{sg}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(c)*(6*sqrt(2)*(A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 3*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-sin(-
1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
 - 2*(sqrt(2)*B*a^(7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - sqrt(2)*A*a^(
7/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 5*sqrt(2)*B*a^(7/2)*c^3*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2)/a^6 + (5*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 9*sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*(3*sqrt(2)*A*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*
sqrt(2)*B*sqrt(a)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.189.1 Optimal result . . . 1623
3.189.2 Mathematica [A] (verified) . . . 1623
3.189.3 Rubi [A] (verified) . . . 1624
3.189.4 Maple [A] (verified) . . . 1628
3.189.5 Fricas [F] . . . 1628
3.189.6 Sympy [F(-1)] . . . 1629
3.189.7 Maxima [B] (verification not implemented) . . . 1629
3.189.8 Giac [A] (verification not implemented) . . . 1630
3.189.9 Mupad [F(-1)] . . . 1630

3.189.1 Optimal result

Integrand size = 40, antiderivative size = 211

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
1/4*(A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)
-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+(A-5
*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+1/2*(A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a
*sin(f*x+e))^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{5}}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A +
2*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.189.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3451$$

$$-\frac{(A-5B)\int \frac{(c-c\sin(e+fx))^{5/2}}{(\sin(e+fx)a+a)^{3/2}}\,dx}{4a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-5B)\int \frac{(c-c\sin(e+fx))^{5/2}}{(\sin(e+fx)a+a)^{3/2}}\,dx}{4a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3218$$

$$-\frac{(A-5B)\left(-\frac{2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}\,dx}{a} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-5B)\left(-\frac{2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3219$$

$$-\frac{(A-5B)\left(-\frac{2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-5B)\left(-\frac{2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3216$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3146$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 16$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((A - 5*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f
*(a + a*Sin[e + f*x])^(3/2))) - (2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e
+ f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e
+ f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/a))/(4*a)
```

3.189.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.189.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.74

method	result
default	$-\frac{c^2\sec(fx+e)\left(2A(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-A(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-B(\sin^3(fx+e))-10B(\cos^2(f$
parts	$\frac{A\sec(fx+e)\left((\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+4\ln($ $f(1+\sin(fx+e$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-c^2/a^2/f*sec(f*x+e)*(2*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos
(f*x+e)^2*ln(2/(1+cos(f*x+e)))-B*sin(f*x+e)^3-10*B*cos(f*x+e)^2*ln(-cot(f*
x+e)+csc(f*x+e)+1)+5*B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*sin(f*x+e)^2*A-
4*A*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*A*sin(f*x+e)*ln(2/(1+cos(f*x
+e)))-8*B*sin(f*x+e)^2+20*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-10*B*s
in(f*x+e)*ln(2/(1+cos(f*x+e)))-4*A*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*A*ln(2/(
1+cos(f*x+e)))-5*B*sin(f*x+e)+20*B*ln(-cot(f*x+e)+csc(f*x+e)+1)-10*B*ln(2/
(1+cos(f*x+e))))*(-c*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)
))^(1/2)
```

3.189.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*
sin(f*x + e)), x)
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(189) = 378.

Time = 0.47 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\frac{8\sqrt{a}c^{\frac{5}{2}}\sin(fx+e)^2}{\left(a^3+\frac{4a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x +
 e) + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) +
 c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c
^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/
2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.41

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{4\sqrt{2}Bc^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\sqrt{2}(}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/4*sqrt(2)*(4*sqrt(2)*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^2/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*
sqrt(2)*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*sqrt(a)*c
^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*log(-32*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^2 + 32)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt
(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*sqrt(a)*c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 4*(A*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) - 2*B*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))))*sqrt(c)/f
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{5/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.190.1 Optimal result 1631
3.190.2 Mathematica [A] (verified) 1631
3.190.3 Rubi [A] (verified) 1632
3.190.4 Maple [A] (verified) 1635
3.190.5 Fricas [F] 1635
3.190.6 Sympy [F] 1636
3.190.7 Maxima [F] 1636
3.190.8 Giac [A] (verification not implemented) 1636
3.190.9 Mupad [F(-1)] 1637

3.190.1 Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output
```
-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)-B*c^
2*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-B*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

3.190.2 Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2
*(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
a*(1 + Sin[e + f*x]))^(5/2))
```

3.190.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{B\int \frac{(c-c\sin(e+fx))^{3/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{B\int \frac{(c-c\sin(e+fx))^{3/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3218$$

$$\frac{B\left(-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a} - \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{B\left(-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a} - \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3216$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3146$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 16$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input

```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output

```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) + (B*(-((c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c
*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])^(3/2))))/a
```

3.190.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.190.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.44

method	result
default	$\frac{c\sec(fx+e)\left(-B(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(fx+e)}\right)+2B(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)+2B(\sin^2(fx+e))+2B\sin(fx+$
parts	$\frac{A\tan(fx+e)\sqrt{-c(\sin(fx+e)-1)}\,c}{f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2}+\frac{B\sec(fx+e)\left(2(\cos^2(fx+e))\ln(-\cot(fx+e)+\csc(fx+e)+1)-(\cos^2(fx+e))\ln\left(\frac{2}{1+\cos(f$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
c/a^2/f*sec(f*x+e)*(-B*cos(f*x+e)^2*ln(2/(1+cos(f*x+e)))+2*B*cos(f*x+e)^2*
ln(-cot(f*x+e)+csc(f*x+e)+1)+2*B*sin(f*x+e)^2+2*B*sin(f*x+e)*ln(2/(1+cos(f
*x+e)))-4*B*sin(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*sin(f*x+e)+B*sin(f*x
+e)+2*B*ln(2/(1+cos(f*x+e)))-4*B*ln(-cot(f*x+e)+csc(f*x+e)+1))*(-c*(sin(f*
x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

3.190.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3
 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

3.190.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\,(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f
*x) + 1))**(5/2), x)
```

3.190.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.38

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{4\sqrt{2}Bc\log\left(-2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.\right.\right.}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/8*sqrt(2)*sqrt(c)*(4*sqrt(2)*B*c*log(-2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
 + 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))) - sqrt(2)*(A*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 5*B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*(A*sqrt(a)*c*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sqrt(a)*c*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/((sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.191.1 Optimal result 1638
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3.191.1 Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-B*
c*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{a(1+\sin(e+fx))}(A+B+2B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{2a^3 f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^(5/2),x]
```

output
```
-1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin
[e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5)
```

3.191.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{c - c\sin(e+fx)}(A + B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3450$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{5/2}}dx + \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{\sqrt{c - c\sin(e+fx)}}{(\sin(e+fx)a+a)^{5/2}}dx + \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}$$

$$\downarrow 3217$$

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/2*((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin
[e + f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c -
c*Sin[e + f*x]])
```

3.191.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.191.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\tan(fx+e)(A\sin(fx+e)+B\sin(fx+e)+2A)\sqrt{-c(\sin(fx+e)-1)}}{2a^2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}}$	70
parts	$-\frac{A\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-2\tan(fx+e)-\sec(fx+e))}{2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2} - \frac{B\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)-\sec(fx+e))}{2f(1+\sin(fx+e))\sqrt{a(1+\sin(fx+e))}\,a^2}$	128

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/a^2/f*tan(f*x+e)*(A*sin(f*x+e)+B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(
1/2)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(2\,B\sin(fx+e)+A+B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin}}{2\left(a^3 f\cos(fx+e)^3-2\,a^3 f\cos(fx+e)\sin(fx+e)-2\,a\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*
f*cos(f*x + e))
```

3.191.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sin(e+fx)-1\right)}(A+B\sin(e+fx))}{\left(a\left(\sin(e+fx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
 + 1))**(5/2), x)
```

3.191.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
 a)^(5/2), x)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(4\,B\sqrt{a}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\right.\right.\right.}{8\,a^3 f\cos\left(-\frac{1}{4}\,\pi+\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
1/8*(4*B*sqrt(a)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + A*sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*sqrt(a)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^3*f*cos(-1/4*pi + 1/2*f*x + 
1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

3.191.9 Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\sqrt{-c\,(\sin(e+fx)-1)}\left(A\sin(2e+2fx)+3B\sin(2e+2fx)-2A\left(2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)-3B\left(\right.\right.}{a^2 f\sqrt{a\,(\sin(e+fx)+1)}\,\left(-8\sin(e+fx)^2+4\sin(e+fx)+2\sin(2e+2fx\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
-(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*
x) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + 
 B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2
)*(4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + 
f*x)^2 + 8))
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

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3.192.1 Optimal result

Integrand size = 40, antiderivative size = 151

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output

```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*
(A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*(A+
B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right) + s}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
+ f*x]])
```

3.192.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3222$$

$$\frac{(A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3220$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 4257$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]) + ((A + B)*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt
[c - c*Sin[e + f*x]]) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/(2*a)
```

3.192.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.192.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(133) = 266$.

Time = 2.94 (sec) , antiderivative size = 772, normalized size of antiderivative = 5.11

method	result	size
default	Expression too large to display	772
parts	Expression too large to display	816

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/4/a^2/f*(A*sin(f*x+e)-A+B+B*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(
f*x+e)+1)+2*A*sin(f*x+e)*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*A*ln(cs
c(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f*x+e)+2*B*sin(f*x+e)*cos(f*x+e)*ln(
-cot(f*x+e)+csc(f*x+e)+1)-2*B*ln(csc(f*x+e)-cot(f*x+e)-1)*sin(f*x+e)*cos(f
*x+e)+2*A*cos(f*x+e)-B*sin(f*x+e)-B*cos(f*x+e)^2+A*cos(f*x+e)^2-2*A*cos(f*
x+e)*ln(csc(f*x+e)-cot(f*x+e)-1)+2*A*sin(f*x+e)*cos(f*x+e)^2+3*A*sin(f*x+e
)*cos(f*x+e)-B*cos(f*x+e)*sin(f*x+e)+A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x
+e)+csc(f*x+e)+1)-B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)+A*cos(f*x+e)^
3*ln(csc(f*x+e)-cot(f*x+e)-1)+B*cos(f*x+e)^3*ln(csc(f*x+e)-cot(f*x+e)-1)-A
*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*A*cos(f*x+e)^3-B*sin(f*x+e)*co
s(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-2*B*cos(f*x+e)*ln(csc(f*x+e)-cot(f*
x+e)-1)+2*A*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+2*B*cos(f*x+e)*ln(-cot
(f*x+e)+csc(f*x+e)+1)-A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1
)+B*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-A*cos(f*x+e)^3*ln(-cot(f*x+e
)+csc(f*x+e)+1)-B*cos(f*x+e)^3*ln(-cot(f*x+e)+csc(f*x+e)+1)+A*cos(f*x+e)^2
*ln(-cot(f*x+e)+csc(f*x+e)+1))/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)-cos(f*x
+e)-2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{((A+B)\cos(fx+e)^3-2(A+B)\cos(fx+e)\sin(fx}{}\right.$$

$$-\frac{((A+B)\cos(fx+e)^3-2(A+B)\cos(fx+e)\sin(fx+e)-2(A+B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-}}{}\right.}{4\left(a^3cf\cos(fx+e)^3-2a^3cf\cos(fx+\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
[1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 + B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
 + e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(
f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x +
e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(
-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
/(a*c*cos(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3
*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

3.192.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A + B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(
e + f*x) - 1))), x)
```

3.192.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c)), x)
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.40

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\,(A+B)\log\left(-4096\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+4096\right)}{a^{\frac{5}{2}}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4}{a^{\frac{5}{2}}\sqrt{c}\,\mathrm{sgn}(\cos($$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
1/16*(2*(A + B)*log(-4096*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 4096)/(a^(5/2
)*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 
1/2*e))) - 4*(A + B)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(5/2)*sqr
t(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))) + (2*(A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + A*sqrt
(a) - B*sqrt(a))/(a^3*sqrt(c)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

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3.193.1 Optimal result

Integrand size = 40, antiderivative size = 208

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx &= \\ -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} &\\ -\frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} &\\ +\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} &\\ +\frac{(3A+B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} &\end{aligned}$$

output

```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-1/8*
(3*A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/8*(
3*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*
(3*A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right) +}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2
 + (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*
x])^(3/2))
```

3.193.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3451$$

$$\frac{(3A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx}{4a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3222$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3222$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3220$$

$$\frac{(3A+B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{4a(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 4257$$

$$\frac{(3A+B)\left(\frac{\cos(e+fx)\operatorname{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{4a(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) + ((3*A + B)*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*
(c - c*Sin[e + f*x])^(3/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/a))/(4*a)
```

3.193.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.193.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sec(fx+e)\left(3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(\csc(fx+e)-\cot(fx+e)-1)-3A\sin(fx+e)\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1\right.}{\left.}$
parts	$-\frac{A\sec(fx+e)\left(-3\left(\cos^2(fx+e)\right)\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)+3\left(\cos^2(fx+e)\right)\sin(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-\right.}{8fca^2(1+\sin(fx+e))\sqrt{}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/8/a^2/c/f*sec(f*x+e)*(3*A*sin(f*x+e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x
+e)-1)-3*A*sin(f*x+e)*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)+B*sin(f*x+
e)*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*sin(f*x+e)*cos(f*x+e)^2*ln(-
cot(f*x+e)+csc(f*x+e)+1)+2*sin(f*x+e)^3*A+3*A*cos(f*x+e)^2*ln(csc(f*x+e)-c
ot(f*x+e)-1)-3*A*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)+1)-2*B*sin(f*x+e)^
3+B*cos(f*x+e)^2*ln(csc(f*x+e)-cot(f*x+e)-1)-B*cos(f*x+e)^2*ln(-cot(f*x+e)
+csc(f*x+e)+1)-sin(f*x+e)^2*A-3*B*sin(f*x+e)^2-5*A*sin(f*x+e)+B*sin(f*x+e)
)/(1+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.01

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{((3\,A+B)\cos(fx+e)^3\sin(fx+e)+(3\,A+B)\cos\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
[1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^
3) - 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*s
in(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*s
in(f*x + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x +
e))) + ((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*s
in(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)]
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

3.193.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2)), x)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2\,(3\,A\sqrt{a}+B\sqrt{a})\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^3c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)} - \frac{4\,(3$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
1/32*(2*(3*A*sqrt(a) + B*sqrt(a))*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 +
1)/(a^3*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))) - 4*(3*A*sqrt(a) + B*sqrt(a))*log(abs(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)))/(a^3*c^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + (2*(3*A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f
*x + 1/2*e)^4 - (3*A*sqrt(a) + B*sqrt(a))*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2
 - A*sqrt(a) + B*sqrt(a))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^3*c^(3
/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

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3.194.8 Giac [F(-2)] 1665
3.194.9 Mupad [F(-1)] 1665

3.194.1 Optimal result

Integrand size = 40, antiderivative size = 245

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx &= \\ -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} &\\ -\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} &\\ +\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} &\\ +\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} &\\ +\frac{3A\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} &\end{aligned}$$

output

```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*
A*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f
*x+e)/a^2/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*cos(f*x+e)
/a^2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*arctanh(sin(f
*x+e))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sec^3(e+fx)\left(16B - 9A\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right) - 12$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(5/2)),x]
```

output
```
(Sec[e + f*x]^3*(16*B - 9*A*Log[1 - Tan[(e + f*x)/2]] - 12*A*Cos[2*(e + f*
x)]*(Log[1 - Tan[(e + f*x)/2]] - Log[1 + Tan[(e + f*x)/2]]) - 3*A*Cos[4*(e
 + f*x)]*(Log[1 - Tan[(e + f*x)/2]] - Log[1 + Tan[(e + f*x)/2]]) + 9*A*Log
[1 + Tan[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x)]))/(64*a^
2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

3.194.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{A\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx}{a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{A\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx}{a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{A\left(\frac{3\int\frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{3\int\frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{A\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3222$$

$$\frac{A\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e-}{2f(a\sin(e+fx)+a)^{3/}}$$

$$\frac{a}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2a}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3220$$

$$\frac{A\left(\frac{3\left(\frac{\cos(e+fx)\int\sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2a}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{\cos(e+f}{2f(a\sin(e+fx)+a)^{3/2}(}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{A\left(\frac{3\left(\frac{\cos(e+fx)\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2a}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{\cos(e+f}{2f(a\sin(e+fx)+a)^{3/2}(}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 4257$$

$$\frac{A\left(\frac{3\left(\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2a}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{\cos(e+}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)) + (A*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(5/2)) + (3*(Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/(2*c)))/(2*a)))/a
```

3.194.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.194.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

method	result
default	$\frac{3A\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-3A\cos(fx+e)\ln(\csc(fx+e)-\cot(fx+e)-1)-2\cos(fx+e)B+3\tan(fx+e)A+2\tan($ $8a^2c^2f\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}$
parts	$\frac{A(3\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-3\ln(\csc(fx+e)-\cot(fx+e)-1)\cos(fx+e)+3\tan(fx+e)+2(\sec^2(fx+e))\tan(fx+}$ $8f\sqrt{-c(\sin(fx+e)-1)}\sqrt{a(1+\sin(fx+e))}\,a^2c^2$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/8/a^2/c^2/f/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)*(3*A*cos(
f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)-3*A*cos(f*x+e)*ln(csc(f*x+e)-cot(f*x+e
)-1)-2*cos(f*x+e)*B+3*tan(f*x+e)*A+2*tan(f*x+e)*sec(f*x+e)^2*A+2*sec(f*x+e
)^3*B)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[\frac{3\sqrt{ac}A\cos(fx+e)^5\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f
*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*s
in(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e)
+ 2*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(
f*x + e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)
^5 - ((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.194.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2)), x)
```

3.194.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

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3.195.1 Optimal result

Integrand size = 36, antiderivative size = 174

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$= \frac{2^{\frac{1}{2}+n}c(B(m-n)+A(1+m+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n),\frac{1}{2}(3+2m),\frac{1}{2}(\right.}{f(1+2m)(1+m+n}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)}$$

output
```
2^(1/2+n)*c*(B*(m-n)+A*(1+m+n))*cos(f*x+e)*hypergeom([1/2-n, 1/2+m],[3/2+m
],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f
*x+e))^(-1+n)/f/(1+2*m)/(1+m+n)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f
*x+e))^n/f/(1+m+n)
```

3.195.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$= \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^n, x]
```

3.195.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n dx$$

$$\downarrow 3452$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^n dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 3042$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^n dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 3224$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{n-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 3042$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{n-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 3168$$

$$\frac{c^2\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int(c-}{f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 80$$

$$\frac{c^2 2^{n-\frac{1}{2}}\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}(c-c\sin(}{f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\downarrow 79$$

$$\frac{c2^{n+\frac{1}{2}}\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)+\frac{1}{2}(2m+1)}(}{f(2m-}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*(1 + m
 + n))) + (2^(1/2 + n)*c*(A + (B*(m - n))/(1 + m + n))*Cos[e + f*x]*Hyperg
eometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(
1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-
1/2 + (-1 - 2*m)/2 + m + n)*(c + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)
/2))/(f*(1 + 2*m))
```

3.195.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.195.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

3.195.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

3.195.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^n (A + B\sin(e + fx)) \, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e
 + f*x)), x)
```

3.195.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^n, x)
```

3.195.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^n, x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^n \, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

3.196.1 Optimal result 1672
3.196.2 Mathematica [C] (warning: unable to verify) 1672
3.196.3 Rubi [A] (verified) 1673
3.196.4 Maple [F] 1676
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3.196.8 Giac [F] 1678
3.196.9 Mupad [F(-1)] 1678

3.196.1 Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{2^{\frac{1}{2}+m}a^4c^3(B(3-m)-A(4+m))\cos^7(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1}{2}-m,\frac{9}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1-}{7f(4+m)}$$
$$-\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

output
```
1/7*2^(1/2+m)*a^4*c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m
],[9/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)
/f/(4+m)-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)
```

3.196.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.95 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.17

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx =$$
$$-\frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^3(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{10(4A-3B)\,\text{Hypergeometric2}}{}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
-1/16*((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^3*(Cos[e + f*x] + I*(
1 + Sin[e + f*x]))*((-10*(4*A - 3*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*
Cos[e + f*x] - Sin[e + f*x]])/m + (2*(15*A - 13*B)*Hypergeometric2F1[1, 2
+ m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*
x]))/(-1 + m) + (2*(15*A - 13*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x
] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (4*(3*A - 4*B
)*Hypergeometric2F1[1, -1 + m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos
[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/(2 + m) + (4*(3*A - 4*B)*Hypergeometr
ic2F1[1, 3 + m, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] +
I*Sin[2*(e + f*x)]))/(-2 + m) - ((2*I)*(A - 3*B)*Hypergeometric2F1[1, -2 +
 m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] - I*Sin[3*(e
+ f*x)]))/(3 + m) + ((2*I)*(A - 3*B)*Hypergeometric2F1[1, 4 + m, 4 - m, I*
Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]))/(-3
+ m) + (B*Hypergeometric2F1[1, -3 + m, -3 - m, I*Cos[e + f*x] - Sin[e + f*
x]]*(Cos[4*(e + f*x)] - I*Sin[4*(e + f*x)]))/(4 + m) + (B*Hypergeometric2F
1[1, 5 + m, 5 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[4*(e + f*x)] + I*Si
n[4*(e + f*x)]))/(-4 + m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)
```

3.196.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e+fx))^3 (a\sin(e+fx)+a)^m (A + B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c - c\sin(e+fx))^3 (a\sin(e+fx)+a)^m (A + B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3 \int \cos^6(e+fx)(\sin(e+fx)a+a)^{m-3}(A + B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(\sin(e+fx)a+a)^{m-3}(A+B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$a^3c^3\left(\left(A-\frac{B(3-m)}{m+4}\right)\int \cos^6(e+fx)(\sin(e+fx)a+a)^{m-3}dx-\frac{B\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f(m+4)}\right)$$

$$\downarrow 3042$$

$$a^3c^3\left(\left(A-\frac{B(3-m)}{m+4}\right)\int \cos(e+fx)^6(\sin(e+fx)a+a)^{m-3}dx-\frac{B\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f(m+4)}\right)$$

$$\downarrow 3168$$

$$a^3c^3\left(\frac{a^2\left(A-\frac{B(3-m)}{m+4}\right)\cos^7(e+fx)\int (a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin(e+fx)}{f(a-a\sin(e+fx))^{7/2}(a\sin(e+fx)+a)^{7/2}}-\frac{B\cos^7(e+fx}{}\right.$$

$$\downarrow 80$$

$$a^3c^3\left(\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(3-m)}{m+4}\right)\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\int \left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{f(a-a\sin(e+fx))^{7/2}}\right.$$

$$\downarrow 79$$

$$a^3c^3\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(3-m)}{m+4}\right)\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1}{2}\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^3*c^3*(-1/7*(2^(1/2 + m)*a*(A - (B*(3 - m))/(4 + m))*Cos[e + f*x]^7*Hype
rgeometric2F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^
(1/2 - m)*(a + a*Sin[e + f*x])^(-4 + m))/f - (B*Cos[e + f*x]^7*(a + a*Sin[
e + f*x])^(-3 + m))/(f*(4 + m)))
```

3.196.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.196.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^3 \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

3.196.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)^3 (a\sin(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A -
B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*s
in(f*x + e) + a)^m, x)
```

3.196.6 Sympy [F]

$$\begin{aligned}
&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= -c^3\left(\int \left(-A(a\sin(e+fx)+a)^m\right)\,dx + \int 3A(a\sin(e+fx)+a)^m\sin(e+fx)\,dx\right. \\
&\qquad + \int \left(-3A(a\sin(e+fx)+a)^m\sin^2(e+fx)\right)\,dx \\
&\qquad + \int A(a\sin(e+fx)+a)^m\sin^3(e+fx)\,dx \\
&\qquad + \int \left(-B(a\sin(e+fx)+a)^m\sin(e+fx)\right)\,dx \\
&\qquad + \int 3B(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx \\
&\qquad + \int \left(-3B(a\sin(e+fx)+a)^m\sin^3(e+fx)\right)\,dx \\
&\qquad \left. + \int B(a\sin(e+fx)+a)^m\sin^4(e+fx)\,dx\right)
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(3*A*(a*sin(e + f
*x) + a)**m*sin(e + f*x), x) + Integral(-3*A*(a*sin(e + f*x) + a)**m*sin(e
 + f*x)**2, x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3, x) +
Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(3*B*(a*sin
(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(-3*B*(a*sin(e + f*x) + a)
**m*sin(e + f*x)**3, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)*
*4, x))
```

3.196.7 Maxima [F]

$$\begin{aligned}
&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a
)^m, x)
```

3.196.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)^3 (a\sin(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a
)^m, x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^m\,(c - c\sin(e + fx))^3 \, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

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3.197.2 Mathematica [C] (warning: unable to verify) . . . 1679
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3.197.6 Sympy [F] . . . 1683
3.197.7 Maxima [F] . . . 1684
3.197.8 Giac [F] . . . 1684
3.197.9 Mupad [F(-1)] . . . 1685

3.197.1 Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{2^{\frac{1}{2}+m}a^3c^2(B(2-m)-A(3+m))\cos^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1-}{5f(3+m)}$$
$$-\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

output
```
1/5*2^(1/2+m)*a^3*c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m
],[7/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)
/f/(3+m)-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)
```

3.197.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.06

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{ic^2(a(1+\sin(e+fx)))^m(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{4i(3A-2B)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+fx}{m}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
((I/8)*c^2*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*
(((-4*I)*(3*A - 2*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - S
in[e + f*x]])/m - ((8*A - 7*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x]
- Sin[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((8*A - 7*B)*Hy
pergeometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[e + f
*x] + I*Sin[e + f*x]))/(-1 + m) + ((2*I)*(A - 2*B)*Hypergeometric2F1[1, 3
+ m, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e
+ f*x)]))/(-2 + m) + (2*(A - 2*B)*Hypergeometric2F1[1, -1 + m, -1 - m, I*C
os[e + f*x] - Sin[e + f*x]]*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]))/(2 +
m) - (B*Hypergeometric2F1[1, -2 + m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]
]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]))/(3 + m) + (B*Hypergeometric2F1[
1, 4 + m, 4 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[
3*(e + f*x)]))/(-3 + m)))/f
```

3.197.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e+fx))^2 (a\sin(e+fx)+a)^m (A + B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c - c\sin(e+fx))^2 (a\sin(e+fx)+a)^m (A + B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^2c^2 \int \cos^4(e+fx)(\sin(e+fx)a+a)^{m-2}(A + B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^2c^2 \int \cos(e+fx)^4(\sin(e+fx)a+a)^{m-2}(A + B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$a^2c^2\left(\left(A-\frac{B(2-m)}{m+3}\right)\int\cos^4(e+fx)(\sin(e+fx)a+a)^{m-2}dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}\right)$$

$$\downarrow 3042$$

$$a^2c^2\left(\left(A-\frac{B(2-m)}{m+3}\right)\int\cos(e+fx)^4(\sin(e+fx)a+a)^{m-2}dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}\right)$$

$$\downarrow 3168$$

$$a^2c^2\left(\frac{a^2\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)\int(a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin(e+fx)}{f(a-a\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^{5/2}}-\frac{B\cos^5(e+fx)}{}\right.$$

$$\downarrow 80$$

$$a^2c^2\left(\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\int\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{f(a-a\sin(e+fx))^{5/2}}\right.$$

$$\downarrow 79$$

$$a^2c^2\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}\right.\right.}{5f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/5*(2^(1/2 + m)*a*(A - (B*(2 - m))/(3 + m))*Cos[e + f*x]^5*Hype
rgeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^
(1/2 - m)*(a + a*Sin[e + f*x])^(-3 + m))/f - (B*Cos[e + f*x]^5*(a + a*Sin[
e + f*x])^(-2 + m))/(f*(3 + m)))
```

3.197.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.197.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c - c\sin(fx+e))^2 \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

3.197.5 Fricas [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^2 \, dx$$
$$= \int (B\sin(fx+e) + A)(c\sin(fx+e) - c)^2 (a\sin(fx+e) + a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
 e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

3.197.6 Sympy [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^2 \, dx$$
$$= c^2 \left(\int A(a\sin(e+fx) + a)^m \, dx + \int (-2A(a\sin(e+fx) + a)^m \sin(e+fx)) \, dx \right.$$
$$+ \int A(a\sin(e+fx) + a)^m \sin^2(e+fx) \, dx + \int B(a\sin(e+fx) + a)^m \sin(e+fx) \, dx$$
$$+ \int (-2B(a\sin(e+fx) + a)^m \sin^2(e+fx)) \, dx$$
$$\left. + \int B(a\sin(e+fx) + a)^m \sin^3(e+fx) \, dx \right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*
x) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f
*x)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integra
l(-2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**3, x))
```

3.197.7 Maxima [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= \int (B\sin(fx+e) + A)(c\sin(fx+e) - c)^2 (a\sin(fx+e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)
^m, x)
```

3.197.8 Giac [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^2\,dx$$
$$= \int (B\sin(fx+e) + A)(c\sin(fx+e) - c)^2 (a\sin(fx+e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)
^m, x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^2\, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^2\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

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3.198.1 Optimal result

Integrand size = 34, antiderivative size = 139

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{2^{\frac{1}{2}+m}a^2c(B(1-m)-A(2+m))\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+}{3f(2+m)}$$
$$-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}$$

output
```
1/3*2^(1/2+m)*a^2*c*(B*(1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],
[5/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f
/(2+m)-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)
```

3.198.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.37

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{2^{-2-m}ce^{ifmx}\left(1-ie^{i(e+fx)}\right)^{-2m}\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^m\left(\frac{iBe^{-i(2e+f(2+m)x)}\,\text{Hypergeometric2F1}(-2-m,-2r}{2+m}\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
-((2^(-2 - m)*c*E^(I*f*m*x)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*
x)))^m*((I*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, I*E^(I*(e + f*x))])/(
E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*(A - B)*Hypergeometric2F1[-1 - m,
-2*m, -m, I*E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*x))*(1 + m)) - (2*A*E^(
I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*E^(I*(e + f*
x))])/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2
*m, 2 - m, I*E^(I*(e + f*x))])/(-1 + m) + (I*B*E^((2*I)*e - I*f*(-2 + m)*x
)*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*E^(I*(e + f*x))])/(-2 + m) + ((4
*I)*A*Hypergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I*f*m*x)*
m) - ((2*I)*B*Hypergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I
*f*m*x)*m))*(-1 + Sin[e + f*x]))/((1 - I*E^(I*(e + f*x)))^(2*m)*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^2))
```

3.198.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e+fx))(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c - c\sin(e+fx))(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\downarrow 3446$$

$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)^{m-1}(A+B\sin(e+fx))dx$$

$$\downarrow 3042$$

$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)^{m-1}(A+B\sin(e+fx))dx$$

$$\downarrow 3339$$

$$ac\left(\left(A-\frac{B(1-m)}{m+2}\right)\int\cos^2(e+fx)(\sin(e+fx)a+a)^{m-1}dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}\right)$$

$$\downarrow 3042$$

$$ac\left(\left(A-\frac{B(1-m)}{m+2}\right)\int\cos(e+fx)^2(\sin(e+fx)a+a)^{m-1}dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}\right)$$

$$\downarrow 3168$$

$$ac\left(\frac{a^2\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)\int\sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin(e+fx)}{f(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{3/2}}-\frac{B\cos^3(e+fx)(a\right.$$

$$\downarrow 80$$

$$ac\left(\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\int\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}\sqrt{}\right.$$

$$\downarrow 79$$

$$ac\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/3*(2^(1/2 + m)*a*(A - (B*(1 - m))/(2 + m))*Cos[e + f*x]^3*Hypergeo
metric2F1[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2
 - m)*(a + a*Sin[e + f*x])^(-2 + m))/f - (B*Cos[e + f*x]^3*(a + a*Sin[e +
f*x])^(-1 + m))/(f*(2 + m)))
```

3.198.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.198.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e))(c - c\sin(fx + e))\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

3.198.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f*x + e) + a)^m, x)
```

3.198.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= -c\left(\int \left(-A(a\sin(e + fx) + a)^m\right)\,dx + \int A(a\sin(e + fx) + a)^m \sin(e + fx)\,dx \right.$$
$$+ \int \left(-B(a\sin(e + fx) + a)^m \sin(e + fx)\right)\,dx$$
$$\left. + \int B(a\sin(e + fx) + a)^m \sin^2(e + fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
-c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
 a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x)
, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

3.198.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^
m, x)
```

3.198.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int -(B\sin(fx + e) + A)(c\sin(fx + e) - c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^
m, x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

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3.199.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}$$
$$-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+$$
$$f(1+m)$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*
hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(
a+a*sin(f*x+e))^m/f/(1+m)
```

3.199.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+\sin$$
$$f$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
(2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin
[e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin
[e + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

3.199.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3042$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3131$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3042$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3130$$

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2}\right.}{f(m+1)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
 A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e +
 f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)
)
```

3.199.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.199.4 Maple [F]

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

3.199.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m (A+B\sin(e+fx))\, dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.199.6 Sympy [F]

$$\begin{aligned}&\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\, dx \\ &= \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))\, dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

3.199.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.199.8 Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

3.200.1 Optimal result 1698
3.200.2 Mathematica [F] 1698
3.200.3 Rubi [A] (verified) 1699
3.200.4 Maple [F] 1701
3.200.5 Fricas [F] 1701
3.200.6 Sympy [F] 1702
3.200.7 Maxima [F] 1702
3.200.8 Giac [F] 1703
3.200.9 Mupad [F(-1)] 1703

3.200.1 Optimal result

Integrand size = 36, antiderivative size = 123

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+f}{cfm}$$
$$-\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm}$$

output
```
2^(1/2+m)*(A*m+B*m+B)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*se
c(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f/m-B*sec(f*x+e)*(a+a
*sin(f*x+e))^(1+m)/a/c/f/m
```

3.200.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]), x]
```

3.200.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^2(e+fx)(\sin(e+fx)a+a)^{m+1}(A+B\sin(e+fx))dx}{ac}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+1}(A+B\sin(e+fx))}{\cos(e+fx)^2}dx}{ac}$$

$$\downarrow 3339$$

$$\frac{\frac{(Am+Bm+B)\int \sec^2(e+fx)(\sin(e+fx)a+a)^{m+1}dx}{m} - \frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$$\downarrow 3042$$

$$\frac{\frac{(Am+Bm+B)\int \frac{(\sin(e+fx)a+a)^{m+1}}{\cos(e+fx)^2}dx}{m} - \frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$$\downarrow 3168$$

$$\frac{\frac{a^2(Am+Bm+B)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{fm} - \frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$$\downarrow 80$$

$$\frac{a^2 2^{m-\frac{1}{2}}(Am+Bm+B)\sec(e+fx)\sqrt{a-a\sin(e+fx)}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\int\frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{fm} \_ \frac{B\sec(e+fx)(}{}$$

$$ac$$

$$\downarrow 79$$

$$\frac{a2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{fm} \_ \frac{B\sec(e+fx)(a\,s}{}$$

$$ac$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
((2^(1/2 + m)*a*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 -
 Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e +
 f*x])^m)/(f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(f*m))/(a*
c)
```

3.200.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))  Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))  Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m  Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.200.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{c-c\sin(fx+e)}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

3.200.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{c\sin(fx+e)-c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

3.200.6 Sympy [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = -\frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin(e+fx)-1}\,dx + \int \frac{B(a\sin(e+fx)+a)^m \sin(e+fx)}{\sin(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
-(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(
a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

3.200.7 Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \int -\frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{c\sin(fx + e) - c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
), x)
```

3.200.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{c\sin(fx+e)-c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

3.201.1 Optimal result 1704
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3.201.1 Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2^{\frac{1}{2}+m}(A(1-m)-B(2+m))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+}{3ac^2f(1-m)}$$
$$+\frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)}$$

output
```
1/3*2^(1/2+m)*(A*(1-m)-B*(2+m))*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin
(f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/
f/(1-m)+B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)
```

3.201.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2, x]
```

3.201.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^4(e+fx)(\sin(e+fx)a+a)^{m+2}(A+B\sin(e+fx))dx}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+2}(A+B\sin(e+fx))}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\downarrow 3339$$

$$\frac{\left(A-\frac{B(m+2)}{1-m}\right)\int \sec^4(e+fx)(\sin(e+fx)a+a)^{m+2}dx+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\downarrow 3042$$

$$\frac{\left(A-\frac{B(m+2)}{1-m}\right)\int \frac{(\sin(e+fx)a+a)^{m+2}}{\cos(e+fx)^4}dx+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\downarrow 3168$$

$$\frac{\frac{a^2\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{3/2}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{5/2}}d\sin(e+fx)}{f}+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\downarrow 80$$

$$\frac{\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\int\frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{5/2}}d\sin(e+fx)}{f}+\frac{B\sec^3(}{}}{a^2c^2}$$

$$\downarrow 79$$

$$\frac{\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{3f}+\frac{B\sec^3(e+f}{}}{a^2c^2}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
((2^(1/2 + m)*a*(A - (B*(2 + m))/(1 - m))*Hypergeometric2F1[-3/2, 1/2 - m,
 -1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(
a + a*Sin[e + f*x])^(1 + m))/(3*f) + (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x]
)^(2 + m))/(f*(1 - m)))/(a^2*c^2)
```

3.201.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))  Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))  Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m  Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.201.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^2}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

3.201.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2
+ 2*c^2*sin(f*x + e) - 2*c^2), x)
```

3.201.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin^2(e+fx)-2\sin(e+fx)+1}\,dx + \int \frac{B(a\sin(e+fx)+a)^m\sin(e+fx)}{\sin^2(e+fx)-2\sin(e+fx)+1}\,dx}{c^2}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1)
, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 -
2*sin(e + f*x) + 1), x))/c**2
```

3.201.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^2, x)
```

3.201.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x)
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

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3.202.1 Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2^{\frac{1}{2}+m}(A(2-m)-B(3+m))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+}{5a^2c^3f(2-m)}$$
$$+\frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)}$$

output
```
1/5*2^(1/2+m)*(A*(2-m)-B*(3+m))*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin
(f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^
3/f/(2-m)+B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)
```

3.202.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3, x]
```

3.202.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\downarrow 3446$$

$$\frac{\int \sec^6(e+fx)(\sin(e+fx)a+a)^{m+3}(A+B\sin(e+fx))dx}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+3}(A+B\sin(e+fx))}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\downarrow 3339$$

$$\frac{\left(A-\frac{B(m+3)}{2-m}\right)\int \sec^6(e+fx)(\sin(e+fx)a+a)^{m+3}dx+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$$\downarrow 3042$$

$$\frac{\left(A-\frac{B(m+3)}{2-m}\right)\int \frac{(\sin(e+fx)a+a)^{m+3}}{\cos(e+fx)^6}dx+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$$\downarrow 3168$$

$$\frac{\frac{a^2\left(A-\frac{B(m+3)}{2-m}\right)\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^{5/2}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{7/2}}d\sin(e+fx)}{f}+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$$\downarrow 80$$

$$\frac{\frac{a^2 2^{m-\frac{1}{2}} \left(A-\frac{B(m+3)}{2-m}\right) \sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\int \frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{7/2}} d\sin(e+fx)}{f} + \frac{B\sec^5(}{}}{a^3c^3}$$

$$\downarrow 79$$

$$\frac{\frac{a 2^{m+\frac{1}{2}} \left(A-\frac{B(m+3)}{2-m}\right) \sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{5f} + \frac{B\sec^5(e+f}{}}{a^3c^3}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
((2^(1/2 + m)*a*(A - (B*(3 + m))/(2 - m))*Hypergeometric2F1[-5/2, 1/2 - m,
 -3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(
a + a*Sin[e + f*x])^(2 + m))/(5*f) + (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x]
)^(3 + m))/(f*(2 - m)))/(a^3*c^3)
```

3.202.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))  Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))  Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m  Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.202.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^3} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

3.202.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m (A+B\sin(e+fx))}{(c-c\sin(e+fx))^3} dx$$
$$= \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^3} dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^
2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

3.202.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.202.7 Maxima [F]

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx\\ &= \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^3}\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^3, x)
```

3.202.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^3, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
x)
```

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

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3.203.2 Mathematica [A] (verified) 1716
3.203.3 Rubi [A] (verified) 1717
3.203.4 Maple [F] 1719
3.203.5 Fricas [F] 1719
3.203.6 Sympy [F] 1720
3.203.7 Maxima [F] 1720
3.203.8 Giac [F(-1)] 1720
3.203.9 Mupad [F(-1)] 1721

3.203.1 Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e
))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 21.88 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right.\right.}{2f(1+2m)(3+2m}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

3.203.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3452$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3224$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$\frac{a(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 78$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (a
+ a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]])
```

3.203.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.203.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}dx$$

input

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

3.203.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c*sin(f*x + e) - c), x)
```

3.203.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

3.203.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
 + c), x)
```

3.203.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2), x)
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

3.204.1 Optimal result 1722
3.204.2 Mathematica [A] (verified) 1722
3.204.3 Rubi [A] (verified) 1723
3.204.4 Maple [F] 1725
3.204.5 Fricas [F] 1725
3.204.6 Sympy [F] 1726
3.204.7 Maxima [F] 1726
3.204.8 Giac [F(-1)] 1726
3.204.9 Mupad [F(-1)] 1727

3.204.1 Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e
))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 21.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(c(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right.\right.}{2f(1+2m)(3+2m}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

3.204.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c\sin(e+fx)+c)^m}{\sqrt{a-a\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c\sin(e+fx)+c)^m}{\sqrt{a-a\sin(e+fx)}}dx$$

$$\downarrow 3452$$

$$(A+B)\int \frac{(\sin(e+fx)c+c)^m}{\sqrt{a-a\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)c+c)^m}{\sqrt{a-a\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\downarrow 3224$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)c+c)^{m+\frac{1}{2}}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)c+c)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\downarrow 3146$$

$$\frac{c(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)c+c)^{m-\frac{1}{2}}}{c-c\sin(e+fx)}d(c\sin(e+fx))}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\downarrow 78$$

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)c+c}{2c}\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}} -$$
$$\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (c
+ c*Sin[e + f*x])/(2*c)]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*S
in[e + f*x]])
```

3.204.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.204.4 Maple [F]

$$\int \frac{(A + B\sin(fx+e))(c + c\sin(fx+e))^m}{\sqrt{a - a\sin(fx+e)}}dx$$

input

```
int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

output

```
int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

3.204.5 Fricas [F]

$$\int \frac{(A + B\sin(e+fx))(c + c\sin(e+fx))^m}{\sqrt{a - a\sin(e+fx)}}\,dx$$
$$= \int \frac{(B\sin(fx+e) + A)(c\sin(fx+e) + c)^m}{\sqrt{-a\sin(fx+e) + a}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
 c)^m/(a*sin(f*x + e) - a), x)
```

3.204.6 Sympy [F]

$$\int \frac{(A + B\sin(e + fx))(c + c\sin(e + fx))^m}{\sqrt{a - a\sin(e + fx)}}\,dx = \int \frac{(c(\sin(e + fx) + 1))^m (A + B\sin(e + fx))}{\sqrt{-a(\sin(e + fx) - 1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f
*x) - 1)), x)
```

3.204.7 Maxima [F]

$$\int \frac{(A + B\sin(e + fx))(c + c\sin(e + fx))^m}{\sqrt{a - a\sin(e + fx)}}\,dx = \int \frac{(B\sin(fx + e) + A)(c\sin(fx + e) + c)^m}{\sqrt{-a\sin(fx + e) + a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
 + a), x)
```

3.204.8 Giac [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + c\sin(e + fx))^m}{\sqrt{a - a\sin(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + c\sin(e + fx))^m}{\sqrt{a - a\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(c + c\,\sin(e + f\,x))^m}{\sqrt{a - a\,\sin(e + f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/
2), x)
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

3.205.1 Optimal result 1728
3.205.2 Mathematica [A] (verified) 1729
3.205.3 Rubi [A] (verified) 1729
3.205.4 Maple [F] 1732
3.205.5 Fricas [B] (verification not implemented) 1732
3.205.6 Sympy [F(-1)] 1733
3.205.7 Maxima [B] (verification not implemented) 1733
3.205.8 Giac [F] 1734
3.205.9 Mupad [B] (verification not implemented) 1735

3.205.1 Optimal result

Integrand size = 38, antiderivative size = 275

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(5+2m)(7+2m)\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(7+2m)\left(15+16m+4m^2\right)}$$
$$-\frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)}$$
$$-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)}$$

output

```
-2*c*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(3/2)/f/(4*m^2+24*m+35)-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(5/2)/f/(7+2*m)-64*c^3*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^
m/f/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)-16*c^2*(B*(5-2*m)
-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(7+2*m)
/(4*m^2+16*m+15)
```

3.205.2 Mathematica [A] (verified)

Time = 11.98 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(-1246A+1040B-114$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/2*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*S
qrt[c - c*Sin[e + f*x]]*(-1246*A + 1040*B - 1140*A*m + 664*B*m - 392*A*m^2
 + 256*B*m^2 - 48*A*m^3 + 32*B*m^3 + 2*(3 + 8*m + 4*m^2)*(-4*B*(5 + m) + A
*(7 + 2*m))*Cos[2*(e + f*x)] + (1 + 2*m)*(8*A*(7 + 2*m)^2 - B*(505 + 208*m
 + 28*m^2))*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)] + 46*B*m*Sin[3*(e + f*x)]
 + 36*B*m^2*Sin[3*(e + f*x)] + 8*B*m^3*Sin[3*(e + f*x)]))/(f*(1 + 2*m)*(3
+ 2*m)*(5 + 2*m)*(7 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.205.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$
$$\downarrow 3452$$
$$-\frac{(B(5-2m)-A(2m+7))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{5/2}dx}{2m+7}-$$
$$\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3042$$

$$-\frac{(B(5-2m)-A(2m+7))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{5/2}dx}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3219$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}\right)}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3042$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}\right)}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3219$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+fx)}{}\right.}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3042$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+fx)}{}\right.}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\downarrow 3217$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}\right)}{2m+7}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2))/(f*(
7 + 2*m)) - ((B*(5 - 2*m) - A*(7 + 2*m))*((2*c*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m)) + (8*c*((8*c^2*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x
]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]])/(
f*(3 + 2*m))))/(5 + 2*m)))/(7 + 2*m)
```

3.205.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.205.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c - c\sin(fx+e))^{\frac{5}{2}}\, dx$$

input

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(262) = 524$.

Time $= 0.31$ (sec) , antiderivative size $= 562$, normalized size of antiderivative $= 2.04$

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\, dx = \frac{2\left(\left(8\,Bc^2m^3 + 36\,Bc^2m^2 + 46\,Bc^2m + 15\,Bc^2\right)\cos(fx+e)^4 + 64\,(A+B)c^2m - \left(8\,(A-2\,B\right.\right.$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 6
4*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A
 - 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 +
 (8*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A
- 85*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 +
2*(79*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^
2*m - (8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3
+ 32*(7*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(3
1*A - 63*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*
m^3 + 60*(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(
f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(
16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f
*m^2 + 352*f*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 +
 352*f*m + 105*f)*sin(f*x + e) + 105*f)
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.205.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(262) = 524$.

Time = 0.34 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.64

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
-2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*
(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)
*a^m*c^(5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*
x + e) + 1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^
m*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)
*a^m*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^
m*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(
cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/(
(16*m^4 + 128*m^3 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m
+ 105)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 1)^(5/2)))/f
```

3.205.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \int (B\sin(fx + e) + A)(-c\sin(fx + e) + c)^{\frac{5}{2}}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
 + a)^m, x)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.72

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{\sqrt{c-c\sin(e+fx)}\left(\frac{Bc^2(a+a\sin(e+fx))^m(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i})}{4f(16m^4+128m^3+344m^2+352m+105)}-\frac{c^2e^{e\,3\mathrm{i}+fx\,3\mathrm{i}}(a+a\sin(e+fx))^m(2100A-1575B+1272}{4f(16m^4+128m^3+}\right.$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
-((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*3
6i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^
2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m -
110*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^
2 + 128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x)
)^m*(A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B
*m^2*68i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (
c^2*exp(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 1
84*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 +
 16*m^4 + 105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m
*(A*350i - B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(35
2*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a +
a*sin(e + f*x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 12
8*m^3 + 16*m^4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8
*m + 4*m^2 + 3)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128
*m^3 + 16*m^4 + 105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*
m + 4*m^2 + 3)*(A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*35
2i + m^2*344i + m^3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 1
6*m^4 + 105))
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

3.206.1 Optimal result 1736
3.206.2 Mathematica [A] (verified) 1737
3.206.3 Rubi [A] (verified) 1737
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3.206.6 Sympy [F(-1)] 1740
3.206.7 Maxima [B] (verification not implemented) 1741
3.206.8 Giac [F] 1741
3.206.9 Mupad [B] (verification not implemented) 1742

3.206.1 Optimal result

Integrand size = 38, antiderivative size = 166

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = \frac{4(A-B)c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} - \frac{2(A-3B)c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}} - \frac{2Bc^2\cos(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2f(5+2m)\sqrt{c-c\sin(e+fx)}}$$

output

```
4*(A-B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
-2*(A-3*B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+
e))^(1/2)-2*B*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5+2*m)/(c-c*sin
(f*x+e))^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(50A-39B+\cdots}{f(1+2m)(3+2m)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
 c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3
 + 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)
*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]))
```

3.206.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\downarrow 3452$$

$$-\frac{(B(3-2m)-A(2m+5))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\downarrow 3042$$

$$-\frac{(B(3-2m)-A(2m+5))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\downarrow 3219$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\downarrow 3042$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\downarrow 3217$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(
5 + 2*m)) - ((B*(3 - 2*m) - A*(5 + 2*m))*((8*c^2*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^m)/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]])/(f*(3 + 2*m))))/(5
 + 2*m)
```

3.206.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.206.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{\frac{3}{2}}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.89

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{2\left((4\,Bcm^2+8\,Bcm+3\,Bc)\cos(fx+e)^3+8\,(A+B)cm+(4\,Acm^2+12\,(A-B)cm+(5\,A$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m
^2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4
*(A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A
 + B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c
 - (4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*
f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*
m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.206.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.00

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx =$$

$$-2\left(\frac{\left(a^m c^{\frac{3}{2}}(2m+5) - \frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)}{\cos(fx+e)+1} - \frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^m c^{\frac{3}{2}}(2m+5)\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right) Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right) - m\log\left(\frac{\sin(fx}{(\cos(fx+}\right.}\right.$$

$$\left.\left.\frac{}{(4m^2+8m+3)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
-2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x +
 e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m
*c^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)
) - 2*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(c
os(f*x + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(
2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)
*(2*m + 9)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/
((8*m^3 + 36*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

3.206.8 Giac [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e$$
$$+ fx))^{3/2}\,dx = \int (B\sin(fx+e) + A)(-c\sin(fx+e) + c)^{\frac{3}{2}}(a\sin(fx+e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
 + a)^m, x)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.89

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{c-c\sin(e+fx)}\left(\frac{c\,e^{e\,3i+f\,x\,3i}\,(a+a\sin(e+fx))^m\,(45A-30B+28Am+4Bm+4Am^2)}{f\,(m^3\,8i+m^2\,36i+m\,46i+15i)}+\frac{c\,e^{e\,2i+f\,x\,2i}\,(a+a\sin($$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*
(45*A - 30*B + 28*A*m + 4*B*m + 4*A*m^2))/(f*(m*46i + m^2*36i + m^3*8i + 1
5i)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(A*45i - B*30i + A*m*2
8i + B*m*4i + A*m^2*4i))/(f*(m*46i + m^2*36i + m^3*8i + 15i)) + (B*c*(a +
a*sin(e + f*x))^m*(m*8i + m^2*4i + 3i))/(2*f*(m*46i + m^2*36i + m^3*8i + 1
5i)) + (B*c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3))/(
2*f*(m*46i + m^2*36i + m^3*8i + 15i)) + (c*exp(e*1i + f*x*1i)*(2*m + 1)*(a
 + a*sin(e + f*x))^m*(10*A - 15*B + 4*A*m - 2*B*m))/(2*f*(m*46i + m^2*36i
+ m^3*8i + 15i)) + (c*exp(e*4i + f*x*4i)*(2*m + 1)*(a + a*sin(e + f*x))^m*
(A*10i - B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i)))
)/(exp(e*3i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/
(m*46i + m^2*36i + m^3*8i + 15i))
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

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3.207.1 Optimal result

Integrand size = 38, antiderivative size = 104

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{2Bc\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+2
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(-2B+A(3+2m)+B($$
$$f(1+2m)(3+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
 c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 +
2*m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.207.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c - c\sin(e+fx)}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{c - c\sin(e+fx)}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\downarrow 3450$$

$$(A-B)\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx + \frac{B\int(\sin(e+fx)a+a)^{m+1}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx + \frac{B\int(\sin(e+fx)a+a)^{m+1}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\downarrow 3217$$

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}} + \frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
(2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*
Sin[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3
 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

3.207.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

3.207.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) \sqrt{c - c\sin(fx+e)}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx)) \sqrt{c - c\sin(e+fx)}\,dx =$$

$$-\frac{2\left((2\,Bm + B)\cos(fx+e)^2 - 2\,(A+B)m - (2\,Am + 3\,A - 2\,B)\cos(fx+e) - (2\,(A+B)m + (2\,B\right.}{4\,fm^2 + 8\,fm + (4\,fm^2 + 8\,fm + 3\,f)\cos(fx+}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
-2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
 + e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) -
3*A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m
 + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x
+ e) + 3*f)
```

3.207.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m \sqrt{-c(\sin(e + fx) - 1)}(A + B\sin(e + fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(100) = 200$.

Time $= 0.34$ (sec) , antiderivative size $= 323$, normalized size of antiderivative $= 3.11$

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx =$$
$$-\frac{2\left(\frac{2\left(\frac{2a^m\sqrt{c}m\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^m\sqrt{c}m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-a^m\sqrt{c}-\frac{a^m\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+8m+\frac{\left(4m^2+8m+3\right)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+3\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{(}{}\right.}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
-2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2
 + 8*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f
*x + e) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)))/f
```

3.207.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{-c\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m, x)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx =$$
$$-\frac{(a(\sin(e + fx) + 1))^m \sqrt{-c(\sin(e + fx) - 1)}\,(6A\cos(e + fx) - 4B\cos(e + fx) + B\sin(2e + 2$$
$$f(\sin(e + fx) - 1)(4m^2 + 8m + 3)$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x)
 - 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(
2*e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))
```

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

3.208.1 Optimal result 1748
3.208.2 Mathematica [A] (verified) 1748
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3.208.4 Maple [F] 1751
3.208.5 Fricas [F] 1751
3.208.6 Sympy [F] 1752
3.208.7 Maxima [F] 1752
3.208.8 Giac [F(-1)] 1752
3.208.9 Mupad [F(-1)] 1753

3.208.1 Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e
))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right.\right.}{2f(1+2m)(3+2m}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

3.208.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\downarrow 3452$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3224$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 3146$$

$$\frac{a(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\downarrow 78$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}} - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (a
+ a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]])
```

3.208.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.208.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}dx$$

input

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

3.208.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c*sin(f*x + e) - c), x)
```

3.208.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

3.208.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
 + c), x)
```

3.208.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2), x)
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

3.209.1 Optimal result 1754
3.209.2 Mathematica [A] (verified) 1754
3.209.3 Rubi [A] (verified) 1755
3.209.4 Maple [F] 1757
3.209.5 Fricas [F] 1758
3.209.6 Sympy [F] 1758
3.209.7 Maxima [F] 1758
3.209.8 Giac [F(-2)] 1759
3.209.9 Mupad [F(-1)] 1759

3.209.1 Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}}$$
$$+\frac{(A(1-2m)-B(3+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\s}{4cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1
-2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e
))*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 25.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(B(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)\right.}{4cf(1+2m)(-1+}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
-1/4*(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(B*(3 + 2*m)*Hypergeometric2F1
[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 2*(B + 2
*B*m + Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(A - A
*Sin[e + f*x]))))/(c*f*(1 + 2*m)*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]])
```

3.209.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3451, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3451$$

$$\frac{(A(1-2m)-B(2m+3))\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(A(1-2m)-B(2m+3))\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3224$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{4c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{4c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 3146$$

$$\frac{a(A(1-2m)-B(2m+3))\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{4cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\downarrow 78$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3
/2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2
+ m, 3/2 + m, (a + a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(4*c*f*(
1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

3.209.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.209.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{3}{2}}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

3.209.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

3.209.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{(-c\,(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

3.209.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^(3/2), x)
```

3.209.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,1,0,0,0,0,0]%%%}+%%%{1,[0,0,1,1,1,0,0,0,0]%%%} /
%%%{16,[0
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/
2), x)
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

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3.210.2 Mathematica [A] (verified) 1760
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3.210.1 Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}}$$
$$+\frac{(A(3-2m)-B(5+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\s}{16c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(
3-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 33.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{\cos(e+fx)\left(B(5+2m)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\si\right.}{16c^2(f+}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
-1/16*(Cos[e + f*x]*(B*(5 + 2*m)*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1
 + Sin[e + f*x])/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(B + 2*B*m
 + A*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Si
n[e + f*x])^2))*(a*(1 + Sin[e + f*x]))^m)/(c^2*(f + 2*f*m)*(-1 + Sin[e + f
*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

3.210.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3451, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3451$$

$$\frac{(A(3-2m)-B(2m+5))\int \frac{(\sin(e+fx)a+a)^m}{(c-c\sin(e+fx))^{3/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(A(3-2m)-B(2m+5))\int \frac{(\sin(e+fx)a+a)^m}{(c-c\sin(e+fx))^{3/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3224$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)\int \sec^3(e+fx)(\sin(e+fx)a+a)^{m+\frac{3}{2}}dx}{8ac^2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{3}{2}}}{\cos(e+fx)^3}dx}{8ac^2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 3146$$

$$\frac{a^2(A(3-2m)-B(2m+5))\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{8c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\downarrow 78$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(2,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5
/2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2
+ m, 3/2 + m, (a + a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(16*c^2*
f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

3.210.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.210.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

3.210.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x
 + e)), x)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.210.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^(5/2), x)
```

3.210.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument Value
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/
2), x)
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

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3.211.9 Mupad [B] (verification not implemented) 1772

3.211.1 Optimal result

Integrand size = 40, antiderivative size = 267

$$\begin{aligned}
&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)} \\
&\quad + \frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)} \\
&\quad + \frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{c^2f(7+2m)\left(15+16m+4m^2\right)} \\
&\quad + \frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^3f(5+2m)(7+2m)\left(3+8m+4m^2\right)}
\end{aligned}$$

output

```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m)/f/(7+2*m)+(3*A
-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/c/f/(4*m
^2+24*m+35)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e
))^(-2-m)/c^2/f/(8*m^3+60*m^2+142*m+105)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a
*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^3/f/(16*m^4+128*m^3+344*m^2+352*m
+105)
```

3.211.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$= \frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\left(B(13+16m+4m^2)-2A(18+41m+24m^2+\right.}{ac^4f(1+2m}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-4 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B*(13 + 16*m + 4*m^2) - 2*A*
(18 + 41*m + 24*m^2 + 4*m^3) + (13 + 16*m + 4*m^2)*(3*A - 2*B*(2 + m))*Sin
[e + f*x] + 4*(2 + m)*(-3*A + 2*B*(2 + m))*Sin[e + f*x]^2 + (6*A - 4*B*(2
+ m))*Sin[e + f*x]^3))/(a*c^4*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(-
1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^m)
```

3.211.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-4}\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-4}dx$$
$$\downarrow 3451$$
$$\frac{(3A-2B(m+2))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-3}dx}{c(2m+7)}+$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$
$$\downarrow 3042$$

$$\frac{(3A-2B(m+2))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-3}dx}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$$\downarrow 3222$$

$$\frac{(3A-2B(m+2))\left(\frac{2\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}\right)}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$$\downarrow 3042$$

$$\frac{(3A-2B(m+2))\left(\frac{2\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}\right)}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$$\downarrow 3222$$

$$\frac{(3A-2B(m+2))\left(\frac{2\left(\frac{\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+\right.}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$$\downarrow 3042$$

$$\frac{(3A-2B(m+2))\left(\frac{2\left(\frac{\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+\right.}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}+\frac{(3A-2B(m+2))\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}+\frac{2\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{\cos(e+fx)(}{c(2m+5)}\right.\right.}{c(2m+7)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)
)/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*((Cos[e + f*x]*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 + 2*m)) + (2*((Cos[e + f*x]*(a + a
*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m)) + (Cos[e + f
*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(
3 + 2*m))))/(c*(5 + 2*m))))/(c*(7 + 2*m))
```

3.211.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.211.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c - c\sin(fx+e))^{-4-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.77

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{-4-m}\, dx$$
$$= \frac{\left(4\left(2Bm^2 - (3A - 8B)m - 6A + 8B\right)\cos(fx+e)^3 + \left(8Am^3 + 12(4A - B)m^2 + 2(47A - 24B)m +\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="fricas")
```

output
```
(4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4
*A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m -
3*A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*
m - 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*si
n(f*x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105
*f)
```

3.211.6 Sympy [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{-4-m}\, dx$$
$$= \int (a(\sin(e+fx)+1))^m (-c(\sin(e+fx)-1))^{-m-4} (A + B\sin(e+fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 4)*(A +
B*sin(e + f*x)), x)
```

3.211.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-4}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 4), x)
```

3.211.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-4}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 4), x)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$

$$= -\frac{\sin(4e+4fx)\,(a+a\sin(e+fx))^m\,(4B-3A+2Bm)\,1\mathrm{i}}{4f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\cos(e+fx)\,(a+a\sin(e+fx))^m\,(A\,168\mathrm{i}-B\,84\mathrm{i}+A\,m\,340\mathrm{i}-B\,m\,96\mathrm{i}+A\,m^2\,192\mathrm{i}+A\,m^3\,32\mathrm{i}-B}{4f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\sin(2e+2fx)\,(a+a\sin(e+fx))^m\,(2m^2+8m+7)\,(4B-3A+2Bm)\,1\mathrm{i}}{f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$
$$+\frac{\cos(3e+3fx)\,(m+2)\,(a+a\sin(e+fx))^m\,(-A\,3\mathrm{i}+B\,4\mathrm{i}+B\,m\,2\mathrm{i})}{f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 4),x)
```

output
```
(cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i
+ A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(
m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a
*sin(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)
*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a +
 a*sin(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin
(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos
(3*e + 3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c
 - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)
)
```

3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$

3.212.1 Optimal result 1773
3.212.2 Mathematica [A] (verified) 1774
3.212.3 Rubi [A] (verified) 1774
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3.212.1 Optimal result

Integrand size = 40, antiderivative size = 191

$$\begin{aligned}
&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx \\
&= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{cf(3+2m)(5+2m)} \\
&\quad + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^2f(5+2m)\left(3+8m+4m^2\right)}
\end{aligned}$$

output

```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+(2*A
-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m
^2+16*m+15)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(-1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

3.212.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\left(-B(3+2m)+A(7+12m+4m^2)+(3+2m\right.}{ac^3f(1+2m)(3+2m)(5+2m)(-1+\sin(e}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-3 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(-(B*(3 + 2*m)) + A*(7 + 12*m
 + 4*m^2) + (3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x] + (2*A - B*(3 + 2*
m))*Sin[e + f*x]^2))/(a*c^3*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 + Sin[e +
f*x])^2*(c - c*Sin[e + f*x])^m)
```

3.212.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-3}\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-3}dx$$
$$\downarrow 3451$$
$$\frac{(2A-B(2m+3))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$
$$\downarrow 3042$$

$$\frac{(2A-B(2m+3))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\downarrow 3222$$

$$\frac{(2A-B(2m+3))\left(\frac{\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\downarrow 3042$$

$$\frac{(2A-B(2m+3))\left(\frac{\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}+\frac{(2A-B(2m+3))\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}\right)}{c(2m+5)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)
)/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*((Cos[e + f*x]*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m)) + (Cos[e + f*x]*(a + a*Sin
[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))))/(c
*(5 + 2*m))
```

3.212.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.212.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c - c\sin(fx+e))^{-3-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \frac{\left((2\,Bm-2\,A+3\,B)\cos(fx+e)^3+(4\,Bm^2-4\,(A-3\,B)m-6\,A+9\,B)\cos(fx+e)\sin(fx+e)+(4\right.}{8\,fm^3+36\,fm^2+4}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="fricas")
```

output
```
((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B
)*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*
x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)
```

3.212.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \int (a(\sin(e+fx)+1))^m\,(-c(\sin(e+fx)-1))^{-m-3}\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 3)*(A +
B*sin(e + f*x)), x)
```

3.212.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 3), x)
```

3.212.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 3), x)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.25

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m}\,dx =$$
$$-\frac{(a(\sin(e + fx) + 1))^m (30A\cos(e + fx) - 15B\cos(e + fx) - 2A\cos(3e + 3fx) + 3B\cos(3e +$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 3),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*co
s(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2
*e + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(
e + f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e
+ 2*f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*
m + 36*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e +
 3*f*x) - 10))
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

3.213.1 Optimal result 1779
3.213.2 Mathematica [A] (verified) 1779
3.213.3 Rubi [A] (verified) 1780
3.213.4 Maple [F] 1781
3.213.5 Fricas [A] (verification not implemented) 1782
3.213.6 Sympy [F] 1782
3.213.7 Maxima [F] 1782
3.213.8 Giac [F] 1783
3.213.9 Mupad [B] (verification not implemented) 1783

3.213.1 Optimal result

Integrand size = 40, antiderivative size = 114

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx\\&=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)}\\&\quad+\frac{(A-2B(1+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{cf(1+2m)(3+2m)}\end{aligned}$$

output
```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+(A-2
*B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2
+8*m+3)
```

3.213.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx\\&=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}(B-2A(1+m)+(A-2B(1+m))\sin(e+f}{ac^2f(1+2m)(3+2m)(-1+\sin(e+fx))}\end{aligned}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-2 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B - 2*A*(1 + m) + (A - 2*B*(
1 + m))*Sin[e + f*x]))/(a*c^2*f*(1 + 2*m)*(3 + 2*m)*(-1 + Sin[e + f*x])*(c
 - c*Sin[e + f*x])^m)
```

3.213.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-2}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-2}dx$$

$$\downarrow 3451$$

$$\frac{(A-2B(m+1))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

$$\downarrow 3042$$

$$\frac{(A-2B(m+1))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

$$\downarrow 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)
)/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

3.213.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.213.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{-2-m} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\frac{((2\,Bm-A+2\,B)\cos(fx+e)\sin(fx+e)+(2\,Am+2\,A-B)\cos(fx+e))(a\sin(fx+e)+a)^m(-cs}{4\,fm^2+8\,fm+3\,f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="fricas")
```

output
```
((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
 e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*
m + 3*f)
```

3.213.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\int (a(\sin(e+fx)+1))^m\,(-c(\sin(e+fx)-1))^{-m-2}\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 2)*(A +
B*sin(e + f*x)), x)
```

3.213.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 2), x)
```

3.213.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-2-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 2), x)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-2-m}\,dx =$$
$$-\frac{(a(\sin(e + fx) + 1))^m (4A\cos(e + fx) - 2B\cos(e + fx) - A\sin(2e + 2fx) + 2B\sin(2e + 2f}{c^2 f(-c(\sin(e + fx) - 1))^m (4m^2 + 8m + 3)(4\sin(e + fx) + \cos}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*
e + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2
*f*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x)
 + cos(2*e + 2*f*x) - 3))
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

3.214.1 Optimal result 1784
3.214.2 Mathematica [C] (verified) 1784
3.214.3 Rubi [A] (verified) 1785
3.214.4 Maple [F] 1788
3.214.5 Fricas [F] 1788
3.214.6 Sympy [F] 1789
3.214.7 Maxima [F] 1789
3.214.8 Giac [F] 1789
3.214.9 Mupad [F(-1)] 1790

3.214.1 Optimal result

Integrand size = 40, antiderivative size = 163

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$
$$- \frac{2^{\frac{1}{2}-m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)(1-s}{f(1+2m)}$$

output
```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)-2^(1
/2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1
-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

3.214.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.90

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$= \frac{2^{-1-m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-1-m)-2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-1-m}\left(\frac{1-}{}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-1 - m),x]
```

output
```
(2^(-1 - m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - m) - 2*m)*(a*(
1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(-1 - m)*((1 - Tan[(e + f*x)/2])
/Sqrt[(1 + Cos[e + f*x])^(-1)])^(2*m)*(I*B*(1 + 2*m)*Hypergeometric2F1[1,
-2*m, 1 - 2*m, ((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1
+ Tan[(e + f*x)/2]) - I*B*(1 + 2*m)*Hypergeometric2F1[1, -2*m, 1 - 2*m, (I
*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2])
- 2*(A + B)*m*(1 + Tan[(e + f*x)/2])))/(f*m*(1 + 2*m)*((1 - Tan[(e + f*x)/
2])/Sqrt[Sec[(e + f*x)/2]^2])^(2*m)*(-1 + Tan[(e + f*x)/2]))
```

3.214.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-1}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-1}dx$$

$$\downarrow 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx}{c}$$

$$\downarrow 3224$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} -$$
$$\frac{B\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{-2m}dx}{c}$$

$$\downarrow 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} -$$
$$\frac{B\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{-2m}dx}{c}$$

$$\downarrow 3168$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} -$$
$$\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int(c-c\sin(e+fx)$$
$$f$$

$$\downarrow 80$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} -$$
$$\frac{Bc2^{-m-\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+c)$$
$$f$$

$$\downarrow 79$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} -$$
$$\frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+c)^{\frac{1}{2}($$
$$f(2m+1)$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/
2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2
 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2)*(c
 + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(f*(1 + 2*m))
```

3.214.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

3.214.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{-1-m}\, dx$$

input

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

output

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

3.214.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-1}\, dx$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

3.214.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^{-m-1} (A + B\sin(e + fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 1)*(A +
B*sin(e + f*x)), x)
```

3.214.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-1}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 1), x)
```

3.214.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m-1}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 1), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\, dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c - c\,\sin(e + f\,x))^{m+1}}\, dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 1), x)
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

3.215.1 Optimal result 1791
3.215.2 Mathematica [C] (verified) 1791
3.215.3 Rubi [A] (verified) 1792
3.215.4 Maple [F] 1795
3.215.5 Fricas [F] 1795
3.215.6 Sympy [F(-1)] 1796
3.215.7 Maxima [F] 1796
3.215.8 Giac [F] 1796
3.215.9 Mupad [F(-1)] 1797

3.215.1 Optimal result

Integrand size = 38, antiderivative size = 158

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$= \frac{2^{\frac{1}{2}-m}c(A+2Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx)\right.}{f(1+2m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f}$$

output
```
2^(1/2-m)*c*(2*B*m+A)*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*
sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1
-m)/f/(1+2*m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/((c-c*sin(f*x+e))^m)
```

3.215.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.54

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx =$$
$$-\frac{i2^{-1-m}\left(a\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)^m(c-c\sin(e+fx))^{-m}(\cosh(m\log(2))+\sinh(m\log($$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^m,x]
```

output
```
((-I)*2^(-1 - m)*(a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)^m*(Cosh[m*Log
[2]] + Sinh[m*Log[2]])*((A*Hypergeometric2F1[1, -2*m, 1 - 2*m, -((Cos[e +
f*x] + I*(-1 + Sin[e + f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))])/m -
 (2*B*Hypergeometric2F1[2, 1 - 2*m, 2 - 2*m, -((Cos[e + f*x] + I*(-1 + Sin
[e + f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))]*(Cos[e + f*x] + I*(-1
+ Sin[e + f*x])))/((-1 + 2*m)*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))) + ((B
*Hypergeometric2F1[2*m, 1 + 2*m, 2 + 2*m, (1 - I*Cos[e + f*x] + Sin[e + f*
x])/2]*(Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x]))^(2*m)*(1 - I*Cos[e +
 f*x] + Sin[e + f*x]))/(1 + 2*m) - (A*Hypergeometric2F1[-2*m, -2*m, 1 - 2*
m, (1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*(Cosh[m*Log[16]] + Sinh[m*Log[16
]]))/(4^m*m))/(1 - I*Cos[e + f*x] + Sin[e + f*x])^(2*m)))/(f*(c - c*Sin[e
+ f*x])^m)
```

3.215.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}dx$$

$$\downarrow 3452$$

$$(A+2Bm)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 3042$$

$$(A+2Bm)\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 3224$$

$$(A+2Bm)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 3042$$

$$(A+2Bm)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 3168$$

$$\frac{c^2(A+2Bm)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int(c-c\sin(e+fx))^{-m}dx}{f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 80$$

$$\frac{c^2 2^{-m-\frac{1}{2}}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)-\frac{1}{2}}\int(1-\sin(e+fx))^{-m-\frac{1}{2}}dx}{f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\downarrow 79$$

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)-\frac{1}{2}}}{f(2m+1)}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x
]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)) + (2
^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2)*(c + c*Sin
[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(f*(1 + 2*m))
```

3.215.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.215.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (-c(\sin(fx+e)-1))^{-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

3.215.5 Fricas [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{-m}\, dx$$
$$= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
```

output
```
Timed out
```

3.215.7 Maxima [F]

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx\\ &= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^m, x)
```

3.215.8 Giac [F]

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx\\ &= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^m, x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-m}\, dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c - c\,\sin(e + f\,x))^m}\, dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
x)
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

3.216.1 Optimal result . . . 1798
3.216.2 Mathematica [F] . . . 1798
3.216.3 Rubi [A] (verified) . . . 1799
3.216.4 Maple [F] . . . 1802
3.216.5 Fricas [F] . . . 1802
3.216.6 Sympy [F] . . . 1802
3.216.7 Maxima [F] . . . 1803
3.216.8 Giac [F] . . . 1803
3.216.9 Mupad [F(-1)] . . . 1803

3.216.1 Optimal result

Integrand size = 40, antiderivative size = 170

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$= \frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+si\right.}{f(1+2m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f}$$

output
```
2^(1/2-m)*c^2*(2*A-B*(1-2*m))*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m]
,1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*
x+e))^(-1-m)/f/(1+2*m)-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)
)^(1-m)/f
```

3.216.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$= \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(1 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(1 - m), x]
```

3.216.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}dx$$

$$\downarrow 3452$$

$$\frac{1}{2}(2A-B(1-2m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{1-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 3042$$

$$\frac{1}{2}(2A-B(1-2m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{1-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 3224$$

$$\frac{1}{2}(2A-B(1-2m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{1-2m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 3042$$

$$\frac{1}{2}(2A-B(1-2m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{1-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 3168$$

$$\frac{c^2(2A-B(1-2m))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int}{2f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 80$$

$$\frac{c^2 2^{m-\frac{3}{2}}(2A-B(1-2m))\cos(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c}{f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\downarrow 79$$

$$-\frac{c2^{m-\frac{1}{2}}(2A-B(1-2m))\cos(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}(3}{f(}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

input

```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 -
m),x]
```

output

```
-1/2*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/
f - (2^(-1/2 + m)*c*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(1
- 2*m)/2, (3 - 2*m)/2, (5 - 2*m)/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x
])^(1/2 - m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^((-1 - 2*m)/2 + (
3 - 2*m)/2 + m)*(c + c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2 + m))/(f*(3 - 2*
m))
```

3.216.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.216.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{1-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

3.216.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+1}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

3.216.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^{1-m} (A + B\sin(e + fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(1 - m)*(A + B
*sin(e + f*x)), x)
```

3.216.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+1}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 1), x)
```

3.216.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+1}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 1), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^{1-m}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
m),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
m), x)
```

3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

3.217.1 Optimal result . . . 1805
3.217.2 Mathematica [F] . . . 1805
3.217.3 Rubi [A] (verified) . . . 1806
3.217.4 Maple [F] . . . 1809
3.217.5 Fricas [F] . . . 1809
3.217.6 Sympy [F] . . . 1809
3.217.7 Maxima [F] . . . 1810
3.217.8 Giac [F] . . . 1810
3.217.9 Mupad [F(-1)] . . . 1810

3.217.1 Optimal result

Integrand size = 40, antiderivative size = 173

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+si\right.}{3f(1+2m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f}$$

output
```
1/3*2^(5/2-m)*c^3*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([1/2+m, -3/2+m],[3/
2+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*si
n(f*x+e))^(-1-m)/f/(1+2*m)-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*
x+e))^(2-m)/f
```

3.217.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(2 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(2 - m), x]
```

3.217.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}dx$$

$$\downarrow 3452$$

$$\frac{1}{3}(3A-2B(1-m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{2-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A-2B(1-m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{2-m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 3224$$

$$\frac{1}{3}(3A-2B(1-m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{2-2m}dx - \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A-2B(1-m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{2-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 3168$$

$$\frac{c^2(3A-2B(1-m))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int}{3f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 80$$

$$\frac{c^3 2^{\frac{3}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m-\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}}(c}{3f}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\downarrow 79$$

$$\frac{c^2 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m-\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}}(c}{3f(2m}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 -
m),x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
f + (2^(5/2 - m)*c^2*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-
3 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^(-1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1/2 + (-1 - 2
*m)/2)*(c + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(3*f*(1 + 2*m))
```

3.217.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

3.217.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{2-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

3.217.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+2}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

3.217.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^{2-m} (A + B\sin(e + fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(2 - m)*(A + B
*sin(e + f*x)), x)
```

3.217.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+2}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 2), x)
```

3.217.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+2}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{2-m}\, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^{2-m}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
m),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
m), x)
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

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3.218.1 Optimal result

Integrand size = 46, antiderivative size = 34

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

output
```
a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(-3+n)/f
```

3.218.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3B(c-c\sin(e+fx))^n(14\cos(e+fx)-6\cos(3(e+fx))+14\sin(2(e+fx))-\sin(4(e+fx)))}{8f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4
+ n)*Sin[e + f*x]),x]
```

output
```
(a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*S
in[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

3.218.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3446, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(B(3-n)-B(n+4)\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(B(3-n)-B(n+4)\sin(e+fx))(c-c\sin(e+fx))^n dx$$

$$\downarrow 3446$$

$$a^3c^3\int \cos^6(e+fx)(c-c\sin(e+fx))^{n-3}(B(3-n)-B(n+4)\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^3c^3\int \cos(e+fx)^6(c-c\sin(e+fx))^{n-3}(B(3-n)-B(n+4)\sin(e+fx))dx$$

$$\downarrow 3333$$

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*S
in[e + f*x]),x]
```

output
```
(a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

3.218.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.218.4 Maple [A] (verified)

Time = 11.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

method	result	size
parallelrisch	$-\frac{a^3 B(-c(\sin(fx+e)-1))^n(6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e)-14\cos(fx+e))}{8f}$	63

input
```
int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/8*a^3*B*(-c*(sin(f*x+e)-1))^n*(6*cos(3*f*x+3*e)+sin(4*f*x+4*e)-14*sin(2
*f*x+2*e)-14*cos(f*x+e))/f
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int (a + a\sin(e+fx))^3 (c - c\sin(e+fx))^n (B(3-n) - B(4+n)\sin(e+fx))\,dx =$$
$$-\frac{\left(3\,Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e) + \left(Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e)\right)\sin(fx+e)\right)(-c$$
$$f$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="fricas")
```

output
```
-(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 -
4*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

3.218.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.

Time = 65.21 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a + a\sin(e + fx))^3 (c - c\sin(e + fx))^n (B(3 - n) - B(4 + n)\sin(e + fx))\, dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x
+e)),x)
```

output
```
Piecewise((-B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n
*tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*
f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c - 2*c*t
an(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e
/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*t
an(e/2 + f*x/2)**2 + f) - 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14
*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 +
f*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
 f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2
)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f
*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
 1))**n*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)*
*6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c
- 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*t
an(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4
*f*tan(e/2 + f*x/2)**2 + f) + B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 +
f*x/2)**2 + 1))**n/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6...
```

3.218.7 Maxima [F]

$$\int (a + a\sin(e + fx))^3 (c - c\sin(e + fx))^n (B(3 - n) - B(4 + n)\sin(e + fx))\, dx$$
$$= \int -(B(n + 4)\sin(fx + e) + B(n - 3))(a\sin(fx + e) + a)^3 (-c\sin(fx + e) + c)^n\, dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="maxima")
```

output
```
-integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c
*sin(f*x + e) + c)^n, x)
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9586 vs. $2(34) = 68$.

Time = 48.34 (sec) , antiderivative size = 9586, normalized size of antiderivative = 281.94

$$\int (a + a\sin(e + fx))^3 (c - c\sin(e + fx))^n (B(3 - n) - B(4 + n)\sin(e + fx))\, dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="giac")
```

output
```
(B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1
/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*
tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f
*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x
+ 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*tan(1/2*f*x + 1/2*e) - 2*c)*
sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*
f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*
e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*
c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n*
sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2*
f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)
^8 + 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x ...
```

3.218.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\frac{B\,a^3\left(-c\left(\sin\left(e+f\,x\right)-1\right)\right)^n\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\right.\right.}{8\,f}$$

input
```
int(-(B*(n - 3) + B*sin(e + f*x)*(n + 4))*(a + a*sin(e + f*x))^3*(c - c*si
n(e + f*x))^n,x)
```

output
```
(B*a^3*(-c*(sin(e + f*x) - 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 1
4*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

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3.219.8 Giac [B] (verification not implemented) 1822
3.219.9 Mupad [B] (verification not implemented) 1823

3.219.1 Optimal result

Integrand size = 45, antiderivative size = 34

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3+n}}{f}$$

output
```
-a^3*B*c^3*cos(f*x+e)^7*(c+c*sin(f*x+e))^(-3+n)/f
```

3.219.2 Mathematica [A] (verified)

Time = 7.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^n}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4
+ n)*Sin[e + f*x]),x]
```

output
```
-((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

3.219.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 3446, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))^3 (B(n+4)\sin(e+fx) + B(3-n))(c\sin(e+fx)+c)^n \, dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))^3 (B(n+4)\sin(e+fx) + B(3-n))(c\sin(e+fx)+c)^n dx$$

$$\downarrow 3446$$

$$a^3c^3 \int \cos^6(e+fx)(\sin(e+fx)c+c)^{n-3}(B(3-n)+B(n+4)\sin(e+fx))dx$$

$$\downarrow 3042$$

$$a^3c^3 \int \cos(e+fx)^6(\sin(e+fx)c+c)^{n-3}(B(3-n)+B(n+4)\sin(e+fx))dx$$

$$\downarrow 3333$$

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*S
in[e + f*x]),x]
```

output
```
-((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

3.219.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.219.4 Maple [A] (verified)

Time = 13.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

method	result	size
parallelrisch	$-\frac{a^3 B(c(1+\sin(fx+e)))^n(14\cos(fx+e)-6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e))}{8f}$	62

input
```
int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/8*a^3*B*(c*(1+sin(f*x+e)))^n*(14*cos(f*x+e)-6*cos(3*f*x+3*e)+sin(4*f*x+
4*e)-14*sin(2*f*x+2*e))/f
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int (a - a\sin(e+fx))^3 (c + c\sin(e+fx))^n (B(3-n) + B(4+n)\sin(e+fx))\,dx$$
$$= \frac{\left(3\,Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e) - \left(Ba^3\cos(fx+e)^3 - 4\,Ba^3\cos(fx+e)\right)\sin(fx+e)\right)(c\sin($$
$$f$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="fricas")
```

output
```
(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

3.219.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.

Time = 65.37 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a - a\sin(e+fx))^3 (c + c\sin(e+fx))^n (B(3-n) + B(4+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x
+e)),x)
```

output
```
Piecewise((B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f
*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c + 2*c*ta
n(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/
2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*ta
n(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*
x/2)**2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
+ f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*
B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f
*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*
x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)
**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*
x/2)**2 + f) - 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
1))**n*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**
6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c +
 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*ta
n(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*
f*tan(e/2 + f*x/2)**2 + f) - B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**n/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*...
```

3.219.7 Maxima [F]

$$\int (a - a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$= \int -(B(n+4)\sin(fx+e) - B(n-3))(a\sin(fx+e)-a)^3(c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="maxima")
```

output
```
-integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*
sin(f*x + e) + c)^n, x)
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9587 vs. $2(34) = 68$.

Time = 48.82 (sec) , antiderivative size = 9587, normalized size of antiderivative = 281.97

$$\int (a - a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="giac")
```

output
```
-(B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) +
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-
1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c
*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*
f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*n*floor(1/4*sgn(2*c*tan(1/2*f*x
 + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c*tan(1/2*f*x + 1/2*e) - 2*c)
*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2
*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2
*e)) + 1/4*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4
*c*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*n
*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e)^2 + 4*c*tan(1/2
*f*x + 1/2*e)) + 1/4*pi*n*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e
)^8 - 6*B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x...
```

3.219.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int (a - a\sin(e+fx))^3 (c + c\sin(e+fx))^n (B(3-n) + B(4+n)\sin(e+fx))\,dx =$$
$$-\frac{B\,a^3\,(c\,(\sin(e+f\,x)+1))^n\,(14\,\cos(e+f\,x) - 6\,\cos(3\,e+3\,f\,x) - 14\,\sin(2\,e+2\,f\,x) + \sin(4\,e+4\,f}{8\,f}$$

input
```
int(-(B*(n - 3) - B*sin(e + f*x)*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*si
n(e + f*x))^n,x)
```

output
```
-(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 1
4*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$

3.220.1 Optimal result 1824
3.220.2 Mathematica [A] (verified) 1824
3.220.3 Rubi [A] (verified) 1825
3.220.4 Maple [A] (verified) 1826
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3.220.9 Mupad [B] (verification not implemented) 1830

3.220.1 Optimal result

Integrand size = 44, antiderivative size = 33

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f}$$

output
```
a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f
```

3.220.2 Mathematica [A] (verified)

Time = 7.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
 + m)*Sin[e + f*x]),x]
```

output
```
(B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

3.220.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3042, 3446, 25, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e+fx))^3 (a\sin(e+fx)+a)^m (B(m-3) - B(m+4)\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c - c\sin(e+fx))^3 (a\sin(e+fx)+a)^m (B(m-3) - B(m+4)\sin(e+fx))dx$$

$$\downarrow 3446$$

$$a^3c^3 \int -\cos^6(e+fx)(\sin(e+fx)a+a)^{m-3}(B(3-m)+B(m+4)\sin(e+fx))dx$$

$$\downarrow 25$$

$$-a^3c^3 \int \cos^6(e+fx)(\sin(e+fx)a+a)^{m-3}(B(3-m)+B(m+4)\sin(e+fx))dx$$

$$\downarrow 3042$$

$$-a^3c^3 \int \cos(e+fx)^6(\sin(e+fx)a+a)^{m-3}(B(3-m)+B(m+4)\sin(e+fx))dx$$

$$\downarrow 3333$$

$$\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*
Sin[e + f*x]),x]
```

output
```
(a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.220.4 Maple [A] (verified)

Time = 13.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
parallelrisch	$\frac{c^3 B(a(1+\sin(fx+e)))^m(14\cos(fx+e)-6\cos(3fx+3e)+\sin(4fx+4e)-14\sin(2fx+2e))}{8f}$	62

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
1/8*c^3*B*(a*(1+sin(f*x+e)))^m*(14*cos(f*x+e)-6*cos(3*f*x+3*e)+sin(4*f*x+4
*e)-14*sin(2*f*x+2*e))/f
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^3 (B(-3+m) - B(4+m)\sin(e+fx))\,dx =$$
$$-\frac{\left(3\,Bc^3\cos(fx+e)^3 - 4\,Bc^3\cos(fx+e) - \left(Bc^3\cos(fx+e)^3 - 4\,Bc^3\cos(fx+e)\right)\sin(fx+e)\right)(a\sin$$
$$f$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="fricas")
```

output
```
-(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 -
4*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

3.220.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.

Time = 65.47 (sec) , antiderivative size = 898, normalized size of antiderivative = 27.21

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^3 (B(-3+m) - B(4+m)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*
x+e)),x)
```

output
```
Piecewise((-B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m
*tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*
f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a + 2*a*t
an(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e
/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*t
an(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14
*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 +
f*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
 f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2
)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f
*x/2)**2 + f) + 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
 1))**m*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)*
*6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a
+ 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*t
an(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4
*f*tan(e/2 + f*x/2)**2 + f) + B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 +
f*x/2)**2 + 1))**m/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6...
```

3.220.7 Maxima [F]

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^3 (B(-3+m) - B(4+m)\sin(e+fx))\,dx$$
$$= \int (B(m+4)\sin(fx+e) - B(m-3))(c\sin(fx+e) - c)^3 (a\sin(fx+e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*s
in(f*x + e) + a)^m, x)
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9586 vs. 2(33) = 66.

Time = 45.41 (sec) , antiderivative size = 9586, normalized size of antiderivative = 290.48

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^3 (B(-3+m) - B(4+m)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="giac")
```

output
```
(B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1
/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*
tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f
*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x
+ 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*tan(1/2*f*x + 1/2*e) - 2*a)*
sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*
f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*
e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*
a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m*
sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2*
f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)
^8 - 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x ...
```

3.220.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^3 (B(-3+m) - B(4+m)\sin(e+fx))\,dx$$
$$= \frac{B\,c^3\,(a\,(\sin(e+f\,x)+1))^m\,(14\,\cos(e+f\,x) - 6\,\cos(3\,e+3\,f\,x) - 14\,\sin(2\,e+2\,f\,x) + \sin(4\,e+4\,f\,x}{8\,f}$$

input
```
int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin
(e + f*x))^3,x)
```

output
```
(B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

3.221.1 Optimal result 1831
3.221.2 Mathematica [A] (verified) 1831
3.221.3 Rubi [A] (verified) 1832
3.221.4 Maple [A] (verified) 1833
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3.221.9 Mupad [B] (verification not implemented) 1837

3.221.1 Optimal result

Integrand size = 43, antiderivative size = 35

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{-3+m}}{f}$$

output
```
-a^3*B*c^3*cos(f*x+e)^7*(a-a*sin(f*x+e))^(-3+m)/f
```

3.221.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \frac{Bc^3(a-a\sin(e+fx))^m(-14\cos(e+fx)+6\cos(3(e+fx))-14\sin(2(e+fx))+\sin(4(e+fx)))}{8f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
 + m)*Sin[e + f*x]),x]
```

output
```
(B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*
Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

3.221.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 3446, 25, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(e+fx)+c)^3(a-a\sin(e+fx))^m(B(m+4)\sin(e+fx)+B(m-3))\,dx$$

$$\downarrow 3042$$

$$\int (c\sin(e+fx)+c)^3(a-a\sin(e+fx))^m(B(m+4)\sin(e+fx)+B(m-3))dx$$

$$\downarrow 3446$$

$$a^3c^3\int -\cos^6(e+fx)(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\downarrow 25$$

$$-a^3c^3\int \cos^6(e+fx)(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\downarrow 3042$$

$$-a^3c^3\int \cos(e+fx)^6(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\downarrow 3333$$

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*
Sin[e + f*x]),x]
```

output
```
-((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.221.4 Maple [A] (verified)

Time = 11.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{c^3 B(-a(\sin(fx+e)-1))^m(-6\cos(3fx+3e)-\sin(4fx+4e)+14\sin(2fx+2e)+14\cos(fx+e))}{8f}$	65

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
-1/8*c^3*B*(-a*(sin(f*x+e)-1))^m*(-6*cos(3*f*x+3*e)-sin(4*f*x+4*e)+14*sin(
2*f*x+2*e)+14*cos(f*x+e))/f
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.20

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx$$
$$= \frac{\left(3Bc^3\cos(fx+e)^3 - 4Bc^3\cos(fx+e) + \left(Bc^3\cos(fx+e)^3 - 4Bc^3\cos(fx+e)\right)\sin(fx+e)\right)(-a\sin$$
$$f$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="fricas")
```

output
```
(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.

Time = 66.13 (sec) , antiderivative size = 898, normalized size of antiderivative = 25.66

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*
x+e)),x)
```

output
```
Piecewise((B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f
*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a - 2*a*ta
n(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/
2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*ta
n(e/2 + f*x/2)**2 + f) + 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*
x/2)**2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
+ f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*
B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f
*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*
x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)
**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*
x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
1))**m*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**
6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a -
 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*ta
n(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*
f*tan(e/2 + f*x/2)**2 + f) - B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**m/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*...
```

3.221.7 Maxima [F]

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx$$
$$= \int (B(m+4)\sin(fx+e) + B(m-3))(c\sin(fx+e)+c)^3(-a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*
sin(f*x + e) + a)^m, x)
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9587 vs. 2(35) = 70.

Time = 41.81 (sec) , antiderivative size = 9587, normalized size of antiderivative = 273.91

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="giac")
```

output
```
-(B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) -
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-
1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a
*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*
f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)) + pi*m*floor(1/4*sgn(2*a*tan(1/2*f*x
 + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a*tan(1/2*f*x + 1/2*e) - 2*a)
*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2
*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2
*e)) + 1/4*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4
*a*tan(1/2*f*x + 1/2*e)) - 1/4*sgn(tan(1/2*f*x + 1/2*e)) + 1/2) - 1/4*pi*m
*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e)^2 + 4*a*tan(1/2
*f*x + 1/2*e)) + 1/4*pi*m*sgn(tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e
)^8 + 6*B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x...
```

3.221.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx =$$
$$-\frac{B\,c^3\,(-a\,(\sin(e+f\,x)-1))^m\,(14\,\cos(e+f\,x) - 6\,\cos(3\,e+3\,f\,x) + 14\,\sin(2\,e+2\,f\,x) - \sin(4\,e+4}{8\,f}$$

input
```
int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin
(e + f*x))^3,x)
```

output
```
-(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) +
14*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

3.222.1 Optimal result 1838
3.222.2 Mathematica [A] (verified) 1838
3.222.3 Rubi [A] (verified) 1839
3.222.4 Maple [A] (verified) 1840
3.222.5 Fricas [A] (verification not implemented) 1840
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3.222.8 Giac [B] (verification not implemented) 1842
3.222.9 Mupad [B] (verification not implemented) 1843

3.222.1 Optimal result

Integrand size = 47, antiderivative size = 36

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f}$$

output
```
B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f
```

3.222.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1
+ m + n)*Sin[e + f*x]),x]
```

output
```
(B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

3.222.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3042, 3449}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(B(m-n)-B(m+n+1)\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(B(m-n)-B(m+n+1)\sin(e+fx))dx$$

$$\downarrow 3449$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m +
n)*Sin[e + f*x]),x]
```

output
```
(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

3.222.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3449
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

3.222.4 Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{B(a(1+\sin(fx+e)))^m(-c(\sin(fx+e)-1))^n\cos(fx+e)}{f}$	37

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x
,method=_RETURNVERBOSE)
```

output
```
B/f*(a*(1+sin(f*x+e)))^m*(-c*(sin(f*x+e)-1))^n*cos(f*x+e)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \frac{(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n B\cos(fx+e)}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="fricas")
```

output
```
(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

3.222.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= -B\left(\int (-m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n)\,dx\right.$$
$$+\int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\,dx$$
$$+\int (a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx$$
$$+\int m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx$$
$$\left.+\int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f
*x+e)),x)
```

output
```
-B*(Integral(-m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Int
egral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Integral((a
*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral
(m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Int
egral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

3.222.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \int -(B(m+n+1)\sin(fx+e)-B(m-n))(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="maxima")
```

output
```
-integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m
*(-c*sin(f*x + e) + c)^n, x)
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8947 vs. $2(36) = 72$.

Time = 53.15 (sec) , antiderivative size = 8947, normalized size of antiderivative = 248.53

$$\int (a + a\sin(e+fx))^m (c - c\sin(e+fx))^n (B(m-n) - B(1+m+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="giac")
```

output
```
(B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/
2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
+ 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2
) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 +
2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi
*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e
)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x +
1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(
f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/
2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e...
```

3.222.9 Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \frac{B\,\cos(e+f\,x)\,\left(a\left(\sin(e+f\,x)+1\right)\right)^m\left(-c\left(\sin(e+f\,x)-1\right)\right)^n}{f}$$

input
```
int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c
*sin(e + f*x))^n,x)
```

output
```
(B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

3.223.1 Optimal result 1844
3.223.2 Mathematica [A] (verified) 1844
3.223.3 Rubi [A] (verified) 1845
3.223.4 Maple [A] (verified) 1846
3.223.5 Fricas [A] (verification not implemented) 1846
3.223.6 Sympy [F] 1847
3.223.7 Maxima [F] 1847
3.223.8 Giac [B] (verification not implemented) 1848
3.223.9 Mupad [B] (verification not implemented) 1849

3.223.1 Optimal result

Integrand size = 46, antiderivative size = 37

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c+c\sin(e+fx))^n}{f}$$

output
```
-B*cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n/f
```

3.223.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(c(1+\sin(e+fx)))^n(a-a\sin(e+fx))^m}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1
+ m + n)*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

3.223.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3042, 3449}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))^m (c\sin(e+fx)+c)^n (B(m+n+1)\sin(e+fx)+B(m-n))\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))^m (c\sin(e+fx)+c)^n (B(m+n+1)\sin(e+fx)+B(m-n))dx$$

$$\downarrow 3449$$

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m (c\sin(e+fx)+c)^n}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n)*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

3.223.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 3449
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m + n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2^(-1)]
```

3.223.4 Maple [A] (verified)

Time = 9.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$-\frac{B(-a(\sin(fx+e)-1))^m(c(1+\sin(fx+e)))^n\cos(fx+e)}{f}$	38

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x
,method=_RETURNVERBOSE)
```

output
```
-B/f*(-a*(sin(f*x+e)-1))^m*(c*(1+sin(f*x+e)))^n*cos(f*x+e)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^n (B(m-n) + B(1+m+n)\sin(e+fx))\,dx$$
$$= -\frac{(-a\sin(fx+e)+a)^m (c\sin(fx+e)+c)^n B\cos(fx+e)}{f}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="fricas")
```

output
```
-(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

3.223.6 Sympy [F]

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^n (B(m-n) + B(1+m+n)\sin(e+fx))\,dx$$
$$= B\left(\int m(-a\sin(e+fx)+a)^m (c\sin(e+fx)+c)^n\,dx \right.$$
$$+ \int (-n(-a\sin(e+fx)+a)^m (c\sin(e+fx)+c)^n)\,dx$$
$$+ \int (-a\sin(e+fx)+a)^m (c\sin(e+fx)+c)^n \sin(e+fx)\,dx$$
$$+ \int m(-a\sin(e+fx)+a)^m (c\sin(e+fx)+c)^n \sin(e+fx)\,dx$$
$$\left. + \int n(-a\sin(e+fx)+a)^m (c\sin(e+fx)+c)^n \sin(e+fx)\,dx\right)$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f
*x+e)),x)
```

output
```
B*(Integral(m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integ
ral(-n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integral((-a
*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(
m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Inte
gral(n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

3.223.7 Maxima [F]

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^n (B(m-n) + B(1+m+n)\sin(e+fx))\,dx$$
$$= \int (B(m+n+1)\sin(fx+e) + B(m-n))(-a\sin(fx+e)+a)^m (c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m
*(c*sin(f*x + e) + c)^n, x)
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8948 vs. 2(37) = 74.

Time = 51.92 (sec) , antiderivative size = 8948, normalized size of antiderivative = 241.84

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^n (B(m-n) + B(1+m+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="giac")
```

output
```
-(B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1
/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan
(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
 + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) +
2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f
*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 +
 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*p
i*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x +
e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 -
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x +
 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan
(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1
/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*...
```

3.223.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^n (B(m-n) + B(1+m+n)\sin(e+fx))\,dx$$
$$= -\frac{B\,\cos(e+f\,x)\,\left(-a\,(\sin(e+f\,x)-1)\right)^m\,\left(c\,(\sin(e+f\,x)+1)\right)^n}{f}$$

input
```
int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c
*sin(e + f*x))^n,x)
```

output
```
-(B*cos(e + f*x)*(-a*(sin(e + f*x) - 1))^m*(c*(sin(e + f*x) + 1))^n)/f
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.224.1 Optimal result 1850
3.224.2 Mathematica [A] (verified) 1851
3.224.3 Rubi [A] (verified) 1851
3.224.4 Maple [A] (verified) 1852
3.224.5 Fricas [A] (verification not implemented) 1853
3.224.6 Sympy [B] (verification not implemented) 1854
3.224.7 Maxima [A] (verification not implemented) 1854
3.224.8 Giac [A] (verification not implemented) 1855
3.224.9 Mupad [B] (verification not implemented) 1855

3.224.1 Optimal result

Integrand size = 32, antiderivative size = 140

$$\begin{aligned}
&\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx \\
&= \frac{1}{8}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{3a^3A\cos^5(c+dx)}{5d} \\
&\quad - \frac{a^3A\cos^7(c+dx)}{7d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}
\end{aligned}$$

output

```
1/8*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+3/5*a^3*A*cos(d*x+c)^5/d-1/7*a^3*A*co
s(d*x+c)^7/d-1/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/12*a^3*A*cos(d*x+c)*sin(d
*x+c)^3/d+1/3*a^3*A*cos(d*x+c)*sin(d*x+c)^5/d
```

3.224.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^3A(840c+840dx-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+dx))-15\cos(7(c+dx))-2}{6720d}$$

input
```
Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*C
os[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(
c + d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

3.224.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\downarrow 3042$$
$$\int \sin(c+dx)^3(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$
$$\downarrow 3445$$
$$\int \left(-a^3A\sin^7(c+dx)-2a^3A\sin^6(c+dx)+2a^3A\sin^4(c+dx)+a^3A\sin^3(c+dx)\right)dx$$
$$\downarrow 2009$$
$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^3Ax$$

input
```
Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5
*d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*
d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[
c + d*x]^5)/(3*d)
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.224.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

method	result
parallelrisch	$-\frac{A\,a^3(-840dx+1365\cos(dx+c)+15\cos(7dx+7c)-70\sin(6dx+6c)-147\cos(5dx+5c)+210\sin(4dx+4c)+175\cos(3dx+$
risch	$\frac{a^3Ax}{8}-\frac{13a^3A\cos(dx+c)}{64d}-\frac{A\,a^3\cos(7dx+7c)}{448d}+\frac{A\,a^3\sin(6dx+6c)}{96d}+\frac{7A\,a^3\cos(5dx+5c)}{320d}-\frac{A\,a^3\sin(4dx+4c)}{32d}$
derivativedivides	$\frac{A\,a^3\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2A\,a^3\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos$
default	$\frac{A\,a^3\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}-2A\,a^3\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos$
parts	$-\frac{A\,a^3(2+\sin^2(dx+c))\cos(dx+c)}{3d}+\frac{2A\,a^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}-\frac{2A\,a^3\left(-\frac{\left(\sin^5(dx$
norman	$-\frac{44A\,a^3}{105d}+\frac{a^3Ax}{8}-\frac{4A\,a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8A\,a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{44A\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{52A\,a^3\left($

input
```
int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-1/6720*A*a^3*(-840*d*x+1365*cos(d*x+c)+15*cos(7*d*x+7*c)-70*sin(6*d*x+6*c
)-147*cos(5*d*x+5*c)+210*sin(4*d*x+4*c)+175*cos(3*d*x+3*c)+210*sin(2*d*x+2
*c)+1408)/d
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{120\,Aa^3\cos(dx+c)^7-504\,Aa^3\cos(dx+c)^5+560\,Aa^3\cos(dx+c)^3-105\,Aa^3dx-35\left(8\,Aa^3\cos(dx\right.}{840\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*co
s(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d
*x + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

3.224.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(131) = 262$.

Time $= 0.48$ (sec) , antiderivative size $= 440$, normalized size of antiderivative $= 3.14$

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)}{2} \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^3(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a
**3*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*
x)**6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*
a**3*sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c +
d*x)**3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin
(c + d*x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d
 + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos
(c + d*x)**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d
*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, T
rue))
```

3.224.7 Maxima [A] (verification not implemented)

Time $= 0.24$ (sec) , antiderivative size $= 157$, normalized size of antiderivative $= 1.12$

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{96\left(5\cos(dx+c)^7 - 21\cos(dx+c)^5 + 35\cos(dx+c)^3 - 35\cos(dx+c)\right)Aa^3 - 1120\left(\cos(dx+c)^3 -$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35
*cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(
4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))
*A*a^3)/d
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{1}{8}Aa^3x - \frac{Aa^3\cos(7\,dx+7\,c)}{448\,d} + \frac{7\,Aa^3\cos(5\,dx+5\,c)}{320\,d} - \frac{5\,Aa^3\cos(3\,dx+3\,c)}{192\,d}$$
$$- \frac{13\,Aa^3\cos(dx+c)}{64\,d} + \frac{Aa^3\sin(6\,dx+6\,c)}{96\,d} - \frac{Aa^3\sin(4\,dx+4\,c)}{32\,d} - \frac{Aa^3\sin(2\,dx+2\,c)}{32\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c
)/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A
*a^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2
*d*x + 2*c)/d
```

3.224.9 Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.14

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{A\,a^3\left(105\,c - 210\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 2464\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1400\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 4032\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 6790\,\text{ta}\right.}{}$$

input
```
int(sin(c + d*x)^3*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*(105*c - 210*tan(c/2 + (d*x)/2) - 2464*tan(c/2 + (d*x)/2)^2 - 1400*
tan(c/2 + (d*x)/2)^3 - 4032*tan(c/2 + (d*x)/2)^4 + 6790*tan(c/2 + (d*x)/2)
^5 + 2240*tan(c/2 + (d*x)/2)^6 - 14560*tan(c/2 + (d*x)/2)^8 - 6790*tan(c/2
 + (d*x)/2)^9 - 3360*tan(c/2 + (d*x)/2)^10 + 1400*tan(c/2 + (d*x)/2)^11 +
210*tan(c/2 + (d*x)/2)^13 + 105*d*x + 735*tan(c/2 + (d*x)/2)^2*(c + d*x) +
 2205*tan(c/2 + (d*x)/2)^4*(c + d*x) + 3675*tan(c/2 + (d*x)/2)^6*(c + d*x)
 + 3675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 2205*tan(c/2 + (d*x)/2)^10*(c + d
*x) + 735*tan(c/2 + (d*x)/2)^12*(c + d*x) + 105*tan(c/2 + (d*x)/2)^14*(c +
 d*x) - 352))/(840*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.225.1 Optimal result 1857
3.225.2 Mathematica [A] (verified) 1857
3.225.3 Rubi [A] (verified) 1858
3.225.4 Maple [A] (verified) 1859
3.225.5 Fricas [A] (verification not implemented) 1860
3.225.6 Sympy [B] (verification not implemented) 1860
3.225.7 Maxima [A] (verification not implemented) 1861
3.225.8 Giac [A] (verification not implemented) 1861
3.225.9 Mupad [B] (verification not implemented) 1862

3.225.1 Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3}{16}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} - \frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d}$$
$$+ \frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d}$$

output
```
3/16*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+2/5*a^3*A*cos(d*x+c)^5/d-3/16*a^3*A*
cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^3*A*cos
(d*x+c)*sin(d*x+c)^5/d
```

3.225.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^3A(180c+180dx-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))-15\sin(2(c+dx))-45\sin$$
$$960d$$

input
```
Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[
5*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*
x)]))/(960*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \sin(c+dx)^2(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$

$$\downarrow 3445$$

$$\int \left(-a^3A\sin^6(c+dx)-2a^3A\sin^5(c+dx)+2a^3A\sin^3(c+dx)+a^3A\sin^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}a^3Ax$$

input
```
Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)
/(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x
]*Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.225.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{A a^3(-180dx+240\cos(dx+c)-5\sin(6dx+6c)-24\cos(5dx+5c)+45\sin(4dx+4c)+40\cos(3dx+3c)+15\sin(2dx+2c)+2}{960d}$
risch	$\frac{3a^3Ax}{16}-\frac{a^3A\cos(dx+c)}{4d}+\frac{Aa^3\sin(6dx+6c)}{192d}+\frac{Aa^3\cos(5dx+5c)}{40d}-\frac{3Aa^3\sin(4dx+4c)}{64d}-\frac{Aa^3\cos(3dx+3c)}{24d}-$
derivativedivides	$\frac{-Aa^3\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2Aa^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(d}{5}}{d}$
default	$\frac{-Aa^3\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2Aa^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(d}{5}}{d}$
parts	$\frac{Aa^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2Aa^3(2+\sin^2(dx+c))\cos(dx+c)}{3d}+\frac{2Aa^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)}{5d}$
norman	$-\frac{8Aa^3}{15d}+\frac{3a^3Ax}{16}-\frac{16Aa^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16Aa^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{8Aa^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3Aa^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{13Aa^3\left(\tan^3$

input
```
int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-1/960*A*a^3*(-180*d*x+240*cos(d*x+c)-5*sin(6*d*x+6*c)-24*cos(5*d*x+5*c)+4
5*sin(4*d*x+4*c)+40*cos(3*d*x+3*c)+15*sin(2*d*x+2*c)+256)/d
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{96\,Aa^3\cos(dx+c)^5 - 160\,Aa^3\cos(dx+c)^3 + 45\,Aa^3dx + 5\left(8\,Aa^3\cos(dx+c)^5 - 26\,Aa^3\cos(dx+c)^3 +\right.}{240\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
 5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c
))*sin(d*x + c))/d
```

3.225.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(119) = 238.

Time = 0.35 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.97

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{Aa^3x\sin^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{16} \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^2(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*co
s(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3
*x*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*
x)**2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c +
 d*x)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) +
 8*A*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2
*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*si
n(c + d*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**
3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*s
in(c)**2, True))
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{128\left(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c)\right)Aa^3 + 640\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)Aa^3}{}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3
+ 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 +
60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3}{16}Aa^3x + \frac{Aa^3\cos(5\,dx+5\,c)}{40\,d} - \frac{Aa^3\cos(3\,dx+3\,c)}{24\,d} - \frac{Aa^3\cos(dx+c)}{4\,d}$$
$$+ \frac{Aa^3\sin(6\,dx+6\,c)}{192\,d} - \frac{3\,Aa^3\sin(4\,dx+4\,c)}{64\,d} - \frac{Aa^3\sin(2\,dx+2\,c)}{64\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)
/d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^
3*sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

3.225.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.12

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{A\,a^3\left(45\,c-90\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-768\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+130\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+1500\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-1280\tan\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int(sin(c + d*x)^2*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*(45*c - 90*tan(c/2 + (d*x)/2) - 768*tan(c/2 + (d*x)/2)^2 + 130*tan(
c/2 + (d*x)/2)^3 + 1500*tan(c/2 + (d*x)/2)^5 - 1280*tan(c/2 + (d*x)/2)^6 -
 1500*tan(c/2 + (d*x)/2)^7 - 1920*tan(c/2 + (d*x)/2)^8 - 130*tan(c/2 + (d*
x)/2)^9 + 90*tan(c/2 + (d*x)/2)^11 + 45*d*x + 270*tan(c/2 + (d*x)/2)^2*(c
+ d*x) + 675*tan(c/2 + (d*x)/2)^4*(c + d*x) + 900*tan(c/2 + (d*x)/2)^6*(c
+ d*x) + 675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 270*tan(c/2 + (d*x)/2)^10*(c
 + d*x) + 45*tan(c/2 + (d*x)/2)^12*(c + d*x) - 128))/(240*d*(tan(c/2 + (d*
x)/2)^2 + 1)^6)
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.226.1 Optimal result 1863
3.226.2 Mathematica [A] (verified) 1863
3.226.3 Rubi [A] (verified) 1864
3.226.4 Maple [A] (verified) 1865
3.226.5 Fricas [A] (verification not implemented) 1866
3.226.6 Sympy [B] (verification not implemented) 1866
3.226.7 Maxima [A] (verification not implemented) 1867
3.226.8 Giac [A] (verification not implemented) 1867
3.226.9 Mupad [B] (verification not implemented) 1868

3.226.1 Optimal result

Integrand size = 30, antiderivative size = 96

$$\begin{aligned}&\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{1}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos^5(c+dx)}{5d}\\&\quad-\frac{a^3A\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}\end{aligned}$$

output
```
1/4*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+1/5*a^3*A*cos(d*x+c)^5/d-1/4*a^3*A*co
s(d*x+c)*sin(d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d
```

3.226.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\begin{aligned}&\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{a^3A\cos(c+dx)\left(-30\arcsin\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(c+dx)}(-28-15\sin(c+dx)+16\sin^2(c+dx)+3\right.}{60d\sqrt{\cos^2(c+dx)}}\end{aligned}$$

input
```
Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*Cos[c + d*x]*(-30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[Cos
[c + d*x]^2]*(-28 - 15*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 30*Sin[c + d*x]^
3 + 12*Sin[c + d*x]^4)))/(60*d*Sqrt[Cos[c + d*x]^2])
```

3.226.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \sin(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$

$$\downarrow 3445$$

$$\int \left(-a^3A\sin^5(c+dx)-2a^3A\sin^4(c+dx)+2a^3A\sin^2(c+dx)+a^3A\sin(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

input
```
Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d
) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c +
d*x]^3)/(2*d)
```

3.226.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.226.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

method	result
parallelrisch	$-\frac{A a^3(-60dx+90\cos(dx+c)-3\cos(5dx+5c)+15\sin(4dx+4c)+25\cos(3dx+3c)+112)}{240d}$
risch	$\frac{a^3Ax}{4}-\frac{3a^3A\cos(dx+c)}{8d}+\frac{Aa^3\cos(5dx+5c)}{80d}-\frac{Aa^3\sin(4dx+4c)}{16d}-\frac{5Aa^3\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{Aa^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2Aa^3\left(-\cos$ … d
default	$\frac{Aa^3\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2Aa^3\left(-\cos$ … d
parts	$-\frac{a^3A\cos(dx+c)}{d}+\frac{2Aa^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right.}{d}$ …
norman	$-\frac{14Aa^3}{15d}+\frac{a^3Ax}{4}-\frac{4Aa^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2Aa^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8Aa^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8Aa^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{Aa^3\tan\left(\frac{d}{2}\right.}{2d}$ …

input
```
int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
-1/240*A*a^3*(-60*d*x+90*cos(d*x+c)-3*cos(5*d*x+5*c)+15*sin(4*d*x+4*c)+25*
cos(3*d*x+3*c)+112)/d
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{12\,Aa^3\cos(dx+c)^5 - 40\,Aa^3\cos(dx+c)^3 + 15\,Aa^3dx - 15\left(2\,Aa^3\cos(dx+c)^3 - Aa^3\cos(dx+c)\right)\sin(}{60\,d}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 1
5*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

3.226.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(88) = 176.

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.78

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2} + Aa^3x\sin^2(c+dx) - \frac{3Aa^3x\cos^4(c+dx)}{4} + Aa^3x\cos^2(c+dx) - \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin(c) \end{cases}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(
c + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 +
A*a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**
3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*
x)**3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c +
 d*x)*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*
x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{16\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\sin$$
$$240\,d$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="max
ima")
```

output
```
1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
 15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2
*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{1}{4}Aa^3x+\frac{Aa^3\cos(5\,dx+5\,c)}{80\,d}-\frac{5\,Aa^3\cos(3\,dx+3\,c)}{48\,d}$$
$$-\frac{3\,Aa^3\cos(dx+c)}{8\,d}-\frac{Aa^3\sin(4\,dx+4\,c)}{16\,d}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gia
c")
```

output
```
1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/
d - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

3.226.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.04

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{A\,a^3\,x}{4}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-120)}{60}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-160)}{60}\right.}$$

input
```
int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*x)/4 - (tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(
75*c + 75*d*x - 120))/60) + tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/
12 - (A*a^3*(75*c + 75*d*x - 160))/60) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*
c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + tan(c/2 + (d*x)/2)^6
*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3
*tan(c/2 + (d*x)/2))/2 - 3*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c/2 +
(d*x)/2)^7 - (A*a^3*tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 -
 (A*a^3*(15*c + 15*d*x - 56))/60)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.227.1 Optimal result 1869
3.227.2 Mathematica [A] (verified) 1869
3.227.3 Rubi [A] (verified) 1870
3.227.4 Maple [A] (verified) 1872
3.227.5 Fricas [A] (verification not implemented) 1873
3.227.6 Sympy [B] (verification not implemented) 1873
3.227.7 Maxima [A] (verification not implemented) 1874
3.227.8 Giac [A] (verification not implemented) 1874
3.227.9 Mupad [B] (verification not implemented) 1875

3.227.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5}{8}a^3Ax - \frac{5a^3A\cos^3(c+dx)}{12d} + \frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d} - \frac{A\cos^3(c+dx)\,(a^3+a^3\sin(c+dx))}{4d}$$

output
```
5/8*a^3*A*x-5/12*a^3*A*cos(d*x+c)^3/d+5/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/
4*A*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

3.227.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A(60dx-48\cos(c+dx)-16\cos(3(c+dx))+24\sin(2(c+dx))-3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x
)] - 3*Sin[4*(c + d*x)]))/(96*d)
```

3.227.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3215, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$

$$\downarrow 3215$$

$$aA\int \cos^2(c+dx)(\sin(c+dx)a+a)^2dx$$

$$\downarrow 3042$$

$$aA\int \cos(c+dx)^2(\sin(c+dx)a+a)^2dx$$

$$\downarrow 3157$$

$$aA\left(\frac{5}{4}a\int \cos^2(c+dx)(\sin(c+dx)a+a)dx-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow 3042$$

$$aA\left(\frac{5}{4}a\int \cos(c+dx)^2(\sin(c+dx)a+a)dx-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow 3148$$

$$aA\left(\frac{5}{4}a\left(a\int \cos^2(c+dx)dx-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow 3042$$

$$aA\left(\frac{5}{4}a\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow 3115$$

$$aA\left(\frac{5}{4}a\left(a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow 24$$

$$aA\left(\frac{5}{4}a\left(a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

input
```
Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a*A*(-1/4*(Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/d + (5*a*(-1/3*(a*Cos[
c + d*x]^3)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))))/4)
```

3.227.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3215
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c +
 d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((Lt
Q[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.227.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{A a^3(-60dx+48\cos(dx+c)+3\sin(4dx+4c)+16\cos(3dx+3c)-24\sin(2dx+2c)+64)}{96d}$
risch	$\frac{5a^3Ax}{8}-\frac{a^3A\cos(dx+c)}{2d}-\frac{Aa^3\sin(4dx+4c)}{32d}-\frac{Aa^3\cos(3dx+3c)}{6d}+\frac{Aa^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{-Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2Aa^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2Aa^3\cos(dx+c)+Aa^3(dx+$
default	$\frac{-Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2Aa^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2Aa^3\cos(dx+c)+Aa^3(dx+$
parts	$a^3Ax-\frac{2a^3A\cos(dx+c)}{d}+\frac{2Aa^3(2+\sin^2(dx+c))\cos(dx+c)}{3d}-\frac{Aa^3\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}\right.}{d}$
norman	$\frac{-\frac{4Aa^3}{3d}+\frac{5a^3Ax}{8}-\frac{4Aa^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4Aa^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4Aa^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3Aa^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11Aa^3\left(\tan^3\left(\frac{}{4d}\right.\right.}$

input
```
int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/96*A*a^3*(-60*d*x+48*cos(d*x+c)+3*sin(4*d*x+4*c)+16*cos(3*d*x+3*c)-24*s
in(2*d*x+2*c)+64)/d
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int (a + a\sin(c+dx))^3(A - A\sin(c+dx))\,dx =$$
$$-\frac{16\,Aa^3\cos(dx+c)^3 - 15\,Aa^3dx + 3\left(2\,Aa^3\cos(dx+c)^3 - 5\,Aa^3\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3
- 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

3.227.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(78) = 156.

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.39

$$\int (a + a\sin(c+dx))^3(A - A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8} - \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} - \frac{3Aa^3x\cos^4(c+dx)}{8} + Aa^3x + \frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa^3\sin^2(c \\ x(-A\sin(c)+A)(a\sin(c)+a)^3 \end{cases}$$

input
```
integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c +
 d*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*
A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d)
 - 2*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**
3, True))
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int (a + a\sin(c+dx))^3(A - A\sin(c+dx))\,dx =$$
$$-\frac{64\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)Aa^3 + 3\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) - 8\sin(2\,dx+2\,c)\right)Aa^3 - 9}{96\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3
*cos(d*x + c))/d
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (a + a\sin(c+dx))^3(A - A\sin(c+dx))\,dx = \frac{5}{8}\,Aa^3x - \frac{Aa^3\cos(3\,dx+3\,c)}{6\,d} - \frac{Aa^3\cos(dx+c)}{2\,d} - \frac{Aa^3\sin(4\,dx+4\,c)}{32\,d} + \frac{Aa^3\sin(2\,dx+2\,c)}{4\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

output
```
5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/
32*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

3.227.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.05

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5\,A\,a^3\,x}{8}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-32)}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-96)}{24}\right)}$$

input
```
int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(5*A*a^3*x)/8 - (tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*
(60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/
6 - (A*a^3*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c
+ 15*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*tan(c/2 + (d*x)
/2))/4 - (11*A*a^3*tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*tan(c/2 + (d*x)/2)^
5)/4 + (3*A*a^3*tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*
a^3*(15*c + 15*d*x - 32))/24)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.228.1 Optimal result 1876
3.228.2 Mathematica [A] (verified) 1876
3.228.3 Rubi [A] (verified) 1877
3.228.4 Maple [A] (verified) 1878
3.228.5 Fricas [A] (verification not implemented) 1879
3.228.6 Sympy [F] 1879
3.228.7 Maxima [A] (verification not implemented) 1880
3.228.8 Giac [A] (verification not implemented) 1880
3.228.9 Mupad [B] (verification not implemented) 1881

3.228.1 Optimal result

Integrand size = 30, antiderivative size = 76

$$\begin{aligned}&\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&= a^3Ax-\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{a^3A\cos(c+dx)}{d}\\&\quad-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{d}\end{aligned}$$

output
```
a^3*A*x-a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-1/3*a^3*A*cos(d*x+c
)^3/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

3.228.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{a^3A\big(9\cos(c+dx)-\cos(3(c+dx))+6\big(-2c+2dx-2\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big)\big)+2\log\big(\sin\big(\tfrac{1}{2}(c+dx)\big)\big)+\end{aligned}$$
$$12d$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c
 + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)
```

3.228.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)}dx$$

$$\downarrow 3445$$

$$\int \left(a^3(-A)\sin^3(c+dx)-2a^3A\sin^2(c+dx)+a^3A\csc(c+dx)+2a^3A\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}+a^3Ax$$

input
```
Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*
A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.228.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{A a^3\left(12dx+9\cos(dx+c)-\cos(3dx+3c)+6\sin(2dx+2c)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)}{12d}$
derivativedivides	$\frac{\frac{A a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A a^3(dx+c)+A a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}$
default	$\frac{\frac{A a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2A a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A a^3(dx+c)+A a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}$
risch	$a^3Ax+\frac{3A a^3 e^{i(dx+c)}}{8d}+\frac{3A a^3 e^{-i(dx+c)}}{8d}-\frac{A a^3\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{A a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{A a^3\cos(3dx+3c)}{12d}$
norman	$\frac{a^3Ax+a^3Ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4A a^3}{3d}+\frac{4A a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16A a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2A a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2A a^3\left(\tan^3\left(\frac{dx}{2}\right.\right.}{\left(1+\tan\right.}$

input
```
int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/12*A*a^3*(12*d*x+9*cos(d*x+c)-cos(3*d*x+3*c)+6*sin(2*d*x+2*c)+12*ln(tan(
1/2*d*x+1/2*c))+8)/d
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{2\,Aa^3\cos(dx+c)^3-6\,Aa^3dx-6\,Aa^3\cos(dx+c)\sin(dx+c)-6\,Aa^3\cos(dx+c)+3\,Aa^3\log\left(\frac{1}{2}\cos\right.}{6\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x
+ c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^
3*log(-1/2*cos(d*x + c) + 1/2))/d
```

3.228.6 Sympy [F]

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int (-2\sin(c+dx)\csc(c+dx))\,dx+\int 2\sin^3(c+dx)\csc(c+dx)\,dx\right.$$
$$\left.+\int \sin^4(c+dx)\csc(c+dx)\,dx+\int (-\csc(c+dx))\,dx\right)$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*
x)**3*csc(c + d*x), x) + Integral(sin(c + d*x)**4*csc(c + d*x), x) + Integ
ral(-csc(c + d*x), x))
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{2\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3+3\left(2\,dx+2\,c-\sin(2\,dx+2\,c)\right)Aa^3-12\,(dx+c)Aa^3+6\,Aa^3\,l}{6\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x
 + c)))/d
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{3\,(dx+c)Aa^3+3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,A\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gia
c")
```

output
```
1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a
^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1
/2*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.228.9 Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.79

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{-2Aa^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 4Aa^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 2Aa^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{4Aa^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$
$$+ \frac{2Aa^3\,\mathrm{atan}\left(\frac{4A^2a^6}{4A^2a^6-4A^2a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{4A^2a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4A^2a^6-4A^2a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$
$$+ \frac{Aa^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

output
```
((4*A*a^3)/3 + 2*A*a^3*tan(c/2 + (d*x)/2) + 4*A*a^3*tan(c/2 + (d*x)/2)^2 -
 2*A*a^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d
*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*atan((4*A^2*a^6)/(4*A^2*a
^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2)) + (4*A^2*a^6*tan(c/2 + (d*x)/2))/(4*A^2
*a^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2))))/d + (A*a^3*log(tan(c/2 + (d*x)/2)))
/d
```

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.229.1 Optimal result 1882
3.229.2 Mathematica [A] (verified) 1882
3.229.3 Rubi [A] (verified) 1883
3.229.4 Maple [A] (verified) 1884
3.229.5 Fricas [A] (verification not implemented) 1885
3.229.6 Sympy [F] 1885
3.229.7 Maxima [A] (verification not implemented) 1886
3.229.8 Giac [B] (verification not implemented) 1886
3.229.9 Mupad [B] (verification not implemented) 1887

3.229.1 Optimal result

Integrand size = 32, antiderivative size = 79

$$\begin{aligned}&\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=-\frac{1}{2}a^3Ax-\frac{2a^3A\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}\\&\quad-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

output
```
-1/2*a^3*A*x-2*a^3*A*arctanh(cos(d*x+c))/d+2*a^3*A*cos(d*x+c)/d-a^3*A*cot(
d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

3.229.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{a^3A\left(-2c-2dx+8\cos(c)\cos(dx)-4\cot(c+dx)-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\right.}{4d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c +
 d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)]
))/(4*d)
```

3.229.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^2}dx$$

$$\downarrow 3429$$

$$aA\int \cot^2(c+dx)(\sin(c+dx)a+a)^2dx$$

$$\downarrow 3042$$

$$aA\int \frac{(\sin(c+dx)a+a)^2}{\tan(c+dx)^2}dx$$

$$\downarrow 3188$$

$$\frac{A\int \left(\csc^2(c+dx)a^4-\sin^2(c+dx)a^4+2\csc(c+dx)a^4-2\sin(c+dx)a^4\right)dx}{a}$$

$$\downarrow 2009$$

$$\frac{A\left(-\frac{2a^4\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4x}{2}\right)}{a}$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(A*(-1/2*(a^4*x) - (2*a^4*ArcTanh[Cos[c + d*x]])/d + (2*a^4*Cos[c + d*x])/
d - (a^4*Cot[c + d*x])/d + (a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a
```

3.229.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

3.229.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-A a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A a^3\cos(dx+c)+2A a^3\ln(\csc(dx+c)-\cot(dx+c))-A a^3\cot(dx+c)}{d}$
default	$\frac{-A a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A a^3\cos(dx+c)+2A a^3\ln(\csc(dx+c)-\cot(dx+c))-A a^3\cot(dx+c)}{d}$
parallelrisch	$-\frac{a^3\left(-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\cos(dx+c)-\frac{\cos(2dx+2c)}{2}+4\sin(dx+c)-\frac{5}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)+dx\right.}{2d}$
risch	$-\frac{a^3Ax}{2}-\frac{iA a^3e^{2i(dx+c)}}{8d}+\frac{A a^3e^{i(dx+c)}}{d}+\frac{A a^3e^{-i(dx+c)}}{d}+\frac{iA a^3e^{-2i(dx+c)}}{8d}-\frac{2iA a^3}{d(e^{2i(dx+c)}-1)}-\frac{2A a^3\ln($
norman	$\frac{4A a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{A a^3}{2d}+\frac{4A a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12A a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12A a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{A}{}$

input
```
int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-A*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a^3*cos(d*x+c)+
2*A*a^3*ln(csc(d*x+c)-cot(d*x+c))-A*a^3*cot(d*x+c))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{Aa^3\cos(dx+c)^3+2\,Aa^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,Aa^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(d}{2\,d\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x +
c) + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

3.229.6 Sympy [F]

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int \left(-2\sin(c+dx)\csc^2(c+dx)\right)\,dx+\int 2\sin^3(c+dx)\csc^2(c+dx)\,dx\right.$$
$$\left.+\int \sin^4(c+dx)\csc^2(c+dx)\,dx+\int \left(-\csc^2(c+dx)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x
) + Integral(-csc(c + d*x)**2, x))
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))Aa^3+4\,Aa^3(\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1))-8\,Aa^3\cos(}{4\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
 1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c)
)/d
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{(dx+c)Aa^3-4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(Aa^3\tan(\right.}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan
(1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1
/2*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 -
 A*a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.229.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.86

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{A\,a^3\,\mathrm{atan}\left(\frac{A^2\,a^6}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{4\,A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$
$$-\frac{3\,A\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-8\,A\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-8\,A\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+A\,a^3}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+\frac{2\,A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{A\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

output
```
(A*a^3*atan((A^2*a^6)/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2)) - (4*A^2*a^
6*tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2))))/d - (A*a^
3 - 8*A*a^3*tan(c/2 + (d*x)/2) - 8*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*ta
n(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*
tan(c/2 + (d*x)/2)^5)) + (2*A*a^3*log(tan(c/2 + (d*x)/2)))/d + (A*a^3*tan(
c/2 + (d*x)/2))/(2*d)
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.230.1 Optimal result 1888
3.230.2 Mathematica [A] (verified) 1888
3.230.3 Rubi [A] (verified) 1889
3.230.4 Maple [A] (verified) 1890
3.230.5 Fricas [B] (verification not implemented) 1891
3.230.6 Sympy [F] 1891
3.230.7 Maxima [A] (verification not implemented) 1892
3.230.8 Giac [A] (verification not implemented) 1892
3.230.9 Mupad [B] (verification not implemented) 1893

3.230.1 Optimal result

Integrand size = 32, antiderivative size = 78

$$\begin{aligned}&\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=-2a^3Ax-\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}\\&\quad-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}\end{aligned}$$

output
```
-2*a^3*A*x-1/2*a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-2*a^3*A*cot(
d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

3.230.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.82

$$\begin{aligned}&\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=-2a^3Ax+\frac{a^3A\cos(c)\cos(dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}\\&\quad-\frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\\&\quad+\frac{a^3A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3A\sin(c)\sin(dx)}{d}\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A
*Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*
Log[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*S
in[c]*Sin[d*x])/d
```

3.230.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^3}dx$$

$$\downarrow 3445$$

$$\int \left(-a^3A\sin(c+dx)+a^3A\csc^3(c+dx)+2a^3A\csc^2(c+dx)-2a^3A\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d
- (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

3.230.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.230.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{A a^3 \cos(dx+c)-2A a^3(dx+c)-2A a^3 \cot(dx+c)+A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{A a^3 \cos(dx+c)-2A a^3(dx+c)-2A a^3 \cot(dx+c)+A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{A a^3\left(-16dx+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\csc^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)+23\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{8d}$
risch	$-2a^3Ax+\frac{A a^3 e^{i(dx+c)}}{2d}+\frac{A a^3 e^{-i(dx+c)}}{2d}+\frac{A a^3\left(e^{3i(dx+c)}+e^{i(dx+c)}-4ie^{2i(dx+c)}+4i\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{A a^3\ln\left(e^{i(dx+c)}+1\right.}{2d}$
norman	$\frac{A a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{A a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A a^3}{8d}+\frac{3A a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5A a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{27A a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-$

input
```
int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*a^3*cos(d*x+c)-2*A*a^3*(d*x+c)-2*A*a^3*cot(d*x+c)+A*a^3*(-1/2*csc(d
*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(74) = 148$.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{8\,Aa^3dx\cos(dx+c)^2 - 4\,Aa^3\cos(dx+c)^3 - 8\,Aa^3dx - 8\,Aa^3\cos(dx+c)\sin(dx+c) + 2\,Aa^3\cos(dx}{4\,(d\,c}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x -
8*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x
+ c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^
3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

3.230.6 Sympy [F]

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int \left(-2\sin(c+dx)\csc^3(c+dx)\right)\,dx + \int 2\sin^3(c+dx)\csc^3(c+dx)\,dx\right.$$
$$\left. + \int \sin^4(c+dx)\csc^3(c+dx)\,dx + \int \left(-\csc^3(c+dx)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**3, x
) + Integral(-csc(c + d*x)**3, x))
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{8\,(dx+c)Aa^3 - Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - 4\,Aa^3\cos\left(dx+c\right.}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log
(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a
^3/tan(d*x + c))/d
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 16\,(dx+c)Aa^3 + 4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{16}{\tan\left(\frac{1}{2}\,d\right.}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(t
an(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d
*x + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x
 + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.230.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.82

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{A\,a^3\left(\frac{\cos(c+d\,x)}{2}-4\,\mathrm{atan}\left(\frac{\sqrt{17}\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}(4)+\frac{d\,x}{2}\right)}\right)-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\cos\left(2\,c+2\,d\,x\right)+\frac{\cos(3\,c+3\,d\,x)}{2}+}{2\,d\,\left(\cos\left(c\right.\right.}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

output
```
(A*a^3*(cos(c + d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2
+ (d*x)/2)))/(17*cos(c/2 - atan(4) + (d*x)/2))) - log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2))/2 + cos(2*c + 2*d*x) + cos(3*c + 3*d*x)/2 + 2*sin(2*c +
 2*d*x) + 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(1
7*cos(c/2 - atan(4) + (d*x)/2)))*cos(2*c + 2*d*x) + (log(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 - 1))/(2*d*(cos(c + d*x)^2 - 1))
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.231.1 Optimal result 1894
3.231.2 Mathematica [A] (verified) 1894
3.231.3 Rubi [A] (verified) 1895
3.231.4 Maple [A] (verified) 1896
3.231.5 Fricas [B] (verification not implemented) 1897
3.231.6 Sympy [F] 1897
3.231.7 Maxima [A] (verification not implemented) 1898
3.231.8 Giac [A] (verification not implemented) 1898
3.231.9 Mupad [B] (verification not implemented) 1899

3.231.1 Optimal result

Integrand size = 32, antiderivative size = 78

$$\begin{aligned}\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= -a^3Ax+\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)}{d}\\ &\quad-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}\end{aligned}$$

output
```
-a^3*A*x+a^3*A*arctanh(cos(d*x+c))/d-a^3*A*cot(d*x+c)/d-1/3*a^3*A*cot(d*x+
c)^3/d-a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

3.231.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.81

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 -
24*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2
 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])
/2 - 8*Tan[(c + d*x)/2]))/d
```

3.231.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^4}dx$$

$$\downarrow 3445$$

$$\int \left(a^3A\csc^4(c+dx)+2a^3A\csc^3(c+dx)-2a^3A\csc(c+dx)-a^3A\right)dx$$

$$\downarrow 2009$$

$$\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a
^3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

3.231.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.231.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{A a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24dx+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-9\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-24\right.}{24d}$
derivativedivides	$\frac{-A a^3(dx+c)-2A a^3\ln(\csc(dx+c)-\cot(dx+c))+2A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+A a^3\left(-\frac{2}{3}-\right.}{d}$
default	$\frac{-A a^3(dx+c)-2A a^3\ln(\csc(dx+c)-\cot(dx+c))+2A a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+A a^3\left(-\frac{2}{3}-\right.}{d}$
risch	$-a^3Ax+\frac{2A a^3\left(3\,\mathrm{e}^{5i(dx+c)}+6i\mathrm{e}^{2i(dx+c)}-2i-3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{A a^3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}-\frac{A a^3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$
norman	$\frac{2A a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{A a^3}{24d}+\frac{6A a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11A a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{21A a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{A a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-$

input
```
int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/24*A*a^3*(tan(1/2*d*x+1/2*c)^3-cot(1/2*d*x+1/2*c)^3+6*tan(1/2*d*x+1/2*c)
^2-6*cot(1/2*d*x+1/2*c)^2-24*d*x+9*tan(1/2*d*x+1/2*c)-9*cot(1/2*d*x+1/2*c)
-24*ln(tan(1/2*d*x+1/2*c)))/d
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(76) = 152$.

Time $= 0.29$ (sec) , antiderivative size $= 175$, normalized size of antiderivative $= 2.24$

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{4\,Aa^3\cos(dx+c)^3 - 6\,Aa^3\cos(dx+c) - 3\left(Aa^3\cos(dx+c)^2 - Aa^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx$$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x +
 c)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*co
s(d*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x +
 c)^2 - d)*sin(d*x + c))
```

3.231.6 Sympy [F]

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int \left(-2\sin(c+dx)\csc^4(c+dx)\right)\,dx + \int 2\sin^3(c+dx)\csc^4(c+dx)\,dx\right.$$
$$\left. + \int \sin^4(c+dx)\csc^4(c+dx)\,dx + \int \left(-\csc^4(c+dx)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**4, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**4, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**4, x
) + Integral(-csc(c + d*x)**4, x))
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{6\,(dx+c)Aa^3-3\,Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,Aa^3(\log(\cos($$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - l
og(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c)
+ 1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c
)^3)/d
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.92

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,A$$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(
d*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2
*d*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1
/2*c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.231.9 Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.14

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{2}-\frac{A\,a^3\,\cos(3\,c+3\,d\,x)}{6}+\frac{A\,a^3\,\cos(c+d\,x)}{2}-\frac{A\,a^3\,\sin(3\,c+3\,d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^3\,\sin}{\frac{3\,d\,\sin(c+d\,x)}{4}-\frac{d\,\sin}{}}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

output
```
-((A*a^3*sin(2*c + 2*d*x))/2 - (A*a^3*cos(3*c + 3*d*x))/6 + (A*a^3*cos(c +
 d*x))/2 - (A*a^3*sin(3*c + 3*d*x)*atan((2^(1/2)*(cos(c/2 + (d*x)/2) + sin
(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x)/2))))/2 + (3*A*a^3*sin(c + d*x
)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*sin(c + d*x)*at
an((2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 +
 (d*x)/2))))/2 - (A*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c
 + 3*d*x))/4)/((3*d*sin(c + d*x))/4 - (d*sin(3*c + 3*d*x))/4)
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.232.1 Optimal result 1900
3.232.2 Mathematica [B] (verified) 1901
3.232.3 Rubi [A] (verified) 1901
3.232.4 Maple [A] (verified) 1903
3.232.5 Fricas [B] (verification not implemented) 1903
3.232.6 Sympy [F(-1)] 1904
3.232.7 Maxima [A] (verification not implemented) 1904
3.232.8 Giac [B] (verification not implemented) 1905
3.232.9 Mupad [B] (verification not implemented) 1905

3.232.1 Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{5a^3A\operatorname{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}$$

output
```
5/8*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-3/8*a^3*A*cot(d*x
+c)*csc(d*x+c)/d-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d
```

3.232.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. $2(86) = 172$.

Time $= 0.23$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 2.44$

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d} - \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d}\right.$$
$$- \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$
$$+ \frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$
$$\left. + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}\right)$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c +
d*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Co
s[(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x
)/2]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

3.232.3 Rubi [A] (verified)

Time $= 0.35$ (sec) , antiderivative size $= 86$, normalized size of antiderivative $= 1.00$, number of steps used $= 3$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used $= \{3042, 3445, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^5}dx$$

$$\downarrow 3445$$

$$\int \left(a^3 A \csc^5(c+dx) + 2a^3 A \csc^4(c+dx) - 2a^3 A \csc^2(c+dx) - a^3 A \csc(c+dx)\right) dx$$

$$\downarrow 2009$$

$$\frac{5a^3 A \operatorname{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{a^3 A \cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a^3 A \cot(c+dx)\csc(c+dx)}{8d}$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d)
```

3.232.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.232.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-A\,a^3\ln(\csc(dx+c)-\cot(dx+c))+2A\,a^3\cot(dx+c)+2A\,a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\right.\right.}{d}$
default	$\frac{-A\,a^3\ln(\csc(dx+c)-\cot(dx+c))+2A\,a^3\cot(dx+c)+2A\,a^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\right.\right.}{d}$
parallelrisch	$-\frac{a^3\left(\cot^4\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{16\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}{64d}$
risch	$\frac{A\,a^3\left(9\,\mathrm{e}^{7i(dx+c)}-33\,\mathrm{e}^{5i(dx+c)}+48i\mathrm{e}^{6i(dx+c)}-33\,\mathrm{e}^{3i(dx+c)}-48i\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+16i\mathrm{e}^{2i(dx+c)}-16i\right)}{12d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}-\frac{5A\,a^3\ln(e}{}$
norman	$-\frac{A\,a^3}{64d}-\frac{57A\,a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{19A\,a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{27A\,a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{49A\,a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{A\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{12d}$

input
```
int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-A*a^3*ln(csc(d*x+c)-cot(d*x+c))+2*A*a^3*cot(d*x+c)+2*A*a^3*(-2/3-1/3
*csc(d*x+c)^2)*cot(d*x+c)+A*a^3*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*
x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.93

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{32\,Aa^3\cos(dx+c)^3\sin(dx+c)-18\,Aa^3\cos(dx+c)^3+30\,Aa^3\cos(dx+c)-15\left(Aa^3\cos(dx+c)^4-\right.}{48\left(d\,c\right.}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30
*A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 +
A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*co
s(d*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*
d*cos(d*x + c)^2 + d)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3\,Aa^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right) + 24\,Aa^3(\log(\cos(c$$
$$48\,d$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) +
24*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*
x + c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(78) = 156$.

Time $= 0.32$ (sec) , antiderivative size $= 174$, normalized size of antiderivative $= 2.02$

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 - 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*
A*a^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*
tan(1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.232.9 Mupad [B] (verification not implemented)

Time $= 13.20$ (sec) , antiderivative size $= 244$, normalized size of antiderivative $= 2.84$

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-16\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d}{2}\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

output
```
-(A*a^3*(3*cos(c/2 + (d*x)/2)^8 - 3*sin(c/2 + (d*x)/2)^8 - 16*cos(c/2 + (d
*x)/2)*sin(c/2 + (d*x)/2)^7 + 16*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) -
 24*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 48*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^5 - 48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4))/(192*d*cos(c/2 +
 (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.233.1 Optimal result 1906
3.233.2 Mathematica [B] (verified) 1907
3.233.3 Rubi [A] (verified) 1907
3.233.4 Maple [A] (verified) 1909
3.233.5 Fricas [B] (verification not implemented) 1909
3.233.6 Sympy [F(-1)] 1910
3.233.7 Maxima [A] (verification not implemented) 1910
3.233.8 Giac [A] (verification not implemented) 1911
3.233.9 Mupad [B] (verification not implemented) 1911

3.233.1 Optimal result

Integrand size = 32, antiderivative size = 105

$$\begin{aligned}&\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{a^3A\operatorname{arctanh}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c+dx)}{5d}\\&\quad+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}\end{aligned}$$

output

```
1/4*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-1/5*a^3*A*cot(d*x
+c)^5/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^
3/d
```

3.233.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.55

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d} + \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d} - \frac{19\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{480d}\right.$$
$$- \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{160d} + \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$
$$- \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{16d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d}$$
$$\left. + \frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{480d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{160d}\right)$$

input
```
Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot
[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) -
Log[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]
^4/(32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

3.233.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^6}dx$$

$$\downarrow 3429$$

$$a^3A^3\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}dx$$

$$\downarrow 3042$$

$$a^3A^3\int \frac{1}{(A-A\sin(c+dx))^2\tan(c+dx)^6}dx$$

$$\downarrow 3188$$

$$\frac{a^3\int \left(A^4\csc^6(c+dx)+2A^4\csc^5(c+dx)-2A^4\csc^3(c+dx)-A^4\csc^2(c+dx)\right)dx}{A^3}$$

$$\downarrow 2009$$

$$\frac{a^3\left(\frac{A^4\text{arctanh}(\cos(c+dx))}{4d}-\frac{A^4\cot^5(c+dx)}{5d}-\frac{2A^4\cot^3(c+dx)}{3d}-\frac{A^4\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{A^4\cot(c+dx)\csc(c+dx)}{4d}\right)}{A^3}$$

input
```
Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*((A^4*ArcTanh[Cos[c + d*x]])/(4*d) - (2*A^4*Cot[c + d*x]^3)/(3*d) - (
A^4*Cot[c + d*x]^5)/(5*d) + (A^4*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (A^4*C
ot[c + d*x]*Csc[c + d*x]^3)/(2*d)))/A^3
```

3.233.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

3.233.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result
parallelrisch	$-\frac{a^3\left(\cot^5\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\left(\cot^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{25\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\right.}{160d}$
derivativedivides	$\frac{A\,a^3\cot(dx+c)-2A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(d\right.}{d}$
default	$\frac{A\,a^3\cot(dx+c)-2A\,a^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2A\,a^3\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(d\right.}{d}$
risch	$-\frac{A\,a^3\left(-60i\mathrm{e}^{8i(dx+c)}+15\,\mathrm{e}^{9i(dx+c)}+240i\mathrm{e}^{6i(dx+c)}+90\,\mathrm{e}^{7i(dx+c)}-40i\mathrm{e}^{4i(dx+c)}+80i\mathrm{e}^{2i(dx+c)}-90\,\mathrm{e}^{3i(dx+c)}-28i-15\,\mathrm{e}\right.}{30d\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}$

input

```
int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output

```
-1/160*a^3*(cot(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^5+5*cot(1/2*d*x+1/2*c)
^4-5*tan(1/2*d*x+1/2*c)^4+25/3*cot(1/2*d*x+1/2*c)^3-25/3*tan(1/2*d*x+1/2*c
)^3-30*cot(1/2*d*x+1/2*c)+30*tan(1/2*d*x+1/2*c)+40*ln(tan(1/2*d*x+1/2*c)))
*A/d
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.91

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{56\,Aa^3\cos(dx+c)^5-80\,Aa^3\cos(dx+c)^3+15\left(Aa^3\cos(dx+c)^4-2\,Aa^3\cos(dx+c)^2+Aa^3\right)\log\left(\frac{1}{2}\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d
*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*lo
g(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{15\,Aa^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+120\,Aa^3\cot(dx+c)-8\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)Aa^3\cot(dx+c)^5}{120\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2
*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
- 60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 1
0*tan(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.66

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 +
25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 - 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*
A*a^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*
tan(1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.233.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-15\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

output
```
-(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 +
(d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
 - 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 90*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^6 - 90*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 25*c
os(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(480*d*cos(c/2
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

3.234.1 Optimal result 1912
3.234.2 Mathematica [B] (verified) 1913
3.234.3 Rubi [A] (verified) 1913
3.234.4 Maple [A] (verified) 1915
3.234.5 Fricas [B] (verification not implemented) 1915
3.234.6 Sympy [F(-1)] 1916
3.234.7 Maxima [A] (verification not implemented) 1916
3.234.8 Giac [B] (verification not implemented) 1917
3.234.9 Mupad [B] (verification not implemented) 1917

3.234.1 Optimal result

Integrand size = 32, antiderivative size = 130

$$\begin{aligned}&\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{3a^3A\operatorname{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d}\\&\quad-\frac{2a^3A\cot^5(c+dx)}{5d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}\\&\quad-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}\end{aligned}$$

output

```
3/16*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-2/5*a^3*A*cot(d*
x+c)^5/d+3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-5/24*a^3*A*cot(d*x+c)*csc(d*x+
c)^3/d-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d
```

3.234.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 306 vs. $2(130) = 260$.

Time $= 0.29$ (sec) , antiderivative size $= 306$, normalized size of antiderivative $= 2.35$

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d} + \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{240d}\right.$$
$$- \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{80d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$
$$+ \frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}$$
$$+ \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$
$$\left. - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{240d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}\right)$$

input
```
Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Co
t[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (
3*Log[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Se
c[(c + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/
(384*d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(240*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

3.234.3 Rubi [A] (verified)

Time $= 0.40$ (sec) , antiderivative size $= 130$, normalized size of antiderivative $= 1.00$, number of steps used $= 3$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used $= \{3042, 3445, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^7}dx$$

$$\downarrow 3445$$

$$\int \left(a^3A\csc^7(c+dx)+2a^3A\csc^6(c+dx)-2a^3A\csc^4(c+dx)-a^3A\csc^3(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3a^3A\operatorname{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}$$

input
```
Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) -
(2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d
) - (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc
[c + d*x]^5)/(6*d)
```

3.234.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.234.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{25a^3\left(\frac{1536\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}+\left(\sec^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)-\frac{13\cos(3dx+3c)}{150}-\frac{3\cos(5dx+5c)}{50}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)+$
derivativedivides	$\frac{-Aa^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2Aa^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2Aa^3\left(-\frac{8}{15}-\frac{\left(\csc^4(d\right.}{5}$
default	$\frac{-Aa^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2Aa^3\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)+2Aa^3\left(-\frac{8}{15}-\frac{\left(\csc^4(d\right.}{5}$
risch	$-\frac{Aa^3\left(45e^{11i(dx+c)}+65e^{9i(dx+c)}-750e^{7i(dx+c)}+960ie^{8i(dx+c)}-750e^{5i(dx+c)}-640ie^{6i(dx+c)}+65e^{3i(dx+c)}+45e^{i(dx}\right.}{120d\left(e^{2i(dx+c)}-1\right)^6}$

input
```
int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-25/8192*a^3*(1536/25*ln(tan(1/2*d*x+1/2*c))+sec(1/2*d*x+1/2*c)^5*(sec(1/2
*d*x+1/2*c)*(cos(d*x+c)-13/150*cos(3*d*x+3*c)-3/50*cos(5*d*x+5*c))*csc(1/2
*d*x+1/2*c)+256/75*cos(d*x+c)+64/75*cos(3*d*x+3*c)-64/375*cos(5*d*x+5*c))*
csc(1/2*d*x+1/2*c)^5)*A/d
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(118) = 236.

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{90\,Aa^3\cos(dx+c)^5-80\,Aa^3\cos(dx+c)^3-90\,Aa^3\cos(dx+c)-45\left(Aa^3\cos(dx+c)^6-3\,Aa^3\cos(\right.}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d
*x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(
d*x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6
 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d
*x + c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(
d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.59

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{5\,Aa^3\left(\frac{2\left(15\cos(dx+c)^5-40\cos(dx+c)^3+33\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)-1}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c)
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(
d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos
(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3
*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan
(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(118) = 236$.

Time $= 0.32$ (sec) , antiderivative size $= 242$, normalized size of antiderivative $= 1.86$

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$= \frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*
a^3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 24
0*A*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a
^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan
(1/2*d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d
*x + 1/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.234.9 Mupad [B] (verification not implemented)

Time $= 12.87$ (sec) , antiderivative size $= 340$, normalized size of antiderivative $= 2.62$

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

output

```
-(A*a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 +
(d*x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/
2) - 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 2
40*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*si
n(c/2 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos
(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2
+ (d*x)/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2)^6)
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.235.1 Optimal result 1919
3.235.2 Mathematica [C] (verified) 1920
3.235.3 Rubi [A] (verified) 1920
3.235.4 Maple [A] (verified) 1922
3.235.5 Fricas [B] (verification not implemented) 1922
3.235.6 Sympy [B] (verification not implemented) 1923
3.235.7 Maxima [B] (verification not implemented) 1924
3.235.8 Giac [A] (verification not implemented) 1925
3.235.9 Mupad [B] (verification not implemented) 1926

3.235.1 Optimal result

Integrand size = 32, antiderivative size = 129

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19Ax}{2a^3} - \frac{4A\cos(c+dx)}{a^3d} + \frac{A\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} - \frac{199A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
-19/2*A*x/a^3-4*A*cos(d*x+c)/a^3/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a^3/d-2/5*A
*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+41/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
^2-199/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.80 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.64

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\Big(140\sqrt{2}\sqrt{a}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin$$

input
```
Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(140*Sqrt[2]*Sqrt[a]*Hypergeometric
2F1[-3/2, -3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) - 360*ArcSi
n[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])] + Sqrt[a]*Sqrt[1 - Sin[c + d*x]]*
(-308 - 639*Sin[c + d*x] - 433*Sin[c + d*x]^2 - 75*Sin[c + d*x]^3 + 15*Sin
[c + d*x]^4)))/(30*a^(7/2)*d*(1 + Sin[c + d*x])^2)
```

3.235.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(c+dx)^4(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$
$$\downarrow 3445$$
$$\int \left(-\frac{A\sin^2(c+dx)}{a^3}+\frac{4A\sin(c+dx)}{a^3}+\frac{16A}{a^3(\sin(c+dx)+1)}-\frac{9A}{a^3(\sin(c+dx)+1)^2}+\frac{2A}{a^3(\sin(c+dx)+1)^3}-\frac{9}{a}\right.$$

$$\downarrow 2009$$

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19Ax}{2a^3}$$

input
```
Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*
a^3*d*(1 + Sin[c + d*x]))
```

3.235.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.235.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{32A\left(-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\left(\text{ta}\right.}{}\right.}{d\,a^3}$
default	$\frac{32A\left(-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\left(\text{ta}\right.}{}\right.}{d\,a^3}$
risch	$-\frac{19Ax}{2a^3}-\frac{iA\,e^{2i(dx+c)}}{8a^3d}-\frac{2A\,e^{i(dx+c)}}{a^3d}-\frac{2A\,e^{-i(dx+c)}}{a^3d}+\frac{iA\,e^{-2i(dx+c)}}{8a^3d}-\frac{2\left(825iA\,e^{3i(dx+c)}+240A\,e^{4i(dx+c)}-\right.}{15d\,a^3(e}$
parallelrisch	$\frac{\left((76dx-120)\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(380dx-128)\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+(76dx+\frac{3484}{15})\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(-760dx-304)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{8d\,a^3\left(-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+5\sin\left(\frac{3dx}{2}+\frac{3}{2}\right.\right.}$
norman	$-\frac{19Ax\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{95A\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{19A\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{391A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{1919Ax\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{448A}{15ad}-$

input
```
int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
32/d*A/a^3*(-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/
24/(tan(1/2*d*x+1/2*c)+1)^3-5/16/(tan(1/2*d*x+1/2*c)+1)^2-9/16/(tan(1/2*d*
x+1/2*c)+1)-1/16*(1/2*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)^2-1/2*tan(
1/2*d*x+1/2*c)+4)/(1+tan(1/2*d*x+1/2*c)^2)^2-19/32*arctan(tan(1/2*d*x+1/2*
c)))
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(119) = 238.

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.92

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{15\,A\cos(dx+c)^5+90\,A\cos(dx+c)^4+(285\,Adx+683\,A)\cos(dx+c)^3-1140\,Adx+(855\,Adx-52}{30\left(a^3d\cos(dx\right.}$$

input
```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos
(d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*
x + 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 114
0*A*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d
*x + c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3
*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.235.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3614 vs. $2(126) = 252$.

Time = 22.20 (sec) , antiderivative size = 3614, normalized size of antiderivative = 28.02

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output

```
Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(
c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*
x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 +
d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8
+ 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a*
*3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan
(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d
*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d
*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7
 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a
**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*ta
n(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*
tan(c/2 + d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 +
d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**
6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*
a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*t
an(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*t
an(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/...
```

3.235.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(119) = 238$.

Time $= 0.35$ (sec) , antiderivative size $= 715$, normalized size of antiderivative $= 5.54$

$$\int \frac{\sin^4(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
2275*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin
(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 20*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 5*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*(
(105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1)
+ 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3))/d
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{\frac{285\,(dx+c)A}{a^3}+\frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^3(\tan}}{30\,d}$$

input
```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*
x + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^2*a^3) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 +
 1025*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*
(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.235.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.53

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+570)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+2850)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)$$

$$-\frac{19\,A\,x}{2\,a^3}$$

input
```
int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/3
0) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 57
0))/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x +
2850))/30) + tan(c/2 + (d*x)/2)^2*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x
 + 7902))/30) + tan(c/2 + (d*x)/2)^6*(190*A*(c + d*x) - (A*(5700*c + 5700*
d*x + 6650))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 57
00*d*x + 11270))/30) + tan(c/2 + (d*x)/2)^5*(247*A*(c + d*x) - (A*(7410*c
+ 7410*d*x + 10450))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(741
0*c + 7410*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x +
896))/30)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2)
- (19*A*x)/(2*a^3)
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.236.1 Optimal result 1927
3.236.2 Mathematica [A] (verified) 1927
3.236.3 Rubi [A] (verified) 1928
3.236.4 Maple [A] (verified) 1929
3.236.5 Fricas [B] (verification not implemented) 1930
3.236.6 Sympy [B] (verification not implemented) 1930
3.236.7 Maxima [B] (verification not implemented) 1931
3.236.8 Giac [A] (verification not implemented) 1932
3.236.9 Mupad [B] (verification not implemented) 1933

3.236.1 Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4Ax}{a^3} + \frac{A\cos(c+dx)}{a^3 d} + \frac{2A\cos(c+dx)}{5a^3 d(1+\sin(c+dx))^3} - \frac{31A\cos(c+dx)}{15a^3 d(1+\sin(c+dx))^2} + \frac{104A\cos(c+dx)}{15a^3 d(1+\sin(c+dx))}$$

output
```
4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15
*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+
c))
```

3.236.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\sec(c+dx)\left(-120\arcsin\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\sqrt{1-\sin(c+dx)}\sqrt{c}}{15a^{7/2}d(1+}$$

input
```
Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/15*(A*Sec[c + d*x]*(-120*ArcSin[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqr
t[a])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[1 - Sin[c + d*x]]*Sqrt
[a*(1 + Sin[c + d*x])] + Sqrt[a]*(-94 - 128*Sin[c + d*x] + 73*Sin[c + d*x]
^2 + 134*Sin[c + d*x]^3 + 15*Sin[c + d*x]^4)))/(a^(7/2)*d*(1 + Sin[c + d*x
])^2)
```

3.236.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(c+dx)^3(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3445$$

$$\int \left(-\frac{A\sin(c+dx)}{a^3}-\frac{9A}{a^3(\sin(c+dx)+1)}+\frac{7A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}+\frac{4A}{a^3}\right)dx$$

$$\downarrow 2009$$

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

input
```
Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 +
Sin[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (
104*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

3.236.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.236.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{1}{8+8\left(\tan^2\left(\right.\right.\right.}$
default	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{1}{8+8\left(\tan^2\left(\right.\right.\right.}$
risch	$\frac{4Ax}{a^3}+\frac{A\,e^{i(dx+c)}}{2a^3d}+\frac{A\,e^{-i(dx+c)}}{2a^3d}+\frac{2A\left(435ie^{3i(dx+c)}+135\,e^{4i(dx+c)}-385ie^{i(dx+c)}-605\,e^{2i(dx+c)}+104\right)}{15d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
parallelrisch	$\frac{A\left(120dx\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)-600dx\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)+600dx\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+120dx\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)-1200dx\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-1200\right.}{30d\,a^3\left(\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)-5\sin\right.}$
norman	$\frac{164A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{284Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{188A}{15ad}+\frac{40A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{204Ax\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{20A}{}$

input
```
int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
16/d*A/a^3*(1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan(1/2*d*x+1/2*c)+1)^4+1/12
/(tan(1/2*d*x+1/2*c)+1)^3+3/8/(tan(1/2*d*x+1/2*c)+1)^2+1/2/(tan(1/2*d*x+1/
2*c)+1)+1/8/(1+tan(1/2*d*x+1/2*c)^2)+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(97) = 194$.

Time $= 0.27$ (sec) , antiderivative size $= 225$, normalized size of antiderivative $= 2.18$

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{15\,A\cos(dx+c)^4 + (60\,Adx + 149\,A)\cos(dx+c)^3 - 240\,Adx + (180\,Adx - 103\,A)\cos(dx+c)^2 - 3\,(40}{15\left(a^3 d\cos(dx+c)^3 + 3\,a^3 d\cos(dx+c)^2 -\right.}$$

input
```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x
+ (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) +
(15*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*
(40*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*c
os(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.236.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(100) = 200$.

Time $= 12.94$ (sec) , antiderivative size $= 2290$, normalized size of antiderivative $= 22.23$

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output

```
Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 7
5*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d
*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2
 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2
 + d*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**
6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*
a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan
(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*
x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3
+ 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 + 900*A*d*x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*
d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/
2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/
2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/
2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165
*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*
tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 +
 d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 +
d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)...
```

3.236.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(97) = 194$.

Time $= 0.32$ (sec) , antiderivative size $= 543$, normalized size of antiderivative $= 5.27$

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{2\left(3A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{189\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{160\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{75\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+24}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{11a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{15\,\text{arc}}{}\right.\right.}{}$$

input

```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
11*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) +
 145*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*
x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3))/d
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{2\left(\frac{30\,(dx+c)A}{a^3} + \frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} + \frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}\right.}{15\,d}$$

input
```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A
*tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x
 + 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c
) + 1)^5))/d
```

3.236.9 Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.53

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4\,A\,x}{a^3}$$
$$-\frac{\left(20\,A\,(c+d\,x)-\frac{4\,A\,(75\,c+75\,d\,x+30)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(44\,A\,(c+d\,x)-\frac{4\,A\,(165\,c+165\,d\,x+150)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.$$

input
```
int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x +
205))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 3
0))/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 1
50))/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x +
367))/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x +
 320))/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x
+ 385))/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c
/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.237.1 Optimal result 1934
3.237.2 Mathematica [C] (verified) 1934
3.237.3 Rubi [A] (verified) 1935
3.237.4 Maple [C] (verified) 1936
3.237.5 Fricas [B] (verification not implemented) 1937
3.237.6 Sympy [B] (verification not implemented) 1937
3.237.7 Maxima [B] (verification not implemented) 1938
3.237.8 Giac [A] (verification not implemented) 1939
3.237.9 Mupad [B] (verification not implemented) 1939

3.237.1 Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{Ax}{a^3} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

output
```
-A*x/a^3-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+7/5*A*cos(d*x+c)/a^3/d/(1
+sin(d*x+c))^2-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

3.237.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\left(20\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin(c+\right.}{15a^3d(1+\sin(c+dx))^2}$$

input
```
Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(20*Sqrt[2]*Hypergeometric2F1[-3/2,
 -3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d
*x]]*(-4 + 3*Sin[c + d*x] + Sin[c + d*x]^2)))/(15*a^3*d*(1 + Sin[c + d*x])
^2)
```

3.237.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(c+dx)^2(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3445$$

$$\int \left(\frac{4A}{a^3(\sin(c+dx)+1)} - \frac{5A}{a^3(\sin(c+dx)+1)^2} + \frac{2A}{a^3(\sin(c+dx)+1)^3} - \frac{A}{a^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)} + \frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} - \frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{Ax}{a^3}$$

input
```
Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*
(1 + Sin[c + d*x]))
```

3.237.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.237.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{Ax}{a^3}-\frac{2\left(-75A\,e^{2i(dx+c)}+55iA\,e^{3i(dx+c)}+20A\,e^{4i(dx+c)}-45iA\,e^{i(dx+c)}+13A\right)}{5da^3\left(e^{i(dx+c)}+i\right)^5}$
derivativedivides	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{4\left(\tan\left(\frac{d}{}\right.\right.\right.\right.}$
default	$\frac{8A\left(-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{4\left(\tan\left(\frac{d}{}\right.\right.\right.\right.}$
parallelrisch	$-\frac{\left(\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)dx+(5dx+2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10dx+10)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10dx+22)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(5dx+14\right.}{da^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
norman	$\frac{-\frac{Ax}{a}-\frac{16A}{5ad}-\frac{5Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{13Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25Ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{38Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{46Ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{}$

input
```
int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
-A*x/a^3-2/5*(-75*A*exp(2*I*(d*x+c))+55*I*A*exp(3*I*(d*x+c))+20*A*exp(4*I*
(d*x+c))-45*I*A*exp(I*(d*x+c))+13*A)/d/a^3/(exp(I*(d*x+c))+I)^5
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.

Time $= 0.27$ (sec) , antiderivative size $= 204$, normalized size of antiderivative $= 2.29$

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{(5\,Adx+13\,A)\cos(dx+c)^3-20\,Adx+3\,(5\,Adx-2\,A)\cos(dx+c)^2-(10\,Adx+21\,A)\cos(dx+c)}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3\right.}$$

input
```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d
*x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*
cos(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
 c))
```

3.237.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(85) = 170$.

Time $= 7.41$ (sec) , antiderivative size $= 1268$, normalized size of antiderivative $= 14.25$

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output

```
Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25
*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*ta
n(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(
c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)
**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a
**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*
d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2
+ d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) +
 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 +
25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*
tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*ta
n(c/2 + d*x/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)*
*4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a*
*3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5
+ 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*
d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(
c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)
**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a
**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*ta
n(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 +...
```

3.237.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(83) = 166$.

Time $= 0.34$ (sec) , antiderivative size $= 392$, normalized size of antiderivative $= 4.40$

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{2\left(A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{}{a^3+\frac{5a^3}{\cos}}}{15\,d}$$

input

```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3) + 2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5))/d
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= -\frac{\frac{5\,(dx+c)A}{a^3} + \frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

input
```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
 + 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*
A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.237.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\left(5\,A\,(c+d\,x)-\frac{A\,(25\,c+25\,d\,x+10)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(10\,A\,(c+d\,x)-\frac{A\,(50\,c+50\,d\,x+50)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(10\right.}{a^3}$$
$$-\frac{A\,x}{a^3}$$

input
```
int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + tan(c/2
 + (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + tan(c/2 + (d*
x)/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + tan(c/2 + (d*x)/2)
^2*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c
+ 5*d*x + 16))/5)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.238.1 Optimal result 1941
3.238.2 Mathematica [A] (verified) 1941
3.238.3 Rubi [A] (verified) 1942
3.238.4 Maple [A] (verified) 1943
3.238.5 Fricas [B] (verification not implemented) 1943
3.238.6 Sympy [B] (verification not implemented) 1944
3.238.7 Maxima [B] (verification not implemented) 1945
3.238.8 Giac [A] (verification not implemented) 1945
3.238.9 Mupad [B] (verification not implemented) 1946

3.238.1 Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*
x+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

3.238.2 Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)+15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

input
```
Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 1
5*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

3.238.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$

$$\downarrow 3445$$

$$\int \left(-\frac{A}{a^3(\sin(c+dx)+1)}+\frac{3A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}\right)dx$$

$$\downarrow 2009$$

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

input
```
Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(1
5*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c +
d*x]))
```

3.238.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.238.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{2A\left(15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{15da^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
derivativedivides	$\frac{4A\left(\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{da^3}$
default	$\frac{4A\left(\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}\right)}{da^3}$
risch	$\frac{2A\left(15ie^{3i(dx+c)}+15e^{4i(dx+c)}-5ie^{i(dx+c)}-25e^{2i(dx+c)}+4\right)}{15da^3\left(e^{i(dx+c)}+i\right)^5}$
norman	$\frac{-\frac{2A}{15ad}-\frac{14A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{10A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{2A\left(\tan^6\left(\frac{dx}{2}\right.\right.}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

input
```
int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
-2/15*A*(15*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)^2+5*tan(1/2*d*x+1/2*
c)+1)/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(76) = 152$.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{4\,A\cos(dx+c)^3+7\,A\cos(dx+c)^2-3\,A\cos(dx+c)-\left(4\,A\cos(dx+c)^2-3\,A\cos(dx+c)-6\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)\right.\right.}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*co
s(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*
cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.238.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(78) = 156$.

Time $= 4.17$ (sec) , antiderivative size $= 461$, normalized size of antiderivative $= 5.62$

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} + \frac{}{15a^3d\tan^5\left(\frac{c}{2}+ \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} \end{cases}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a
**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*ta
n(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2
 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**
4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan
(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*
x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15
*a**3*d) - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)
**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75
*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c
)/(a*sin(c) + a)**3, True))
```

3.238.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(76) = 152$.

Time = 0.25 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.24

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{2\left(\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)}{(\cos(dx+c)+1}}\right)}{15\,d}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="max
ima")
```

output
```
2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
+ 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gia
c")
```

output
```
-2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(
1/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{2A\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+15\,\mathrm{si}\right.}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

input
```
int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
-(2*A*cos(c/2 + (d*x)/2)^2*(cos(c/2 + (d*x)/2)^3 + 15*sin(c/2 + (d*x)/2)^3
 - 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 5*cos(c/2 + (d*x)/2)^2*sin(
c/2 + (d*x)/2)))/(15*a^3*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.239.1 Optimal result 1947
3.239.2 Mathematica [A] (verified) 1947
3.239.3 Rubi [A] (verified) 1948
3.239.4 Maple [C] (verified) 1949
3.239.5 Fricas [B] (verification not implemented) 1950
3.239.6 Sympy [B] (verification not implemented) 1951
3.239.7 Maxima [B] (verification not implemented) 1951
3.239.8 Giac [A] (verification not implemented) 1952
3.239.9 Mupad [B] (verification not implemented) 1952

3.239.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4} - \frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}$$

output
```
-1/5*a*A*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^4-1/15*A*cos(d*x+c)^3/d/(a+a*sin(
d*x+c))^3
```

3.239.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

input
```
Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
 + (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^5)
```

3.239.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3215, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A - A\sin(c+dx)}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A - A\sin(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\downarrow 3215$$

$$aA\int \frac{\cos^2(c+dx)}{(\sin(c+dx)a+a)^4}dx$$

$$\downarrow 3042$$

$$aA\int \frac{\cos(c+dx)^2}{(\sin(c+dx)a+a)^4}dx$$

$$\downarrow 3151$$

$$aA\left(\frac{\int \frac{\cos^2(c+dx)}{(\sin(c+dx)a+a)^3}dx}{5a} - \frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

$$\downarrow 3042$$

$$aA\left(\frac{\int \frac{\cos(c+dx)^2}{(\sin(c+dx)a+a)^3}dx}{5a} - \frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

$$\downarrow 3150$$

$$aA\left(-\frac{\cos^3(c+dx)}{15ad(a\sin(c+dx)+a)^3} - \frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

input
```
Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

output
```
a*A*(-1/5*Cos[c + d*x]^3/(d*(a + a*Sin[c + d*x])^4) - Cos[c + d*x]^3/(15*a
*d*(a + a*Sin[c + d*x])^3))
```

3.239.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3215
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c +
 d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((Lt
Q[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

3.239.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

method	result
risch	$\frac{2iA\left(-5ie^{2i(dx+c)}+15\,e^{3i(dx+c)}-i-5\,e^{i(dx+c)}\right)}{15da^3\left(e^{i(dx+c)}+i\right)^5}$
parallelrisch	$-\frac{2A\left(15\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+25\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}{15da^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
derivativedivides	$\frac{2A\left(\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}\right)}{da^3}$
default	$\frac{2A\left(\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}\right)}{da^3}$
norman	$\frac{-\frac{8A}{15ad}-\frac{58A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2A\left(\tan^6\left(\frac{dx}{2}\right.\right.}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

input
```
int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*I*A*(-5*I*exp(2*I*(d*x+c))+15*exp(3*I*(d*x+c))-I-5*exp(I*(d*x+c)))/d/
a^3/(exp(I*(d*x+c))+I)^5
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.66

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\cos(dx+c)^3 - 2\,A\cos(dx+c)^2 + 3\,A\cos(dx+c) - \left(A\cos(dx+c)^2 + 3\,A\cos(dx+c) + 6\,A\right.}{15\left(a^3d\cos(dx+c)^3 + 3\,a^3d\cos(dx+c)^2 - 2\,a^3d\cos(dx+c) - 4\,a^3d + \left(a^3d\cos(dx+c)^2 - 2\,a^3d\cos(dx+c)\right.\right.}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*
x + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(
d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.239.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(53) = 106.

Time = 2.32 (sec) , antiderivative size = 573, normalized size of antiderivative = 9.88

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$

$$= \begin{cases} -\frac{30A\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} - \frac{}{15a^3d\tan^5\left(\frac{c}{2}+ \\ \frac{x(-A\sin(c)+A)}{(a\sin(c)+a)^3} \end{cases}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-30*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a
**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*ta
n(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*tan(c/2
 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**
4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*tan(c/2 + d*x/2)**2/(15*a**3*d*
tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 +
 d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) +
 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan
(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*
d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2
 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
 + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

3.239.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(54) = 108.

Time = 0.23 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.67

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{2\left(\frac{A\left(\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+}{(\cos(dx+c)+}\right.}{15\,d}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5))/d
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,A\right.}{15\,a^3 d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*ta
n(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x
 + 1/2*c) + 1)^5)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 12.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+25\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right.}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

input
```
int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

output
```
-(2*A*cos(c/2 + (d*x)/2)*(4*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
 + 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 5*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2) + 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*a^3*d
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.240.1 Optimal result 1954
3.240.2 Mathematica [B] (verified) 1954
3.240.3 Rubi [A] (verified) 1955
3.240.4 Maple [A] (verified) 1956
3.240.5 Fricas [B] (verification not implemented) 1957
3.240.6 Sympy [F] 1957
3.240.7 Maxima [B] (verification not implemented) 1958
3.240.8 Giac [A] (verification not implemented) 1958
3.240.9 Mupad [B] (verification not implemented) 1959

3.240.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\operatorname{arctanh}(\cos(c+dx))}{a^3 d} + \frac{2A\cos(c+dx)}{5a^3 d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{5a^3 d(1+\sin(c+dx))^2} + \frac{8A\cos(c+dx)}{5a^3 d(1+\sin(c+dx))}$$

output
```
-A*arctanh(cos(d*x+c))/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+3/5*A
*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+8/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

3.240.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(98) = 196.

Time = 6.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.19

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(2\cos\left(\frac{c}{2}\right)-2\sin\left(\frac{c}{2}\right)+3\cos\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.$$

input
```
Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c
/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 2*Sin[(d*x)/2]*(-17
+ 4*Cos[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(C
os[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^5)
```

3.240.3 Rubi [A] (verified)

Time $= 0.36$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 1.00$, number of steps used $= 3$, number of rules used $= 3$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used $= \{3042, 3445, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)(A - A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A - A\sin(c+dx)}{\sin(c+dx)(a\sin(c+dx)+a)^3}dx$$

$$\downarrow 3445$$

$$\int \left(-\frac{A}{a^3(\sin(c+dx)+1)} - \frac{A}{a^3(\sin(c+dx)+1)^2} - \frac{2A}{a^3(\sin(c+dx)+1)^3} + \frac{A\csc(c+dx)}{a^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{A\operatorname{arctanh}(\cos(c+dx))}{a^3 d} + \frac{8A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} + \frac{3A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} + \frac{2A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3}$$

input
```
Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Si
n[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*
Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.240.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{d\,a^3}$
default	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{d\,a^3}$
risch	$\frac{2A\left(25ie^{3i(dx+c)}+5\,e^{4i(dx+c)}-35ie^{i(dx+c)}-55\,e^{2i(dx+c)}+8\right)}{5d\,a^3\left(e^{i(dx+c)}+i\right)^5}+\frac{A\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}-\frac{A\ln\left(e^{i(dx+c)}+1\right)}{d\,a^3}$
norman	$\frac{\frac{8A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{18A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{38A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{26A}{5ad}+\frac{22A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{40A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{176A\left(\tan^2\left(\frac{d}{}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
parallelrisch	$\frac{5\left(\left(\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\frac{\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)}{5}+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\frac{\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)}{5}+2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{d\,a^3\left(-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+5\sin\left(\frac{3dx}{2}+\frac{3c}{2}\right)-5\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)-\cos\left(\right.\right.}$

input
```
int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*A/a^3*(16/5/(tan(1/2*d*x+1/2*c)+1)^5-8/(tan(1/2*d*x+1/2*c)+1)^4+12/(ta
n(1/2*d*x+1/2*c)+1)^3-10/(tan(1/2*d*x+1/2*c)+1)^2+8/(tan(1/2*d*x+1/2*c)+1)
+ln(tan(1/2*d*x+1/2*c)))
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(92) = 184$.

Time $= 0.26$ (sec) , antiderivative size $= 310$, normalized size of antiderivative $= 3.16$

$$\int \frac{\csc(c+dx)(A - A\sin(c+dx))}{(a + a\sin(c+dx))^3}\,dx$$
$$= \frac{16\,A\cos(dx+c)^3 - 22\,A\cos(dx+c)^2 - 42\,A\cos(dx+c) - 5\left(A\cos(dx+c)^3 + 3\,A\cos(dx+c)^2 - 2\,A\right.}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) + 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) + 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.240.6 Sympy [F]

$$\int \frac{\csc(c+dx)(A - A\sin(c+dx))}{(a + a\sin(c+dx))^3}\,dx$$
$$= -\frac{A\left(\int \left(-\frac{\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)\,dx + \int \frac{\sin(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.240.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(92) = 184.

Time = 0.22 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.42

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(\frac{20\sin(d}{\cos(dx+}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+}$$
$$15\,d$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="max
ima")
```

output
```
1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3) + 2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 1
0*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5))/d
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{5A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(20A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+55A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+75A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13A\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{5\,d}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gia
c")
```

output
```
1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)
^4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(
1/2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.240.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.03

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(5\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+90\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+150\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+110\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+25\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x
)/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c
/2 + (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (
d*x)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c
/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 +
(d*x)/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.241.1 Optimal result 1960
3.241.2 Mathematica [A] (verified) 1960
3.241.3 Rubi [A] (verified) 1961
3.241.4 Maple [A] (verified) 1963
3.241.5 Fricas [B] (verification not implemented) 1963
3.241.6 Sympy [F] 1964
3.241.7 Maxima [B] (verification not implemented) 1964
3.241.8 Giac [A] (verification not implemented) 1965
3.241.9 Mupad [B] (verification not implemented) 1966

3.241.1 Optimal result

Integrand size = 32, antiderivative size = 113

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4A\operatorname{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} - \frac{104A\cot(c+dx)}{15a^3d(1+\csc(c+dx))}$$

output
```
4*A*arctanh(cos(d*x+c))/a^3/d-94/15*A*cot(d*x+c)/a^3/d+2/5*A*cot(d*x+c)/a^
3/d/(1+sin(d*x+c))^3+13/15*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4*A*cot(d*x
+c)/a^3/d/(1+sin(d*x+c))
```

3.241.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(15\cot\left(\frac{1}{2}(c+dx)\right) - 120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{12}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))}\right.}{30a^3d}$$

input
```
Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c
 + d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*
x)] - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[
(c + d*x)/2]))/(a^3*d)
```

3.241.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^2(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3429$$

$$aA\int \frac{\cot^2(c+dx)}{(\sin(c+dx)a+a)^4}\,dx$$

$$\downarrow 3042$$

$$aA\int \frac{1}{(\sin(c+dx)a+a)^4\tan(c+dx)^2}\,dx$$

$$\downarrow 3188$$

$$\frac{A\int\left(\frac{\csc^2(c+dx)}{a^2}-\frac{4\csc(c+dx)}{a^2}-\frac{16}{a^2(\csc(c+dx)+1)}+\frac{9}{a^2}+\frac{9}{a^2(\csc(c+dx)+1)^2}-\frac{2}{a^2(\csc(c+dx)+1)^3}\right)dx}{a}$$

$$\downarrow 2009$$

$$\frac{A\left(\frac{4\operatorname{arctanh}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{104\cot(c+dx)}{15a^2d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^2d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^2d(\csc(c+dx)+1)^3}\right)}{a}$$

input
```
Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*((4*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c +
d*x])/(5*a^2*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^2*d*(1 + Cs
c[c + d*x])^2) - (104*Cot[c + d*x])/(15*a^2*d*(1 + Csc[c + d*x]))))/a
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

3.241.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{(\tan}\right.}{2d\,a^3}$
default	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{(\tan}\right.}{2d\,a^3}$
parallelrisch	$-\frac{\left(8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+52\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+161\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{649\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}{3}}{2a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
risch	$-\frac{4\left(-320A\,\mathrm{e}^{4i(dx+c)}+150iA\,\mathrm{e}^{5i(dx+c)}+367A\,\mathrm{e}^{2i(dx+c)}-385iA\,\mathrm{e}^{3i(dx+c)}-47A+205iA\,\mathrm{e}^{i(dx+c)}+30A\,\mathrm{e}^{6i(dx+c)}\right)}{15\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^5a^3d}-\frac{4}{}$
norman	$\frac{-\frac{A}{2ad}+\frac{A\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{3811A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}-\frac{893A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{413A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{161A\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{805}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

input
```
int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/2/d*A/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*
c))-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x+1/2*c)+1)^4-88/3/(tan(1/
2*d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(tan(1/2*d*x+1/2*c)+1))
```

3.241.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(107) = 214.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.59

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{94\,A\cos(dx+c)^4+222\,A\cos(dx+c)^3-115\,A\cos(dx+c)^2-237\,A\cos(dx+c)+30\,\left(A\cos(dx+c)^4\right.}{}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 -
237*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d
*x + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*
A*cos(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30
*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x
 + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*s
in(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 1
28*A*cos(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d
*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*
d*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
- 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

3.241.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= -\frac{A\left(\int \left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)\,dx + \int \frac{\sin(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(107) = 214$.

Time $= 0.25$ (sec) , antiderivative size $= 519$, normalized size of antiderivative $= 4.59$

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{3A\left(\frac{\frac{121\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5}{a^3(\cos}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/
(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) +
1))/a^3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x +
 c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\right.}{a^3\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{30\,d}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c
)/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(
135*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/
2*d*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)^5))/d
```

3.241.9 Mupad [B] (verification not implemented)

Time = 15.92 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.86

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d} - \frac{4\,A\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}$$
$$- \frac{37\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 121\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \frac{514\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + \frac{338\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} + \frac{491\,A\tan}{}}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 10\,a^3\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*log(tan(c/2 + (d*x)/2)))/(a^3*d) -
 (A + (491*A*tan(c/2 + (d*x)/2))/15 + (338*A*tan(c/2 + (d*x)/2)^2)/3 + (51
4*A*tan(c/2 + (d*x)/2)^3)/3 + 121*A*tan(c/2 + (d*x)/2)^4 + 37*A*tan(c/2 +
(d*x)/2)^5)/(d*(10*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3
+ 20*a^3*tan(c/2 + (d*x)/2)^4 + 10*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/
2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)))
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.242.1 Optimal result 1967
3.242.2 Mathematica [A] (verified) 1968
3.242.3 Rubi [A] (verified) 1968
3.242.4 Maple [A] (verified) 1970
3.242.5 Fricas [B] (verification not implemented) 1970
3.242.6 Sympy [F] 1971
3.242.7 Maxima [B] (verification not implemented) 1971
3.242.8 Giac [A] (verification not implemented) 1972
3.242.9 Mupad [B] (verification not implemented) 1973

3.242.1 Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19A\operatorname{arctanh}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{29A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{164A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
-19/2*A*arctanh(cos(d*x+c))/a^3/d+4*A*cot(d*x+c)/a^3/d-1/2*A*cot(d*x+c)*cs
c(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+29/15*A*cos(d*x+c)/
a^3/d/(1+sin(d*x+c))^2+164/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

3.242.2 Mathematica [A] (verified)

Time = 2.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.78

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{A\left(240\cot\left(\frac{1}{2}(c+dx)\right) - 15\csc^2\left(\frac{1}{2}(c+dx)\right) - 1140\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 1140\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) +\right.}{}$$

input
```
Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/
2]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*
x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*
x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120
*a^3*d)
```

3.242.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^3(a\sin(c+dx)+a)^3}\,dx$$

$$\downarrow 3445$$

$$\int \left(-\frac{9A}{a^3(\sin(c+dx)+1)} - \frac{5A}{a^3(\sin(c+dx)+1)^2} - \frac{2A}{a^3(\sin(c+dx)+1)^3} + \frac{A\csc^3(c+dx)}{a^3} - \frac{4A\csc^2(c+dx)}{a^3} +\right.$$

$$\downarrow 2009$$

$$-\frac{19A\text{arctanh}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}$$

input
```
Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*
Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Si
n[c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (16
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.242.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\right.}{4da^3}$
default	$\frac{A\left(\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\right.}{4da^3}$
parallelrisch	$-\frac{\left(-76\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-472\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1504\left(\tan^{3}\right.\right.}{8a^3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
risch	$\frac{A\left(1425ie^{7i(dx+c)}+285e^{8i(dx+c)}-5225ie^{5i(dx+c)}-3325e^{6i(dx+c)}+5635ie^{3i(dx+c)}+6423e^{4i(dx+c)}-1955ie^{i(dx+c)}-395\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^5a^3d}$
norman	$\frac{\frac{57A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{A}{8ad}+\frac{11A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{11A\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{1943A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{2627A\left(\tan\right.}{6}$
	$\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2$

input
```
int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/4/d*A/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-8*tan(1/2*d*x+1/2*c)+64/5/(tan(1/2*d
*x+1/2*c)+1)^5-32/(tan(1/2*d*x+1/2*c)+1)^4+208/3/(tan(1/2*d*x+1/2*c)+1)^3-
72/(tan(1/2*d*x+1/2*c)+1)^2+128/(tan(1/2*d*x+1/2*c)+1)-1/2/tan(1/2*d*x+1/2
*c)^2+8/tan(1/2*d*x+1/2*c)+38*ln(tan(1/2*d*x+1/2*c)))
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{896\,A\cos(dx+c)^5-1222\,A\cos(dx+c)^4-3218\,A\cos(dx+c)^3+1168\,A\cos(dx+c)^2+2292\,A\cos(dx$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
 + 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3
*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x
+ c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*c
os(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(
A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*co
s(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*
x + c) + 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*co
s(d*x + c)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos
(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*co
s(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*
a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a
^3*d)*sin(d*x + c))
```

3.242.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int \left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)\,dx + \int \frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.242.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(128) = 256$.

Time $= 0.27$ (sec) , antiderivative size $= 622$, normalized size of antiderivative $= 4.51$

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = 12\,A\left(\frac{\frac{121\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\sin}{a^3(\cos}\right)$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)
/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) +
 1))/a^3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)
/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 78
0*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{15\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + \frac{15\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6} -}{120\,d}$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
 1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) +
15*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16
*(240*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan
(1/2*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*
x + 1/2*c) + 1)^5))/d
```

3.242.9 Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(165\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4234\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+14090\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+19780\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12060\tan\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (
d*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830
*tan(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)
^8 + 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x
)/2)^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan
(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(
c/2 + (d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 114
0*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 +
 (d*x)/2)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

3.243.1 Optimal result 1974
3.243.2 Mathematica [B] (verified) 1975
3.243.3 Rubi [A] (verified) 1975
3.243.4 Maple [A] (verified) 1977
3.243.5 Fricas [B] (verification not implemented) 1977
3.243.6 Sympy [F] 1978
3.243.7 Maxima [B] (verification not implemented) 1979
3.243.8 Giac [A] (verification not implemented) 1979
3.243.9 Mupad [B] (verification not implemented) 1980

3.243.1 Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{18A\operatorname{arctanh}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{93A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

output

```
18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^
3/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^
3-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(
d*x+c))
```

3.243.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 348 vs. $2(153) = 306$.

Time = 6.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.27

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(-\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{\sec^2\left(\frac{1}{2}(\right.}{2}$$

input
```
Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*
x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[
Sin[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*S
in[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c
 + d*x)/2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

3.243.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^4(a\sin(c+dx)+a)^3}\,dx$$
$$\downarrow 3445$$

$$\int \left(\frac{16A}{a^3(\sin(c+dx)+1)} + \frac{7A}{a^3(\sin(c+dx)+1)^2} + \frac{2A}{a^3(\sin(c+dx)+1)^3} + \frac{A\csc^4(c+dx)}{a^3} - \frac{4A\csc^3(c+dx)}{a^3} + \frac{9A}{a^3} \right.$$

$$\downarrow \text{2009}$$

$$\frac{18A\operatorname{arctanh}(\cos(c+dx))}{a^3 d} - \frac{A\cot^3(c+dx)}{3a^3 d} - \frac{10A\cot(c+dx)}{a^3 d} - \frac{93A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} - \frac{13A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} - \frac{2A\cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3 d}$$

input
```
Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Co
t[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*C
os[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d
*(1 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

3.243.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{A\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{160}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\right.}{8d\,a^3}$
default	$\frac{A\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{160}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\right.}{8d\,a^3}$
parallelrisch	$-\frac{\left(432\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-67\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2637\left(\tan^4\left(\right.\right.\right.}{24}$
risch	$-\frac{4\left(675iA\,e^{9i(dx+c)}+135A\,e^{10i(dx+c)}-3150iA\,e^{7i(dx+c)}-1710A\,e^{8i(dx+c)}+5180iA\,e^{5i(dx+c)}+4572A\,e^{6i(dx+c)}-3590i\right.}{15\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^5a^3d}$
norman	$\frac{-\frac{A}{24ad}+\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{17A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{17A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{7A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{A\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{66469A\left(\tan\right.}{12}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$

input
```
int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/8/d*A/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+39*tan(1/2*d*
x+1/2*c)-128/5/(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2*d*x+1/2*c)+1)^4-160/(t
an(1/2*d*x+1/2*c)+1)^3+176/(tan(1/2*d*x+1/2*c)+1)^2-400/(tan(1/2*d*x+1/2*c
)+1)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-39/tan(1/2*d*x+1/2*c)
-144*ln(tan(1/2*d*x+1/2*c)))
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.88

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{424\,A\cos(dx+c)^6+1002\,A\cos(dx+c)^5-944\,A\cos(dx+c)^4-2074\,A\cos(dx+c)^3+531\,A\cos(dx+}{}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4
- 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135
*(A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x
 + c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*
cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x
+ c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*
A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)
^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin
(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 57
8*A*cos(d*x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A
*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*c
os(d*x + c)^5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*
cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 +
3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 +
 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))
```

3.243.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int \left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)\,dx + \int \frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.243.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(145) = 290.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.61

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(c
os(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) -
A*((20*sin(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 4777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 5)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(
d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.39

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{5\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} + \frac{48\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}{}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
 1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/
(a^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*ta
n(1/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x +
1/2*c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x
+ 1/2*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2
*c))/a^9)/d
```

3.243.9 Mupad [B] (verification not implemented)

Time = 15.98 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{A\left(335\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7559\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+24610\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+33170\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

output
```
-(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*
x)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*
tan(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)
^8 - 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*
x)/2)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(t
an(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*ta
n(c/2 + (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 +
10800*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d
*x)/2))*tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/
2 + (d*x)/2) + 1)^5)
```

3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

3.244.1 Optimal result 1981
3.244.2 Mathematica [A] (verified) 1982
3.244.3 Rubi [A] (verified) 1982
3.244.4 Maple [A] (verified) 1985
3.244.5 Fricas [A] (verification not implemented) 1986
3.244.6 Sympy [B] (verification not implemented) 1986
3.244.7 Maxima [A] (verification not implemented) 1987
3.244.8 Giac [A] (verification not implemented) 1988
3.244.9 Mupad [B] (verification not implemented) 1989

3.244.1 Optimal result

Integrand size = 33, antiderivative size = 327

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+12c^2d+12cd^2+3d^3\right)\right)x \\
&\quad -\frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df} \\
&\quad -\frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\cos(e+fx)\sin(e+fx)}{120f} \\
&\quad -\frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)d))\cos(e+fx)(c+d\sin(e+fx))^2}{60df} \\
&\quad +\frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} \\
&\quad -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}
\end{aligned}$$

output

```
1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))
*x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*
d^2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B
*(6*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+
4*B)*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*
c-5*(A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin
(f*x+e))^4/d/f
```

3.244.2 Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.82

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{a(1+\sin(e+fx))\left(-60(2A(4c^3+12c^2d+9cd^2+3d^3)+B(8c^3+18c^2d+18cd^2+5d^3))\cos(e+fx)+1\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(
8*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d)
 + B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)
] + 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d +
12*c*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2
*(e + f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)
```

3.244.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$
$$\downarrow 3447$$
$$\int (c+d\sin(e+fx))^3\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$
$$\downarrow 3042$$

$$\int (c+d\sin(e+fx))^3\left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (c+d\sin(e+fx))^3(a(5A+4B)d-a(Bc-5(A+B)d)\sin(e+fx))dx}{5d} -$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))^3(a(5A+4B)d-a(Bc-5(A+B)d)\sin(e+fx))dx}{5d} -$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\downarrow 3232$$

$$\frac{\frac{1}{4}\int (c+d\sin(e+fx))^2\left(ad(20Ac+13Bc+15Ad+15Bd)+a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\sin(e+fx\right.}{5d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}\int (c+d\sin(e+fx))^2\left(ad(20Ac+13Bc+15Ad+15Bd)+a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\sin(e+fx\right.}{5d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\downarrow 3232$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (c+d\sin(e+fx))\left(ad\left(60Ac^2+33Bc^2+75Adc+75Bdc+40Ad^2+32Bd^2\right)+a\left(5Ad\left(6c^2+20dc+9d^2\right)-\right.\right.\right.}{aB\cos(e+fx)(c+d\sin(e+fx))^4}$$
$$\frac{}{5df}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (c+d\sin(e+fx))\left(ad\left(60Ac^2+33Bc^2+75Adc+75Bdc+40Ad^2+32Bd^2\right)+a\left(5Ad\left(6c^2+20dc+9d^2\right)-\right.\right.\right.}{aB\cos(e+fx)(c+d\sin(e+fx))^4}$$
$$\frac{}{5df}$$

$$\downarrow 3213$$

3.244. $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(-\frac{ad\left(5Ad\left(6c^2+20cd+9d^2\right)-B\left(6c^3-30c^2d-71cd^2-45d^3\right)\right)\sin(e+fx)\cos(e+fx)}{2f}+\frac{15}{2}adx\left(A\left(8c^3+12c^2d+12cd^2+3d^3\right)+B\right.\right.\right.}{}$$

$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/5*(a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(d*f) + ((a*(B*c - 5*(A + B
)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f) + (-1/3*(a*(4*(5*A + 4*B)*
d^2 - 3*c*(B*c - 5*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/f + ((
15*a*d*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*
c*d^2 + 3*d^3))*x)/2 - (2*a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) -
 B*(3*c^4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/f -
(a*d*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45
*d^3))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/4)/(5*d)
```

3.244.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.244.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

method	result
parts	$\frac{(Aad^3+3Bacd^2+d^3Ba)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(Ac^3a+3Aac^2d+Bac^3)\cos(fx+e)}{f}$
parallelrisch	$\frac{\left(((-A-B)d^3-3d^2c(A+B)-3c^2d(A+B)-Bc^3)\sin(2fx+2e)+\left(\frac{\left(A+\frac{5B}{4}\right)d^2}{3}+dc(A+B)+Bc^2\right)d\cos(3fx+3e)+\frac{d^2(3B}{}\right.}{}$
derivativedivides	$\frac{-A\cos(fx+e)ac^3+3Aac^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aacd^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)+Aad^3\left(-\frac{\left(\sin^3(fx+e\right.}{}$
default	$\frac{-A\cos(fx+e)ac^3+3Aac^2d\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aacd^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)+Aad^3\left(-\frac{\left(\sin^3(fx+e\right.}{}$
risch	$-\frac{3\sin(2fx+2e)Aac^2d}{4f}-\frac{3\sin(2fx+2e)Aacd^2}{4f}-\frac{3\sin(2fx+2e)Bac^2d}{4f}-\frac{3\sin(2fx+2e)Bacd^2}{4f}+Aac^3x+$
norman	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
(A*a*d^3+3*B*a*c*d^2+B*a*d^3)/f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)-(A*a*c^3+3*A*a*c^2*d+B*a*c^3)/f*cos(f*x+e)-1/3*(3*A*a*
c*d^2+A*a*d^3+3*B*a*c^2*d+3*B*a*c*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)+(3*A*
a*c^2*d+3*A*a*c*d^2+B*a*c^3+3*B*a*c^2*d)/f*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2
*f*x+1/2*e)+A*a*c^3*x-1/5*d^3*B*a/f*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*co
s(f*x+e)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.73

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx =$$
$$-\frac{24\,Bad^3\cos(fx+e)^5 - 40\,(3\,Bac^2d + 3\,(A+B)acd^2 + (A+2\,B)ad^3)\cos(fx+e)^3 - 15\,(4\,(2\,A+B)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 +
(A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2
*d + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3
*(A + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2
*(3*B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*
c^2*d + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e
))/f
```

3.244.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.05

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output

```
Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f
*x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*
x)**2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*
cos(e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d*
*2*cos(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x
)/f - 3*A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f
*x)**3/(3*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2
 - B*a*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*
B*a*c**2*d*x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*
c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e +
f*x)/(2*f) - 2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)*
*4/8 + 9*B*a*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*c
os(e + f*x)**4/8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*
a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) - 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*
x)**4/8 + 3*B*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*co
s(e + f*x)**4/8 - B*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*...
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.24

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx$$
$$= \frac{480\,(fx + e)Aac^3 + 120\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Bac^3 + 360\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aac^2d +}{}$$

input

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^
3 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 -
 3*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*
d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d
^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2
+ 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos
(f*x + e)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4
*e) - 8*sin(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3
*cos(f*x + e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.94

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx = -\frac{Bad^3\cos(5fx+5e)}{80f}$$
$$+\frac{1}{8}\left(8Aac^3 + 4Bac^3 + 12Aac^2d + 12Bac^2d + 12Aacd^2 + 9Bacd^2 + 3Aad^3 + 3Bad^3\right)x$$
$$+\frac{(12Bac^2d + 12Aacd^2 + 12Bacd^2 + 4Aad^3 + 5Bad^3)\cos(3fx+3e)}{48f}$$
$$-\frac{(8Aac^3 + 8Bac^3 + 24Aac^2d + 18Bac^2d + 18Aacd^2 + 18Bacd^2 + 6Aad^3 + 5Bad^3)\cos(fx+e)}{8f}$$
$$+\frac{(3Bacd^2 + Aad^3 + Bad^3)\sin(4fx+4e)}{32f}$$
$$-\frac{(Bac^3 + 3Aac^2d + 3Bac^2d + 3Aacd^2 + 3Bacd^2 + Aad^3 + Bad^3)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2
*d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x
+ 1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3
)*cos(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*
c^2*d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/
f + 1/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c
^3 + 3*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a
*d^3)*sin(2*f*x + 2*e)/f
```

3.244.9 Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.54

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$= \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(8Ac^3+3Ad^3+4Bc^3+3Bd^3+12Acd^2+12Ac^2d+9Bcd^2+12Bc^2d\right)}{4\left(2Aac^3+\frac{3Aad^3}{4}+Bac^3+\frac{3Bad^3}{4}+3Aacd^2+3Aac^2d+\frac{9Bacd^2}{4}+3Bac^2d\right)}\right)\left(8Ac^3+3Ad^3+4Bc^3+3B\right.}{4f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aad^3}{4}+Bac^3+\frac{3Bad^3}{4}+3Aacd^2+3Aac^2d+\frac{9Bacd^2}{4}+3Bac^2d\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

output
```
(a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*
A*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3
)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/
4 + 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 1
2*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d
^3)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2
)/4 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c
^2*d) + tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16
*B*a*d^3)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) +
 tan(e/2 + (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*
d^3)/3 + 28*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(
e/2 + (f*x)/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*
a*c^2*d + 12*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)
/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4
 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a
*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - ta
n(e/2 + (f*x)/2)^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^
2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3
)/3 + 2*B*a*c^3 + (16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^
2 + 4*B*a*c^2*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 +...
```

3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

3.245.1 Optimal result 1990
3.245.2 Mathematica [A] (verified) 1991
3.245.3 Rubi [A] (verified) 1991
3.245.4 Maple [A] (verified) 1994
3.245.5 Fricas [A] (verification not implemented) 1994
3.245.6 Sympy [B] (verification not implemented) 1995
3.245.7 Maxima [A] (verification not implemented) 1996
3.245.8 Giac [A] (verification not implemented) 1996
3.245.9 Mupad [B] (verification not implemented) 1997

3.245.1 Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)x \\
&\quad - \frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df} \\
&\quad - \frac{a(3(4A+3B)d^2-2c(Bc-4(A+B)d))\cos(e+fx)\sin(e+fx)}{24f} \\
&\quad + \frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} \\
&\quad - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}
\end{aligned}$$

output

```
1/8*a*(4*A*(2*c^2+2*c*d+d^2)+B*(4*c^2+8*c*d+3*d^2))*x-1/6*a*(4*A*d*(c^2+3*
c*d+d^2)-B*(c^3-4*c^2*d-8*c*d^2-4*d^3))*cos(f*x+e)/d/f-1/24*a*(3*(4*A+3*B)
*d^2-2*c*(B*c-4*(A+B)*d))*cos(f*x+e)*sin(f*x+e)/f+1/12*a*(B*c-4*(A+B)*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^2/d/f-1/4*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f
```

3.245.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.87

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx$$
$$= \frac{a(1+\sin(e+fx))\left(-24(B(4c^2+6cd+3d^2)+A(4c^2+8cd+3d^2))\cos(e+fx)+8d(Ad+B(2c+d))\cos\right.}{96f\,(}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

output
```
(a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d +
 3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4
*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2
 + A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

3.245.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$
$$\downarrow 3447$$
$$\int (c+d\sin(e+fx))^2\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$
$$\downarrow 3042$$
$$\int (c+d\sin(e+fx))^2\left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (c+d\sin(e+fx))^2(a(4A+3B)d-a(Bc-4(A+B)d)\sin(e+fx))dx}{4d} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))^2(a(4A+3B)d-a(Bc-4(A+B)d)\sin(e+fx))dx}{4d} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$$\downarrow 3232$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(ad(12Ac+7Bc+8Ad+8Bd)-a\left(2Bc^2-8(A+B)dc-3(4A+3B)d^2\right)\sin(e+fx)\right)dx}{4d} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(ad(12Ac+7Bc+8Ad+8Bd)-a\left(2Bc^2-8(A+B)dc-3(4A+3B)d^2\right)\sin(e+fx)\right)dx}{4d} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$$\downarrow 3213$$

$$\frac{\frac{1}{3}\left(\frac{ad(-8cd(A+B)-3d^2(4A+3B)+2Bc^2)\sin(e+fx)\cos(e+fx)}{2f}+\frac{3}{2}adx\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-\frac{2a(4Ad(}{4d} - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/4*(a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(d*f) + ((a*(B*c - 4*(A + B
)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((3*a*d*(4*A*(2*c^2 + 2*
c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/2 - (2*a*(4*A*d*(c^2 + 3*c*d +
d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/f + (a*d*(2*B*c^
2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3
)/(4*d)
```

3.245.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.245.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{(Aad^2+2Bacd+d^2Ba)\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3f}-\frac{\left(ac^2A+2Aacd+Bac^2\right)\cos(fx+e)}{f}+\frac{(2Aacd+Aad^2+Bac$
parallelrisch	$\frac{\left(\left((-3B-3A)d^2-6dc(A+B)-3Bc^2\right)\sin(2fx+2e)+(2Bc+(A+B)d)d\cos(3fx+3e)+\frac{3d^2B\sin(4fx+4e)}{8}+\left((-9A-9B)d^2$
derivativedivides	$\frac{-ac^2A\cos(fx+e)+2Aacd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aad^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Bac^2\left(-\frac{\cos(fx+e)\sin(fx+$
default	$\frac{-ac^2A\cos(fx+e)+2Aacd\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aad^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Bac^2\left(-\frac{\cos(fx+e)\sin(fx+$
risch	$Aac^2x+Aacdx+\frac{Aad^2x}{2}+\frac{Bac^2x}{2}+Bacdx+\frac{3Bad^2x}{8}-\frac{a\cos(fx+e)Ac^2}{f}-\frac{2a\cos(fx+e)Acd}{f}-$
norman	$\frac{(ac^2A+Aacd+\frac{1}{2}Aad^2+\frac{1}{2}Bac^2+Bacd+\frac{3}{8}d^2Ba)x+(ac^2A+Aacd+\frac{1}{2}Aad^2+\frac{1}{2}Bac^2+Bacd+\frac{3}{8}d^2Ba)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right.$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*(A*a*d^2+2*B*a*c*d+B*a*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)-(A*a*c^2+2*
A*a*c*d+B*a*c^2)/f*cos(f*x+e)+(2*A*a*c*d+A*a*d^2+B*a*c^2+2*B*a*c*d)/f*(-1/
2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+A*a*c^2*x+d^2*B*a/f*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.75

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{8\left(2\,Bacd+(A+B)ad^2\right)\cos\left(fx+e\right)^3+3\left(4\left(2\,A+B\right)ac^2+8\left(A+B\right)acd+\left(4\,A+3\,B\right)ad^2\right)fx-24\left((A$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2
+ 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)
*a*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*
a*c^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(201) = 402$.

Time $= 0.23$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 2.68$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$

$$= \begin{cases} Aac^2x - \frac{Aac^2\cos(e+fx)}{f} + Aacdx\sin^2(e+fx) + Aacdx\cos^2(e+fx) - \frac{Aacd\sin(e+fx)\cos(e+fx)}{f} - \frac{2Aacd\cos(e} \\ x(A + B\sin(e))(c + d\sin(e))^2(a\sin(e) + a) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**
2 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a*c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f
*x)**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e
+ f*x)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e +
 f*x)/(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*
d*x*cos(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*s
in(e + f*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*
x*sin(e + f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B
*a*d**2*x*cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - B*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B
*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a), True))
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.24

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx$$
$$= \frac{96\,(fx+e)Aac^2 + 24\,(2\,fx + 2\,e - \sin(2\,fx+2\,e))Bac^2 + 48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aacd + 64\,(c}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(
f*x + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))
*A*a*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e
 + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e
) - 96*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx$$
$$= \frac{Bad^2\sin(4\,fx+4\,e)}{32\,f} + \frac{1}{8}\left(8\,Aac^2 + 4\,Bac^2 + 8\,Aacd + 8\,Bacd + 4\,Aad^2 + 3\,Bad^2\right)x$$
$$+ \frac{(2\,Bacd + Aad^2 + Bad^2)\cos(3\,fx+3\,e)}{12\,f}$$
$$- \frac{(4\,Aac^2 + 4\,Bac^2 + 8\,Aacd + 6\,Bacd + 3\,Aad^2 + 3\,Bad^2)\cos(fx+e)}{4\,f}$$
$$- \frac{(Bac^2 + 2\,Aacd + 2\,Bacd + Aad^2 + Bad^2)\sin(2\,fx+2\,e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
 8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^
2)*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d
 + 3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*
a*c*d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

3.245.9 Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.57

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$= \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8Bcd\right)}{4\left(2Aac^2+Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2Bacd\right)}\right)\left(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8B\right.}{4f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(2Aac^2+2Bac^2+4Aacd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2B\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

output
```
(a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A
*c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a
*c*d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B
*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) +
 tan(e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a
*c*d) + tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^
2 + 12*A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^
2)/3 + 6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2
 + (f*x)/2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
+ tan(e/2 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2
*B*a*c*d) - tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A
*a*c*d + 2*B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/
3 + 4*A*a*c*d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f
*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

3.246.1 Optimal result 1998
3.246.2 Mathematica [A] (verified) 1998
3.246.3 Rubi [A] (verified) 1999
3.246.4 Maple [A] (verified) 2001
3.246.5 Fricas [A] (verification not implemented) 2001
3.246.6 Sympy [B] (verification not implemented) 2002
3.246.7 Maxima [A] (verification not implemented) 2002
3.246.8 Giac [A] (verification not implemented) 2003
3.246.9 Mupad [B] (verification not implemented) 2003

3.246.1 Optimal result

Integrand size = 31, antiderivative size = 111

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{1}{2}a(B(c+d)+A(2c+d))x - \frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}$$
$$- \frac{a(3Bc+3Ad-Bd)\cos(e+fx)\sin(e+fx)}{6f} - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af}$$

output
```
1/2*a*(B*(c+d)+A*(2*c+d))*x-1/3*a*(3*A*(c+d)+B*(3*c+d))*cos(f*x+e)/f-1/6*a
*(3*A*d+3*B*c-B*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*B*d*cos(f*x+e)*(a+a*sin(f*x
+e))^2/a/f
```

3.246.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{a(12Acfx+6Bcfx+6Adfx+6Bdfx-3(4A(c+d)+B(4c+3d))\cos(e+fx)+Bd\cos(3(e+fx))-}{12f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4
*c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] -
3*A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

3.246.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3447, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\downarrow 3447$$

$$\int (a\sin(e+fx)+a)\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (\sin(e+fx)a+a)(a(3Ac+2Bd)+a(3Bc+3Ad-Bd)\sin(e+fx))dx}{3a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)(a(3Ac+2Bd)+a(3Bc+3Ad-Bd)\sin(e+fx))dx}{3a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

$$\downarrow 3213$$

$$\frac{-\frac{a^2(3A(c+d)+B(3c+d))\cos(e+fx)}{f} - \frac{a^2(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{2f} + \frac{3}{2}a^2x(A(2c+d)+B(c+d))}{3a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/3*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(a*f) + ((3*a^2*(B*(c + d)
+ A*(2*c + d))*x)/2 - (a^2*(3*A*(c + d) + B*(3*c + d))*Cos[e + f*x])/f - (
a^2*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(2*f))/(3*a)
```

3.246.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.246.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

method	result
parts	$\frac{(Aad+Bac+dBa)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(Aac+Aad+Bac)\cos(fx+e)}{f}+aAcx-\frac{dBa(2+\sin^2(fx+e}{3f}$
parallelrisch	$-\frac{\left((B(c+d)+dA)\sin(2fx+2e)-\frac{Bd\cos(3fx+3e)}{3}+((4c+3d)B+4A(c+d))\cos(fx+e)+(-2cfx-2dfx+4c+\frac{8}{3}d)B-4\left(\left(\frac{f}{2}\right.\right.\right.}{4f}$
derivativedivides	$\frac{-\frac{dBa\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+dBa\left(-\frac{co}{}\right.}{f}$
default	$\frac{-\frac{dBa\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aad\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+dBa\left(-\frac{co}{}\right.}{f}$
risch	$aAcx+\frac{Aadx}{2}+\frac{Bacx}{2}+\frac{Badx}{2}-\frac{a\cos(fx+e)Ac}{f}-\frac{a\cos(fx+e)dA}{f}-\frac{a\cos(fx+e)Bc}{f}-\frac{3a\cos(fx+e)dB}{4f}$
norman	$\frac{(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}dBa)x+(Aac+\frac{1}{2}Aad+\frac{1}{2}Bac+\frac{1}{2}dBa)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3Aac+\frac{3}{2}Aad+\frac{3}{2}Bac+\frac{3}{2}dBa)x\left(\tan\right.}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
(A*a*d+B*a*c+B*a*d)/f*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)-(A*a*c+A*
a*d+B*a*c)/f*cos(f*x+e)+a*A*c*x-1/3*d*B*a/f*(2+sin(f*x+e)^2)*cos(f*x+e)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{2\,Bad\cos(fx+e)^3+3\left((2\,A+B)ac+(A+B)ad\right)fx-3\left(Bac+(A+B)ad\right)\cos(fx+e)\sin(fx+e)+}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a
*c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d
)*cos(f*x + e))/f
```

3.246.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.50

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \begin{cases} Aacx - \frac{Aac\cos(e+fx)}{f} + \frac{Aadx\sin^2(e+fx)}{2} + \frac{Aadx\cos^2(e+fx)}{2} - \frac{Aad\sin(e+fx)\cos(e+fx)}{2f} - \frac{Aad\cos(e+fx)}{f} + \frac{Bacx\sin^2(e+} \\ x(A + B\sin(e))(c + d\sin(e))(a\sin(e) + a) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*
a*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*co
s(e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a
*c*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e
+ f*x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*
x)/f - B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*
f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{12\,(fx + e)Aac + 3\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Bac + 3\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aad + 4\,(\cos(fx}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2
*f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))
*B*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e)
- 12*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))\,dx$$
$$= \frac{Bad\cos(3fx+3e)}{12f} + \frac{1}{2}(2Aac + Bac + Aad + Bad)x$$
$$- \frac{(4Aac + 4Bac + 4Aad + 3Bad)\cos(fx+e)}{4f} - \frac{(Bac + Aad + Bad)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x -
1/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c +
A*a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

3.246.9 Mupad [B] (verification not implemented)

Time = 12.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))\,dx =$$
$$- \frac{\frac{3Aad\sin(2e+2fx)}{2} - \frac{Bad\cos(3e+3fx)}{2} + \frac{3Bac\sin(2e+2fx)}{2} + \frac{3Bad\sin(2e+2fx)}{2} + 6Aac\cos(e+fx) + 6Aa}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

output
```
-((3*A*a*d*sin(2*e + 2*f*x))/2 - (B*a*d*cos(3*e + 3*f*x))/2 + (3*B*a*c*sin
(2*e + 2*f*x))/2 + (3*B*a*d*sin(2*e + 2*f*x))/2 + 6*A*a*c*cos(e + f*x) + 6
*A*a*d*cos(e + f*x) + 6*B*a*c*cos(e + f*x) + (9*B*a*d*cos(e + f*x))/2 - 6*
A*a*c*f*x - 3*A*a*d*f*x - 3*B*a*c*f*x - 3*B*a*d*f*x)/(6*f)
```

3.247 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx$

3.247.1 Optimal result 2004
3.247.2 Mathematica [A] (verified) 2004
3.247.3 Rubi [A] (verified) 2005
3.247.4 Maple [A] (verified) 2006
3.247.5 Fricas [A] (verification not implemented) 2006
3.247.6 Sympy [B] (verification not implemented) 2007
3.247.7 Maxima [A] (verification not implemented) 2007
3.247.8 Giac [A] (verification not implemented) 2008
3.247.9 Mupad [B] (verification not implemented) 2008

3.247.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \frac{1}{2}a(2A + B)x - \frac{a(A + B)\cos(e + fx)}{f} - \frac{aB\cos(e + fx)\sin(e + fx)}{2f}$$

output
```
1/2*a*(2*A+B)*x-a*(A+B)*cos(f*x+e)/f-1/2*a*B*cos(f*x+e)*sin(f*x+e)/f
```

3.247.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \frac{a(2Be + 4Afx + 2Bfx - 4(A + B)\cos(e + fx) - B\sin(2(e + fx)))}{4f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)
]))/(4*f)
```

3.247.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))dx$$

$$\downarrow 3213$$

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
 f*x])/(2*f)
```

3.247.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.247.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{\left(-\frac{B\sin(2fx+2e)}{4}+(-A-B)\cos(fx+e)+fxA+\frac{fxB}{2}+A+B\right)a}{f}$
parts	$axA-\frac{(aA+Ba)\cos(fx+e)}{f}+\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
risch	$axA+\frac{aBx}{2}-\frac{a\cos(fx+e)A}{f}-\frac{a\cos(fx+e)B}{f}-\frac{Ba\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-Ba\cos(fx+e)+aA(fx+e)}{f}$
default	$\frac{Ba\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-aA\cos(fx+e)-Ba\cos(fx+e)+aA(fx+e)}{f}$
norman	$\frac{(aA+\frac{1}{2}Ba)x+(aA+\frac{1}{2}Ba)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2aA+Ba)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{(2aA+2Ba)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{Ba\left(\tan^3\left(\frac{fx}{2}+$
	$\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*B*sin(2*f*x+2*e)+(-A-B)*cos(f*x+e)+f*x*A+1/2*f*x*B+A+B)*a/f
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{(2A+B)afx - Ba\cos(fx+e)\sin(fx+e) - 2(A+B)a\cos(fx+e)}{2f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
 + e))/f
```

3.247.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time $= 0.09$ (sec) , antiderivative size $= 94$, normalized size of antiderivative $= 1.96$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx$$
$$= \begin{cases} Aax - \frac{Aa\cos(e+fx)}{f} + \frac{Bax\sin^2(e+fx)}{2} + \frac{Bax\cos^2(e+fx)}{2} - \frac{Ba\sin(e+fx)\cos(e+fx)}{2f} - \frac{Ba\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A + B\sin(e))(a\sin(e) + a) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*co
s(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f
, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

3.247.7 Maxima [A] (verification not implemented)

Time $= 0.24$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 1.19$

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx$$
$$= \frac{4\,(fx + e)Aa + (2\,fx + 2\,e - \sin(2\,fx + 2\,e))Ba - 4\,Aa\cos(fx + e) - 4\,Ba\cos(fx + e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*
x + e) - 4*B*a*cos(f*x + e))/f
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))\,dx = \frac{1}{2}(2Aa + Ba)x - \frac{Ba\sin(2fx+2e)}{4f} - \frac{(Aa+Ba)\cos(fx+e)}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e
)/f
```

3.247.9 Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.08

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))\,dx = A\,a\,x - \frac{-B\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3 + (2\,A\,a + 2\,B\,a)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 + B\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right) + 2\,A\,a + 2\,B\,a}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + 2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 + 1\right)} + \frac{B\,a\,x}{2}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

output
```
A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/
2 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(
e/2 + (f*x)/2)^4 + 1)) + (B*a*x)/2
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

3.248.1 Optimal result 2009
3.248.2 Mathematica [C] (verified) 2009
3.248.3 Rubi [A] (verified) 2010
3.248.4 Maple [A] (verified) 2013
3.248.5 Fricas [A] (verification not implemented) 2013
3.248.6 Sympy [B] (verification not implemented) 2014
3.248.7 Maxima [F(-2)] 2015
3.248.8 Giac [A] (verification not implemented) 2015
3.248.9 Mupad [B] (verification not implemented) 2016

3.248.1 Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= -\frac{a(Bc-(A+B)d)x}{d^2} + \frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}f} - \frac{aB\cos(e+fx)}{df}$$

output
```
-a*(B*c-(A+B)*d)*x/d^2-a*B*cos(f*x+e)/d/f+2*a*(c-d)*(-A*d+B*c)*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.00

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \frac{a\left(Adx + B(-c+d)x - \frac{Bd\cos(e)\cos(fx)}{f} + \frac{2(c-d)(Bc-Ad)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d
)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*
x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e])
)/(Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/
f)*(1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

3.248.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3502$$

$$\frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d} - \frac{aB\cos(e+fx)}{df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d} - \frac{aB\cos(e+fx)}{df}$$

$$\downarrow 3214$$

$$\frac{\frac{a(c-d)(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\downarrow 3042$$

$$\frac{\frac{a(c-d)(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\downarrow 3139$$

$$\frac{\frac{2a(c-d)(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\downarrow 1083$$

$$-\frac{\frac{4a(c-d)(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\downarrow 217$$

$$\frac{\frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
(-((a*(B*c - (A + B)*d)*x)/d) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(2*d + 2*c
*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a*B*C
os[e + f*x])/(d*f)
```

3.248.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.248.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2a\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-Bc+dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\left(-Acd+Ad^2+Bc^2-cdB\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}\right)}{f}$
default	$\frac{2a\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-Bc+dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{\left(-Acd+Ad^2+Bc^2-cdB\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}\right)}{f}$
risch	$\frac{xaA}{d}-\frac{xaBc}{d^2}+\frac{xaB}{d}-\frac{Ba\,e^{i(fx+e)}}{2df}-\frac{Ba\,e^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c-d)(c+d)}\,a\ln\left(e^{i(fx+e)}+\frac{ic-\sqrt{-(c-d)(c+d)}}{d}\right)A}{(c+d)fd}-$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*a*(1/d^2*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-B*c+B*d)*arctan(tan(1/2*f
*x+1/2*e)))+(-A*c*d+A*d^2+B*c^2-B*c*d)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c
*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$=\left[-\frac{2\,Bad\cos(fx+e)+2\,(Bac-(A+B)ad)fx-(Bac-Aad)\sqrt{-\frac{c-d}{c+d}}\log\left(-\frac{(2c^2-d^2)\cos(fx+e)^2-2cd\sin(f\right.}{2\,d^2f}\right.$$

$$\left.-\frac{Bad\cos(fx+e)+(Bac-(A+B)ad)fx+(Bac-Aad)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{(c\sin(fx+e)+d)\sqrt{\frac{c-d}{c+d}}}{(c-d)\cos(fx+e)}\right)}{d^2f}\right]$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d
)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*
x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2
)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*
x + e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d
)*f*x + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)
*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

3.248.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5508 vs. $2(82) = 164$.

Time = 113.71 (sec) , antiderivative size = 5508, normalized size of antiderivative = 56.20

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((zoo*x*(A + B*sin(e))*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0)
 & Eq(f, 0)), ((A*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) +
A*a*f*x/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))*tan(e/2 +
f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))/(f*tan(e
/2 + f*x/2)**2 + f) + B*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 +
 f) + B*a*f*x/(f*tan(e/2 + f*x/2)**2 + f) - 2*B*a/(f*tan(e/2 + f*x/2)**2 +
 f))/d, Eq(c, 0)), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x
/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f
*(d**2)**(3/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**
3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) + 2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**
(3/2)) + 2*A*a*d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2)
- f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(
d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f
*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x
*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**
2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)*tan(
e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*
(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**...
```

3.248.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{\frac{(Bac-Aad-Bad)(fx+e)}{d^2}+\frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\left(Bac^2-Aacd-Bacd+Aad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}d^2}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
 1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)
/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/
(sqrt(c^2 - d^2)*d^2))/f
```

3.248.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 3074, normalized size of antiderivative = 31.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)
```

output
```
(2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*a
tan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x
))/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*
f*(c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i +
A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/
2)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i
+ B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x
)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)
*1i + A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^
2)^(3/2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^
5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)
*(d^2 - c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1
i - B^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(
e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2
- c^2)^(1/2)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^
2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*6i + B^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^
5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2...
```

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

3.249.1 Optimal result . . . 2017
3.249.2 Mathematica [C] (verified) . . . 2017
3.249.3 Rubi [A] (verified) . . . 2018
3.249.4 Maple [A] (verified) . . . 2021
3.249.5 Fricas [B] (verification not implemented) . . . 2021
3.249.6 Sympy [F(-1)] . . . 2022
3.249.7 Maxima [F(-2)] . . . 2022
3.249.8 Giac [A] (verification not implemented) . . . 2023
3.249.9 Mupad [B] (verification not implemented) . . . 2023

3.249.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{aBx}{d^2} + \frac{2a((A+B)(c-d)d^2 - Bc(c^2-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2} f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

output
```
a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*
e))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)
/f/(c+d*sin(f*x+e))
```

3.249.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(1+\sin(e+fx))\left(Bx + \frac{2\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{(c+d)\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{(-}{}\right)}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

output
```
(a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[
(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[
c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqr
t[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Co
s[e] + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x])))/(d^2*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2)
```

3.249.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3447, 3042, 3500, 25, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3447$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3042$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3500$$

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int -\frac{a(A+B)(c-d)d+aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(A+B)(c-d)d+aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c^2-d^2)} + \frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(A+B)(c-d)d+aB\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 3214$$

$$\frac{\frac{a(c-d)\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{aBx\left(c^2-d^2\right)}{d}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{a(c-d)\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{aBx\left(c^2-d^2\right)}{d}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 3139$$

$$\frac{\frac{2a(c-d)\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}+\frac{aBx\left(c^2-d^2\right)}{d}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 1083$$

$$\frac{\frac{aBx\left(c^2-d^2\right)}{d}-\frac{4a(c-d)\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 217$$

$$\frac{\frac{2a(c-d)\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}+\frac{aBx\left(c^2-d^2\right)}{d}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

output
```
((a*B*(c^2 - d^2)*x)/d + (2*a*(c - d)*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan
[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))
/(d*(c^2 - d^2)) + (a*(B*c - A*d)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

3.249.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.249.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{2a\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{-\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-cdB+d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}\right)}{f}$
default	$\frac{2a\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^2}+\frac{-\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(dA-Bc)}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-cdB+d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}\right)}{f}$
risch	$\frac{aBx}{d^2}-\frac{2ia(-dA+Bc)\left(id+c\,e^{i(fx+e)}\right)}{d^2(c+d)f\left(-ie^{2i(fx+e)}d+id+2c\,e^{i(fx+e)}\right)}-\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)A}{\sqrt{-c^2+d^2}\,(c+d)f}+\frac{a\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-}}{\sqrt{-c^2+d^2}}\right.}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a*(B/d^2*arctan(tan(1/2*f*x+1/2*e))+1/d^2*((-d^2*(A*d-B*c)/(c+d)/c*tan
(1/2*f*x+1/2*e)-d*(A*d-B*c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x
+1/2*e)+c)+(A*d^2-B*c^2-B*c*d+B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c
*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(119) = 238.

Time = 0.29 (sec) , antiderivative size = 655, normalized size of antiderivative = 5.28

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \left[\frac{2\left(Bac^3d+Bac^2d^2-Bacd^3-Bad^4\right)fx\sin\left(fx+e\right)+2\left(Bac^4+Bac^3d-Bac^2d^2-Bacd^3\right)fx+(Bac\right.}{}\right.$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
[1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
 2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^
2*d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x +
e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e
) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2
 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c
^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^
4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)
, ((B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a
*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (
A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqr
t(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e)))
 + (B*a*c^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3
 + c^2*d^4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c
*d^5)*f)]
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.249.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{(fx+e)Ba}{d^2} - \frac{2\,(Bac^2+Bacd-Aad^2-Bad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}} + \frac{2\,(Bacd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Aad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Bad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Bac^2+Bacd-Aad^2-Bad^2)}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2
*e) - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(
c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.249.9 Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 5102, normalized size of antiderivative = 41.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)
```

output

```
(2*B*a*atan(((B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*
d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4
+ 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 -
2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5)
)/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 +
2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d
^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))
/(2*c*d^3 + d^4 + c^2*d^2) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2
*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^
3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^
2))/d^2 + (B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))
/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2
*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B
^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(
2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^
4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*
a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*
d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d
^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^...
```

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

3.250.1 Optimal result . . . 2025
3.250.2 Mathematica [C] (verified) . . . 2025
3.250.3 Rubi [A] (verified) . . . 2026
3.250.4 Maple [B] (verified) . . . 2030
3.250.5 Fricas [B] (verification not implemented) . . . 2030
3.250.6 Sympy [F(-1)] . . . 2031
3.250.7 Maxima [F(-2)] . . . 2032
3.250.8 Giac [B] (verification not implemented) . . . 2032
3.250.9 Mupad [B] (verification not implemented) . . . 2033

3.250.1 Optimal result

Integrand size = 33, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \frac{a(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))}$$

output

```
a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(
c+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+
e))^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f
/(c+d*sin(f*x+e))
```

3.250.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.96

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= a(1+\sin(e+fx))\left(\frac{4(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{(2c^2+d^2)(A}{4(c-d)}\right.$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

output
```
(a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]
*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^
2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d
 - 2*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2)
)*Cos[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d
^2))*Sin[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e +
 f*x]))/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2)
```

3.250.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3447, 3042, 3500, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3042$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3500$$

$$\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int -\frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}$$

$$\downarrow 25$$

$$\frac{\int \frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$\frac{-\frac{\int -\frac{a(c-d)d(2Ac+Bc-Ad-2Bd)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{a(c-d)d(2Ac+Bc-Ad-2Bd)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\frac{ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\frac{ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 3139$$

$$\frac{2ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c^2-d^2)} - \frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))} + \frac{2d\left(c^2-d^2\right)}{\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}}$$

$$\downarrow 1083$$

$$-\frac{4ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c^2-d^2)} - \frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))} + \frac{2d\left(c^2-d^2\right)}{\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}}$$

$$\downarrow 217$$

$$\frac{2ad(c-d)(2Ac-Ad+Bc-2Bd)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}} - \frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))} + \frac{2d\left(c^2-d^2\right)}{\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}}$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) + ((2*
a*(c - d)*d*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2]
)/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*(A*(c - 2*d)*d + B*(c^2
 + 2*c*d - 2*d^2))*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x])))/(2*d*(c
^2 - d^2))
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(167) = 334.

Time = 1.10 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.41

method	result
derivativedivides	$2a\left(\frac{-\frac{\left(3c^2dA-2d^2cA-2Ad^3-Bc^3+2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+c^2d-d^2c-d^3\right)}-\frac{\left(2Ac^4-2Ac^3d+3Ac^2d^2-4Acd^3-2Ad^4+2Bc^4-Bc^3d+4Bc^2d^2-\right.}{2\left(c^3+c^2d-d^2c-d^3\right)c^2}}{\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2c\right.}\right.$
default	$2a\left(\frac{-\frac{\left(3c^2dA-2d^2cA-2Ad^3-Bc^3+2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^3+c^2d-d^2c-d^3\right)}-\frac{\left(2Ac^4-2Ac^3d+3Ac^2d^2-4Acd^3-2Ad^4+2Bc^4-Bc^3d+4Bc^2d^2-\right.}{2\left(c^3+c^2d-d^2c-d^3\right)c^2}}{\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2c\right.}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a*((-1/2*(3*A*c^2*d-2*A*c*d^2-2*A*d^3-B*c^3+2*B*c^2*d)/c/(c^3+c^2*d-c*
d^2-d^3)*tan(1/2*f*x+1/2*e)^3-1/2*(2*A*c^4-2*A*c^3*d+3*A*c^2*d^2-4*A*c*d^3
-2*A*d^4+2*B*c^4-B*c^3*d+4*B*c^2*d^2-2*B*c*d^3)/(c^3+c^2*d-c*d^2-d^3)/c^2*
tan(1/2*f*x+1/2*e)^2-1/2*(5*A*c^2*d-6*A*c*d^2-2*A*d^3+B*c^3+6*B*c^2*d-4*B*
c*d^2)/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-1/2*(2*A*c^2-2*A*c*d-A*d
^2+2*B*c^2-B*c*d)/(c^3+c^2*d-c*d^2-d^3))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1
/2*f*x+1/2*e)+c)^2+1/2*(2*A*c-A*d+B*c-2*B*d)/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(167) = 334.

Time = 0.30 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.49

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
[1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a
*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (
A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^
2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c
*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x
 + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
d^2)) + 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (
2*A + B)*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4
- 2*c^2*d^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d
^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2
- c^4*d^3 - c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)
*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*co
s(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)
*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x
 + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2
 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*
(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*
d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 +
 c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3...
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.250.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(167) = 334.

Time = 0.33 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.24

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{(2\,Aac+Bac-Aad-2\,Bad)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^3+c^2d-cd^2-d^3)\sqrt{c^2-d^2}} + \frac{Bac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,Aac^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Bac^3d}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output

```
((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn
(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d
- c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*
c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*
a*c^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^
3*d*tan(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*
d^2*tan(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*
c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*
d^4*tan(1/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(
1/2*f*x + 1/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/
2*f*x + 1/2*e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2
*d^2)/((c^5 + c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*t
an(1/2*f*x + 1/2*e) + c)^2))/f
```

3.250.9 Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{A\,a\,d^2-2\,A\,a\,c^2-2\,B\,a\,c^2+2\,A\,a\,c\,d+B\,a\,c\,d}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^3-B\,c^3+6\,A\,c\,d^2-5\,A\,c^2\,d+4\,B\,c\,d^2-6\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A}{c(-}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+c^2+4\,c\,d\right.}$$

$$-\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,(2\,A\,c-A\,d+B\,c-2\,B\,d)\,\left(-2\,c^3\,d-2\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{2\,(c+d)^{5/2}\,(c-d)^{3/2}\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A\,c-A\,d+B\,c-2\,B\,d)}{(c+d)^{5/2}\,(c-d)^{3/2}}\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}{2\,A\,a\,c-A\,a\,d+B\,a\,c-2\,B\,a\,d}\right)\,(2\,A\,c-}{f\,(c+d)^{5/2}\,(c-d)^{3/2}}$$

input

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)
```

output

```
- ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d
- c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^
2*d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2
 + (f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c
*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2
- 2*A*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))
/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2
 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*
(2*A*c - A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c
 + d)^(5/2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f
*x)/2)*(2*A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2
- c^2*d - c^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d +
B*c - 2*B*d))/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

3.251.1 Optimal result 2035
3.251.2 Mathematica [A] (verified) 2036
3.251.3 Rubi [A] (verified) 2037
3.251.4 Maple [A] (verified) 2041
3.251.5 Fricas [A] (verification not implemented) 2042
3.251.6 Sympy [B] (verification not implemented) 2043
3.251.7 Maxima [A] (verification not implemented) 2043
3.251.8 Giac [A] (verification not implemented) 2044
3.251.9 Mupad [B] (verification not implemented) 2045

3.251.1 Optimal result

Integrand size = 35, antiderivative size = 464

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)x \\
&\quad+\frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+64d^5))\,c}{60d^2f} \\
&\quad+\frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\cos(e+fx)\sin(e+} \\
&\quad+\frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f} \\
&\quad+\frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f} \\
&\quad+\frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} \\
&\quad-\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}
\end{aligned}$$

output
```
1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11
*d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^
5-12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/24
0*a^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2
+284*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*
d-12*d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^
2/d^2/f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^4/d^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

3.251.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{a^2\cos(e+fx)\left(60(6A(4c^3+8c^2d+7cd^2+2d^3)+B(16c^3+42c^2d+36cd^2+11d^3))\arcsin\left(\frac{\sqrt{1-\sin(e+f}}{\sqrt{2}}\right.\right.$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^3,x]
```

output
```
-1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*
(16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqr
t[2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*
B*c^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*
(5*c^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2
*(e + f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Si
n[e + f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c
^2*d*Sin[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x]
+ 540*A*d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e +
f*x)] - 90*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*
d^3*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)
])))/(f*Sqrt[Cos[e + f*x]^2])
```

3.251.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$

$$\downarrow 3455$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3(a(6Ad+B(c+4d))-a(2Bc-6Ad-7Bd)\sin(e+fx))dx}{6d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3(a(6Ad+B(c+4d))-a(2Bc-6Ad-7Bd)\sin(e+fx))dx}{6d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3447$$

$$\frac{\int (c+d\sin(e+fx))^3\left(-\left((2Bc-6Ad-7Bd)\sin^2(e+fx)a^2\right)+(6Ad+B(c+4d))a^2+\left(a^2(6Ad+B(c+4d))-\right.\right.}{6d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))^3\left(-\left((2Bc-6Ad-7Bd)\sin(e+fx)^2a^2\right)+(6Ad+B(c+4d))a^2+\left(a^2(6Ad+B(c+4d))-\right.\right.}{6d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3502$$

$$\frac{\frac{\int -(c+d\sin(e+fx))^3\left(3d(Bc-18Ad-16Bd)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e+fx)a^2\right)dx}{5d}+\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 25$$
$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\int(c+d\sin(e+fx))^3\left(3d(B(c-16d)-18Ad)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e+\right.}{5d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 3042$$
$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\int(c+d\sin(e+fx))^3\left(3d(B(c-16d)-18Ad)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e+\right.}{5d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 3232$$
$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\frac{1}{4}\int -3(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-1\right.\right.\right.}{6d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 27$$
$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\int(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-12\right.\right.\right.}{6d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 3042$$
$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\int(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-12\right.\right.\right.}{6d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$
$$\downarrow 3232$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df} - \frac{-\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))(a^2d(6Ad(31c^2+50dc+24d^2)-B(2c^3-132dc^2-267d^2c-128d^3))\right.}{}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3042$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df} - \frac{-\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))(a^2d(6Ad(31c^2+50dc+24d^2)-B(2c^3-132dc^2-267d^2c-128d^3))\right.}{}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\downarrow 3213$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df} - \frac{-\frac{a^2\left(6Ad(c-10d)-B\left(2c^2-12cd+55d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4f}-\frac{3}{4}\left(\frac{a^2\left(6Ad\left(c^2-10cd-12d^2\right.\right.\right.}{}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/6*(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(d*f
) + ((a^2*(2*B*c - 6*A*d - 7*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*
d*f) - (-1/4*(a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^3)/f - (3*((a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2)
 - B*(2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*
x])^2)/(3*f) + ((15*a^2*d^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(
16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*x)/2 + (2*a^2*(6*A*d*(c^4 - 10*c^3
*d - 44*c^2*d^2 - 40*c*d^3 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 +
208*c^2*d^3 + 216*c*d^4 + 64*d^5))*Cos[e + f*x])/f + (a^2*d*(6*A*d*(2*c^3
 - 20*c^2*d - 57*c*d^2 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c
*d^3 + 165*d^4))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3))/4)/(5*d))/(6*d)
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.251.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a^2\left(\left(\left(-2A-\frac{31B}{16}\right)d^3-6d^2c(A+B)-6c^2d(A+B)-c^3(A+2B)\right)\sin(2fx+2e)+\left(\left(\frac{3A}{4}+\frac{5B}{6}\right)d^3+2\left(\frac{9B}{8}+A\right)cd^2+c^2(A+2$
parts	$-\frac{(Aa^2d^3+3Ba^2d^2c+2Ba^2d^3)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}-\frac{(2Aa^2c^3+3Aa^2c^2d+Ba^2c^3)\cos(f$
derivativedivides	$\frac{Aa^2c^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aa^2c^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)+3Aa^2d^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos}{4}\right.}{}$
default	$\frac{Aa^2c^3\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aa^2c^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)+3Aa^2d^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos}{4}\right.}{}$
risch	$-\frac{3a^2d^2\cos(5fx+5e)Bc}{80f}+\frac{3\sin(4fx+4e)Aa^2d^2c}{32f}+\frac{3\sin(4fx+4e)Ba^2c^2d}{32f}+\frac{3\sin(4fx+4e)Ba^2d^2c}{16f}+\frac{a^2\cos(3,}{}$
norman	Expression too large to display

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/4*a^2*(((-2*A-31/16*B)*d^3-6*d^2*c*(A+B)-6*c^2*d*(A+B)-c^3*(A+2*B))*sin(
2*f*x+2*e)+((3/4*A+5/6*B)*d^3+2*(9/8*B+A)*c*d^2+c^2*(A+2*B)*d+1/3*B*c^3)*c
os(3*f*x+3*e)+3/8*((2/3*A+5/6*B)*d^2+c*(A+2*B)*d+B*c^2)*d*sin(4*f*x+4*e)-1
/20*((A+2*B)*d+3*B*c)*d^2*cos(5*f*x+5*e)-1/48*B*d^3*sin(6*f*x+6*e)+((-5*B-
11/2*A)*d^3-18*(11/12*B+A)*c*d^2-21*(A+6/7*B)*c^2*d-8*c^3*(A+7/8*B))*cos(f
*x+e)+(-64/15*B+3*f*x*A+11/4*f*x*B-24/5*A)*d^3+21/2*c*(f*x*A+6/7*f*x*B-32/
21*A-48/35*B)*d^2+12*c^2*(f*x*A+7/8*f*x*B-5/3*A-4/3*B)*d+6*c^3*(f*x*A+2/3*
f*x*B-4/3*A-10/9*B))/f
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.78

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx =$$
$$-\frac{48\,(3\,Ba^2cd^2 + (A+2\,B)a^2d^3)\cos(fx+e)^5 - 80\,(Ba^2c^3 + 3\,(A+2\,B)a^2c^2d + 3\,(2\,A+3\,B)a^2cd^2 + (}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*
c^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^
3)*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d +
6*(7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^
3 + 3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x +
 e) + 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2
*c*d^2 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 +
6*(8*A + 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))/f
```

3.251.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1865 vs. 2(450) = 900.

Time = 0.52 (sec) , antiderivative size = 1865, normalized size of antiderivative = 4.02

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2
/2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**
2*c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*
d*x*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A
*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3
/f - 3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8
+ 9*A*a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*
sin(e + f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*
x*cos(e + f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f)
- 6*A*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2
*f) - 4*A*a**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/
4 + 3*A*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*co
s(e + f*x)**4/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d*
*3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(
e + f*x)**3/(3*f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*
d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(1
5*f) - 2*A*a**2*d**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2
 + B*a**2*c**3*x*cos(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x
)/f - B*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f...
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx$$
$$= \frac{240\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2c^3 + 960\,(fx+e)Aa^2c^3 + 320\left(\cos(fx+e)^3 - 3\,\cos(fx+e)\right)Ba^2c}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^
2*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e
 - sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^2*c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*
e))*B*a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*
(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*
(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*
cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*co
s(f*x + e))*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*co
s(f*x + e))*A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 +
60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x +
2*e))*B*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*d^3 - 1920*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) -
2880*A*a^2*c^2*d*cos(f*x + e))/f
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{Ba^2d^3\sin(6fx+6e)}{192f}$$
$$+\frac{1}{16}\left(24Aa^2c^3+16Ba^2c^3+48Aa^2c^2d+42Ba^2c^2d+42Aa^2cd^2+36Ba^2cd^2+12Aa^2d^3+11Ba^2d^3\right)x$$
$$-\frac{\left(3Ba^2cd^2+Aa^2d^3+2Ba^2d^3\right)\cos(5fx+5e)}{80f}$$
$$+\frac{\left(4Ba^2c^3+12Aa^2c^2d+24Ba^2c^2d+24Aa^2cd^2+27Ba^2cd^2+9Aa^2d^3+10Ba^2d^3\right)\cos(3fx+3e)}{48f}$$
$$-\frac{\left(16Aa^2c^3+14Ba^2c^3+42Aa^2c^2d+36Ba^2c^2d+36Aa^2cd^2+33Ba^2cd^2+11Aa^2d^3+10Ba^2d^3\right)\cos(}{8f}$$
$$+\frac{\left(6Ba^2c^2d+6Aa^2cd^2+12Ba^2cd^2+4Aa^2d^3+5Ba^2d^3\right)\sin(4fx+4e)}{64f}$$
$$-\frac{\left(16Aa^2c^3+32Ba^2c^3+96Aa^2c^2d+96Ba^2c^2d+96Aa^2cd^2+96Ba^2cd^2+32Aa^2d^3+31Ba^2d^3\right)\sin(}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 +
48*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a
^2*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*
cos(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d +
 24*A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x +
 3*e)/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2
*d + 36*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*
x + e)/f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*
d^3 + 5*B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3
+ 96*A*a^2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A
*a^2*d^3 + 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

3.251.9 Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 1291, normalized size of antiderivative = 2.78

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

output

```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d
^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))/(8*(3*A*a^2*c^3 +
 (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6
*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 + (21*B*a^2*c^2*d)/4)))*(24*A*c^3 + 12*A*
d^3 + 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^
2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c
^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^
2)/2 + (21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*(20*A*a^2*c^3 + 4*A*a^2*
d^3 + 14*B*a^2*c^3 + 24*A*a^2*c*d^2 + 42*A*a^2*c^2*d + 12*B*a^2*c*d^2 + 24
*B*a^2*c^2*d) - tan(e/2 + (f*x)/2)^11*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a
^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*
c*d^2)/2 + (21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^5*(2*A*a^2*c^3 + 7*A*a
^2*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^
2*d + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) - tan(e/2 + (f*x)/2)^7*(2*A*a^2
*c^3 + 7*A*a^2*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 +
 12*A*a^2*c^2*d + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) + tan(e/2 + (f*x)/2
)^3*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24 + (
87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2*d)
/4) - tan(e/2 + (f*x)/2)^9*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 +
 (187*B*a^2*d^3)/24 + (87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c...
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

3.252.1 Optimal result 2047
3.252.2 Mathematica [A] (verified) 2048
3.252.3 Rubi [A] (verified) 2048
3.252.4 Maple [A] (verified) 2052
3.252.5 Fricas [A] (verification not implemented) 2053
3.252.6 Sympy [B] (verification not implemented) 2053
3.252.7 Maxima [A] (verification not implemented) 2054
3.252.8 Giac [A] (verification not implemented) 2055
3.252.9 Mupad [B] (verification not implemented) 2056

3.252.1 Optimal result

Integrand size = 35, antiderivative size = 336

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)x \\
&\quad + \frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f} \\
&\quad + \frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\cos(e+fx)\sin(e+fx)}{120df} \\
&\quad + \frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f} \\
&\quad + \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f} \\
&\quad - \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}
\end{aligned}
$$

output

```
1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5
*A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*
d^4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*
c^2*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-
2*B*(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*
(c-3*d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a
^2*sin(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

3.252.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{a^2\cos(e+fx)\left(60(2B(4c^2+7cd+3d^2)+A(12c^2+16cd+7d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+f}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
-1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16
*c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^
2]*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2
- 8*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*
d^2*Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 4
80*A*c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] +
270*B*d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f
*x)] - 30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

3.252.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 25, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$
$$\downarrow 3455$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2(a(5Ad+B(c+3d))-a(2B(c-3d)-5Ad)\sin(e+fx))dx}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2(a(5Ad+B(c+3d))-a(2B(c-3d)-5Ad)\sin(e+fx))dx}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3447$$

$$\frac{\int (c+d\sin(e+fx))^2\left(-\left((2B(c-3d)-5Ad)\sin^2(e+fx)a^2\right)+(5Ad+B(c+3d))a^2+\left(a^2(5Ad+B(c+3d))-\right.\right.}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))^2\left(-\left((2B(c-3d)-5Ad)\sin(e+fx)^2a^2\right)+(5Ad+B(c+3d))a^2+\left(a^2(5Ad+B(c+3d))-\right.\right.}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3502$$

$$\frac{\int -(c+d\sin(e+fx))^2\left(d(2Bc-35Ad-30Bd)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx)a^2\right)dx}{4d} + \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e}{4df}$$
$$\frac{}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 25$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{\int (c+d\sin(e+fx))^2\left(d(2B(c-15d)-35Ad)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx)a\right.}{4d}$$
$$\frac{}{5d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3042$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{\int (c+d\sin(e+fx))^2\left(d(2B(c-15d)-35Ad)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx)a\right.}{4d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df} \Big|_{5d}$$

$$\downarrow 3232$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{\frac{1}{3}\int -((c+d\sin(e+fx))(a^2d(5Ad(19c+16d)-B(2c^2-70dc-72d^2))-a^2(5Ad(2c^2-16dc-21c}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 25$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{-\frac{1}{3}\int (c+d\sin(e+fx))(a^2d(5Ad(19c+16d)-B(2c^2-70dc-72d^2))-a^2(5Ad(2c^2-16dc-21d^2}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3042$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{-\frac{1}{3}\int (c+d\sin(e+fx))(a^2d(5Ad(19c+16d)-B(2c^2-70dc-72d^2))-a^2(5Ad(2c^2-16dc-21d^2}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\downarrow 3213$$

$$\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df} - \frac{\frac{1}{3}\left(-\frac{15}{2}a^2d^2x(12Ac^2+16Acd+7Ad^2+8Bc^2+14Bcd+6Bd^2)-\frac{a^2d\left(5Ad\left(2c^2-16cd-21d^2\right)-B\left(\right.\right.\right.}{}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/5*(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(d*f
) + ((a^2*(2*B*(c - 3*d) - 5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*
d*f) - (-1/3*(a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2))*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/f + ((-15*a^2*d^2*(12*A*c^2 + 8*B*c^2 + 16*A*c
*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/2 - (2*a^2*(5*A*d*(c^3 - 8*c^2*d - 2
0*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^2*d^2 + 40*c*d^3 + 18*d^4))*C
os[e + f*x])/f - (a^2*d*(5*A*d*(2*c^2 - 16*c*d - 21*d^2) - B*(4*c^3 - 20*c
^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/(4*d))/(5*
d)
```

3.252.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.252.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{\left(\left((-3B-3A)d^2-6dc(A+B)-\frac{3c^2(A+2B)}{2}\right)\sin(2fx+2e)+\left(\left(\frac{9B}{8}+A\right)d^2+c(A+2B)d+\frac{Bc^2}{2}\right)\cos(3fx+3e)+\frac{3((A+2B)d$
parts	$\frac{(Aa^2d^2+2Ba^2cd+2Ba^2d^2)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(2Aa^2c^2+2Aa^2cd+Ba^2c^2)\cos($
risch	$-\frac{7a^2\cos(fx+e)Acd}{2f}-\frac{3a^2\cos(fx+e)cdB}{f}+\frac{\sin(4fx+4e)Ba^2cd}{16f}+\frac{a^2\cos(3fx+3e)Acd}{6f}+\frac{a^2\cos(3fx+3e)cdB}{3f}$
derivativedivides	$\frac{Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2Aa^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aa^2d^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+$
default	$\frac{Aa^2c^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2Aa^2cd\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Aa^2d^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+$
norman	$\frac{\left(\frac{3}{2}Aa^2c^2+2Aa^2cd+\frac{7}{8}Aa^2d^2+Ba^2c^2+\frac{7}{4}Ba^2cd+\frac{3}{4}Ba^2d^2\right)x+\left(15Aa^2c^2+20Aa^2cd+\frac{35}{4}Aa^2d^2+10Ba^2c^2+\frac{35}{2}Ba^2cd\right.$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/6*(((-3*B-3*A)*d^2-6*d*c*(A+B)-3/2*c^2*(A+2*B))*sin(2*f*x+2*e)+((9/8*B+A
)*d^2+c*(A+2*B)*d+1/2*B*c^2)*cos(3*f*x+3*e)+3/16*((A+2*B)*d+2*B*c)*d*sin(4
*f*x+4*e)-3/40*B*d^2*cos(5*f*x+5*e)+((-9*A-33/4*B)*d^2-21*(A+6/7*B)*c*d-12
*c^2*(A+7/8*B))*cos(f*x+e)+(-36/5*B+21/4*f*x*A+9/2*f*x*B-8*A)*d^2+12*c*(f*
x*A+7/8*f*x*B-5/3*A-4/3*B)*d+9*c^2*(f*x*A+2/3*f*x*B-4/3*A-10/9*B))*a^2/f
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.73

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx =$$
$$-\frac{24\,Ba^2d^2\cos(fx+e)^5 - 40\,(Ba^2c^2 + 2\,(A+2\,B)a^2cd + (2\,A+3\,B)a^2d^2)\cos(fx+e)^3 - 15\,(4\,(3\,A+$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d
+ (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A
 + 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A +
 B)*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*
B)*a^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d
+ (9*A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

3.252.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(330) = 660$.

Time = 0.35 (sec) , antiderivative size = 1129, normalized size of antiderivative = 3.36

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2
/2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**
2*c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*co
s(e + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d
*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a*
*2*c*d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*
A*a**2*d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a*
*2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2
*cos(e + f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**
2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*
f) + B*a**2*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**
2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f
*x)/f - 2*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f +
 3*B*a**2*c*d*x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e +
 f*x)**2/2 + B*a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4
/4 + B*a**2*c*d*x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f
*x)/(4*f) - 4*B*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin
(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f -
 8*B*a**2*c*d*cos(e + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4...
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.42

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx$$
$$= \frac{120\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2c^2 + 480\,(fx+e)Aa^2c^2 + 160\left(\cos(fx+e)^3 - 3\,\cos(fx+e)\right)Ba^2c}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^
2*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e
 - sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^2*c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*
c*d + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*
cos(f*x + e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 9
60*A*a^2*c^2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos
(f*x + e))/f
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.91

$$
\begin{aligned}
&\int (a + a\sin(e+fx))^2 (A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx \\
&\quad = -\frac{Ba^2d^2\cos(5fx+5e)}{80f} \\
&\qquad + \frac{1}{8}\left(12Aa^2c^2 + 8Ba^2c^2 + 16Aa^2cd + 14Ba^2cd + 7Aa^2d^2 + 6Ba^2d^2\right)x \\
&\qquad + \frac{(4Ba^2c^2 + 8Aa^2cd + 16Ba^2cd + 8Aa^2d^2 + 9Ba^2d^2)\cos(3fx+3e)}{48f} \\
&\qquad - \frac{(16Aa^2c^2 + 14Ba^2c^2 + 28Aa^2cd + 24Ba^2cd + 12Aa^2d^2 + 11Ba^2d^2)\cos(fx+e)}{8f} \\
&\qquad + \frac{(2Ba^2cd + Aa^2d^2 + 2Ba^2d^2)\sin(4fx+4e)}{32f} \\
&\qquad - \frac{(Aa^2c^2 + 2Ba^2c^2 + 4Aa^2cd + 4Ba^2cd + 2Aa^2d^2 + 2Ba^2d^2)\sin(2fx+2e)}{4f}
\end{aligned}
$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*
A*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^
2 + 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*
e)/f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12
*A*a^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2
+ 2*B*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2
*c*d + 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

3.252.9 Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.28

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$= \frac{a^2\operatorname{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(12Ac^2+7Ad^2+8Bc^2+6Bd^2+16Acd+14Bcd\right)}{4\left(3Aa^2c^2+\frac{7Aa^2d^2}{4}+2Ba^2c^2+\frac{3Ba^2d^2}{2}+4Aa^2cd+\frac{7Ba^2cd}{2}\right)}\right)\left(12Ac^2+7Ad^2+8Bc^2+6Bd^2+16Ac\right.}{4f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(4Aa^2c^2+2Ba^2c^2+4Aa^2cd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aa^2c^2+\frac{7Aa^2d^2}{4}+2Ba^2c^2+\frac{3Ba^2d^2}{2}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

output

```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2
+ 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 +
(3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8
*B*c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*
A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (
7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*
d)/2) - tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 +
(3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2
*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d +
11*B*a^2*c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B
*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^
6*(16*A*a^2*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d
+ 16*B*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 +
(44*B*a^2*c^2)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + t
an(e/2 + (f*x)/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 +
20*B*a^2*d^2 + (128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A
*a^2*d^2)/3 + (10*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16
*B*a^2*c*d)/3)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*t
an(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

3.253.1 Optimal result 2058
3.253.2 Mathematica [A] (verified) 2059
3.253.3 Rubi [A] (verified) 2059
3.253.4 Maple [A] (verified) 2062
3.253.5 Fricas [A] (verification not implemented) 2062
3.253.6 Sympy [B] (verification not implemented) 2063
3.253.7 Maxima [A] (verification not implemented) 2064
3.253.8 Giac [A] (verification not implemented) 2064
3.253.9 Mupad [B] (verification not implemented) 2065

3.253.1 Optimal result

Integrand size = 33, antiderivative size = 166

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx \\
&= \frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x - \frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)}{6f} \\
&\quad - \frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f} \\
&\quad - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}
\end{aligned}$$

output

```
1/8*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*x-1/6*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*co
s(f*x+e)/f-1/24*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/1
2*(4*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*d*cos(f*x+e)*(a+
a*sin(f*x+e))^3/a/f
```

3.253.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$= \frac{\cos(e+fx)\left(-\frac{1}{3}a^3(4Bc+4Ad-Bd)(1+\sin(e+fx))^2 - Bd(a+a\sin(e+fx))^3 - \frac{a^3(12Ac+8Bc+8Ad+7B}{}\right.}{4af}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
(Cos[e + f*x]*(-1/3*(a^3*(4*B*c + 4*A*d - B*d)*(1 + Sin[e + f*x])^2) - B*d
*(a + a*Sin[e + f*x])^3 - (a^3*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*(6*ArcSin[
Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(4 + Sin[e + f*x]))
)/(6*Sqrt[Cos[e + f*x]^2])))/(4*a*f)
```

3.253.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\downarrow 3447$$

$$\int (a\sin(e+fx)+a)^2\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (\sin(e+fx)a+a)^2(a(4Ac+3Bd)+a(4Bc+4Ad-Bd)\sin(e+fx))dx}{4a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^2(a(4Ac+3Bd)+a(4Bc+4Ad-Bd)\sin(e+fx))dx}{4a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\downarrow 3230$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\int (\sin(e+fx)a+a)^2dx - \frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}}{4a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\int (\sin(e+fx)a+a)^2dx - \frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}}{4a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\downarrow 3123$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\left(-\frac{2a^2\cos(e+fx)}{f} - \frac{a^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{3a^2x}{2}\right) - \frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+}{3f}}{4a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/4*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(a*f) + (-1/3*(a*(4*B*c + 4
*A*d - B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/f + (a*(12*A*c + 8*B*c +
8*A*d + 7*B*d)*((3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*S
in[e + f*x])/(2*f)))/3)/(4*a)
```

3.253.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.253.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{\left(((-6c-6d)B-3A(c+2d))\sin(2fx+2e)+(dA+B(c+2d))\cos(3fx+3e)+\frac{3dB\sin(4fx+4e)}{8}+\left((-21c-18d)B-24\left(c+\frac{7d}{8}\right)\right.\right.}{12f}$
parts	$-\frac{(A\,a^2d+B\,a^2c+2a^2dB)\,(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{(2a^2Ac+A\,a^2d+B\,a^2c)\cos(fx+e)}{f}+\frac{(a^2Ac+2A\,a^2d+2B\,a$
risch	$\frac{3A\,a^2cx}{2}+A\,a^2dx+B\,a^2cx+\frac{7B\,a^2dx}{8}-\frac{2a^2\cos(fx+e)Ac}{f}-\frac{7a^2\cos(fx+e)dA}{4f}-\frac{7a^2\cos(fx+e)Bc}{4f}-$
derivativedivides	$\frac{a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-\frac{B\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2dB\left(-\frac{\left(\sin^3\right.\right.}{}$
default	$\frac{a^2Ac\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-\frac{B\,a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2dB\left(-\frac{\left(\sin^3\right.\right.}{}$
norman	$\frac{(\frac{3}{2}a^2Ac+A\,a^2d+B\,a^2c+\frac{7}{8}a^2dB)x+(6a^2Ac+4A\,a^2d+4B\,a^2c+\frac{7}{2}a^2dB)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6a^2Ac+4A\,a^2d+4B\,a^2c+\frac{7}{2}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
1/12*(((-6*c-6*d)*B-3*A*(c+2*d))*sin(2*f*x+2*e)+(d*A+B*(c+2*d))*cos(3*f*x+
3*e)+3/8*d*B*sin(4*f*x+4*e)+((-21*c-18*d)*B-24*(c+7/8*d)*A)*cos(f*x+e)+(12
*c*f*x+21/2*d*f*x-20*c-16*d)*B+18*((2/3*f*x-10/9)*d+c*(f*x-4/3))*A)*a^2/f
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{8\,(Ba^2c+(A+2\,B)a^2d)\cos(fx+e)^3+3\,(4\,(3\,A+2\,B)a^2c+(8\,A+7\,B)a^2d)fx-48\,((A+B)a^2c+(A$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*
c + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x +
e) + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)
*cos(f*x + e))*sin(f*x + e))/f
```

3.253.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(163) = 326$.

Time $= 0.23$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 3.44$

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \begin{cases} \frac{Aa^2cx\sin^2(e+fx)}{2} + \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx - \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^2c\cos(e+fx)}{f} + Aa^2dx\sin^2(e+fx) \\ x(A + B\sin(e))(c + d\sin(e))(a\sin(e) + a)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e +
f*x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*
d*sin(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f -
2*A*a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*si
n(e + f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(
e + f*x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x
)**3/(3*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*
B*a**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/
2 + 3*B*a**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a*
*2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e
 + f*x)/f - 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))\,dx$$
$$= \frac{24\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2c + 96\,(fx+e)Aa^2c + 32\,(\cos(fx+e)^3 - 3\,\cos(fx+e))Ba^2c + 48}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 96*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))\,dx$$
$$= \frac{Ba^2d\sin(4\,fx + 4\,e)}{32\,f} + \frac{1}{8}\left(12\,Aa^2c + 8\,Ba^2c + 8\,Aa^2d + 7\,Ba^2d\right)x$$
$$+ \frac{(Ba^2c + Aa^2d + 2\,Ba^2d)\cos(3\,fx + 3\,e)}{12\,f}$$
$$- \frac{(8\,Aa^2c + 7\,Ba^2c + 7\,Aa^2d + 6\,Ba^2d)\cos(fx + e)}{4\,f}$$
$$- \frac{(Aa^2c + 2\,Ba^2c + 2\,Aa^2d + 2\,Ba^2d)\sin(2\,fx + 2\,e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d
+ 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f -
 1/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*
(A*a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

3.253.9 Mupad [B] (verification not implemented)

Time = 14.99 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.96

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{a^2\operatorname{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(12Ac+8Ad+8Bc+7Bd)}{4\left(3Aa^2c+2Aa^2d+2Ba^2c+\frac{7Ba^2d}{4}\right)}\right)(12Ac+8Ad+8Bc+7Bd)}{4f}$$
$$- \frac{a^2\left(\operatorname{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(12Ac+8Ad+8Bc+7Bd)}{4f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(Aa^2c+2Aa^2d+2Ba^2c+\frac{15Ba^2d}{4}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(Aa^2c+2Aa^2d+2Ba^2c+\frac{7}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

output
```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*
A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c
 + 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A
*d + 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d +
2*B*a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d +
2*B*a^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2
*B*a^2*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d
 + 10*B*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*
d)/3 + (34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c
+ 2*A*a^2*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a
^2*c + (7*B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B
*a^2*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2
 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

3.254.1 Optimal result . . . 2066
3.254.2 Mathematica [A] (verified) . . . 2066
3.254.3 Rubi [A] (verified) . . . 2067
3.254.4 Maple [A] (verified) . . . 2068
3.254.5 Fricas [A] (verification not implemented) . . . 2069
3.254.6 Sympy [B] (verification not implemented) . . . 2069
3.254.7 Maxima [A] (verification not implemented) . . . 2070
3.254.8 Giac [A] (verification not implemented) . . . 2070
3.254.9 Mupad [B] (verification not implemented) . . . 2071

3.254.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\begin{aligned}&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx\\&=\frac{1}{2}a^2(3A+2B)x-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}\\&\quad-\frac{a^2(3A+2B)\cos(e+fx)\sin(e+fx)}{6f}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}\end{aligned}$$

output
```
1/2*a^2*(3*A+2*B)*x-2/3*a^2*(3*A+2*B)*cos(f*x+e)/f-1/6*a^2*(3*A+2*B)*cos(f
*x+e)*sin(f*x+e)/f-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

3.254.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx=$$
$$-\frac{a^2\cos(e+fx)\left(6(3A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(2(6A+5B)+3(A+2B)\sin(e+\right.}{6f\sqrt{\cos^2(e+fx)}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-1/6*(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2
]] + Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*
Sin[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

3.254.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{1}{3}(3A+2B)\int (\sin(e+fx)a+a)^2dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A+2B)\int (\sin(e+fx)a+a)^2dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

$$\downarrow 3123$$

$$\frac{1}{3}(3A+2B)\left(-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/f + ((3*A + 2*B)*((3*a^2*x)/2
 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)))/3
```

3.254.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.254.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^2(3(-A-2B)\sin(2fx+2e)+\cos(3fx+3e)B+3(-8A-7B)\cos(fx+e)+18fxA+12fxB-24A-20B)}{12f}$
parts	$a^2xA+\frac{(Aa^2+2Ba^2)\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(2Aa^2+Ba^2)\cos(fx+e)}{f}-\frac{Ba^2(2+\sin^2(fx+e))\cos($
risch	$\frac{3a^2xA}{2}+a^2xB-\frac{2a^2\cos(fx+e)A}{f}-\frac{7a^2\cos(fx+e)B}{4f}+\frac{Ba^2\cos(3fx+3e)}{12f}-\frac{\sin(2fx+2e)Aa^2}{4f}-\frac{\sin(2fx+$
derivativedivides	$\frac{Aa^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2Aa^2\cos(fx+e)+2Ba^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{f}$
default	$\frac{Aa^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Ba^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2Aa^2\cos(fx+e)+2Ba^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\right.}{f}$
norman	$\frac{(\frac{3}{2}Aa^2+Ba^2)x+(\frac{3}{2}Aa^2+Ba^2)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}Aa^2+3Ba^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(\frac{9}{2}Aa^2+3Ba^2)x\left(\tan^4\left(\frac{fx}{2}+\right.\right.}{\left(1\right.}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*(3*(-A-2*B)*sin(2*f*x+2*e)+cos(3*f*x+3*e)*B+3*(-8*A-7*B)*cos(f*x+
e)+18*f*x*A+12*f*x*B-24*A-20*B)/f
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \frac{2\,Ba^2\cos(fx+e)^3 + 3\,(3\,A+2\,B)a^2fx - 3\,(A+2\,B)a^2\cos(fx+e)\sin(fx+e) - 12\,(A+B)a^2\cos(fx}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(
f*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

3.254.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \begin{cases} \frac{Aa^2x\sin^2(e+fx)}{2} + \frac{Aa^2x\cos^2(e+fx)}{2} + Aa^2x - \frac{Aa^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^2\cos(e+fx)}{f} + Ba^2x\sin^2(e+fx) + B \\ x(A + B\sin(e))\,(a\sin(e) + a)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**
2*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B
*a**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**
2*cos(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f
*x)**3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(
e) + a)**2, True))
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \frac{3\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2 + 12\,(fx+e)Aa^2 + 4\,(\cos(fx+e)^3 - 3\,\cos(fx+e))Ba^2 + 6\,(2\,fx}{12\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \frac{Ba^2\cos(3\,fx + 3\,e)}{12\,f} + \frac{1}{2}\left(3\,Aa^2 + 2\,Ba^2\right)x$$
$$- \frac{(8\,Aa^2 + 7\,Ba^2)\cos(fx+e)}{4\,f} - \frac{(Aa^2 + 2\,Ba^2)\sin(2\,fx + 2\,e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
 7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

3.254.9 Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx =$$

$$-\frac{\frac{3Aa^2\sin(2e+2fx)}{2} - \frac{Ba^2\cos(3e+3fx)}{2} + 3Ba^2\sin(2e+2fx) + 12Aa^2\cos(e+fx) + \frac{21Ba^2\cos(e+fx)}{2} - 9}{6f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

output
```
-((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(
2*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2
*f*x - 6*B*a^2*f*x)/(6*f)
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

3.255.1 Optimal result . . . 2072
3.255.2 Mathematica [A] (verified) . . . 2073
3.255.3 Rubi [A] (verified) . . . 2073
3.255.4 Maple [A] (verified) . . . 2077
3.255.5 Fricas [A] (verification not implemented) . . . 2078
3.255.6 Sympy [F(-1)] . . . 2078
3.255.7 Maxima [F(-2)] . . . 2079
3.255.8 Giac [A] (verification not implemented) . . . 2079
3.255.9 Mupad [B] (verification not implemented) . . . 2080

3.255.1 Optimal result

Integrand size = 35, antiderivative size = 171

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx \\
&= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} \\
&\quad -\frac{2a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}f} \\
&\quad +\frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f}-\frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df}
\end{aligned}$$

output

```
-1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3+1/2*a^2*(-2*A*d+2*B*c
-3*B*d)*cos(f*x+e)/d^2/f-1/2*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/f-2*a^2*(
c-d)^2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(
c^2-d^2)^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{a^2(1+\sin(e+fx))^2\left(2(2Ad(-c+2d)+B(2c^2-4cd+3d^2))(e+fx)-\frac{8(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+f\right.}{\sqrt{c^2-d^2}}\right.}{\sqrt{c^2-d^2}}}{4d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2)
)*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] -
 B*d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
)
```

3.255.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3455$$

$$\frac{\int \frac{(\sin(e+fx)a+a)(a(Bc+2Ad)-a(2Bc-2Ad-3Bd)\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)(a(Bc+2Ad)-a(2Bc-2Ad-3Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-((2Bc-2Ad-3Bd)\sin^2(e+fx)a^2)+(Bc+2Ad)a^2+(a^2(Bc+2Ad)-a^2(2Bc-2Ad-3Bd))\sin(e+fx)}{c+d\sin(e+fx)}dx}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-((2Bc-2Ad-3Bd)\sin(e+fx)^2a^2)+(Bc+2Ad)a^2+(a^2(Bc+2Ad)-a^2(2Bc-2Ad-3Bd))\sin(e+fx)}{c+d\sin(e+fx)}dx}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3502$$

$$\frac{\frac{\int \frac{a^2d(Bc+2Ad)-a^2\left(2A(c-2d)d-B\left(2c^2-4dc+3d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} + \frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{a^2d(Bc+2Ad)-a^2\left(2A(c-2d)d-B\left(2c^2-4dc+3d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} + \frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3214$$

$$\frac{-\frac{2a^2(c-d)^2(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d} + \frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3042$$

$$\frac{-\frac{2a^2(c-d)^2(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d} + \frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d} - \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$$\downarrow 3139$$

$$\frac{-\frac{4a^2(c-d)^2(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}-$$

$$\frac{2d}{\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}}$$

$$\downarrow 1083$$

$$\frac{-\frac{8a^2(c-d)^2(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos}{df}$$

$$\frac{2d}{\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}}$$

$$\downarrow 217$$

$$\frac{-\frac{4a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}-$$

$$\frac{2d}{\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
((-((a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/d) - (4*a^2*(c -
 d)^2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])]
)/(d*Sqrt[c^2 - d^2]*f))/d + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(d
*f))/(2*d) - (B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

3.255.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.255.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{2a^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-cdB+2d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-cdB+2d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2Acd-4A\,d^2-2B\,c\right.}{d^3}\right.}{f}$
default	$\frac{2a^2\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-cdB+2d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-cdB+2d^2B}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{\left(2Acd-4A\,d^2-2B\,c\right.}{d^3}\right.}{f}$
risch	$-\frac{x\,a^2Ac}{d^2}+\frac{2x\,a^2A}{d}+\frac{x\,a^2Bc^2}{d^3}-\frac{2x\,a^2cB}{d^2}+\frac{3x\,a^2B}{2d}-\frac{a^2e^{i(fx+e)}A}{2df}+\frac{a^2e^{i(fx+e)}Bc}{2d^2f}-\frac{a^2e^{i(fx+e)}B}{df}-\frac{a^2}{}$

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
2/f*a^2*(-1/d^3*((-1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(A*d^2-B*c*d+2*B*d^2)*ta
n(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)*d^2+A*d^2-c*d*B+2*d^2*B)/(1+ta
n(1/2*f*x+1/2*e)^2)^2+1/2*(2*A*c*d-4*A*d^2-2*B*c^2+4*B*c*d-3*B*d^2)*arctan
(tan(1/2*f*x+1/2*e)))+(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)/d^
3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.64

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e) - (2\,Ba^2c^2 - 2\,(A+2\,B)a^2cd + (4\,A+3\,B)a^2d^2)fx + (Ba^2c^2 - (A+$$

$$-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e) - (2\,Ba^2c^2 - 2\,(A+2\,B)a^2cd + (4\,A+3\,B)a^2d^2)fx - 2\,(Ba^2c^2 - (A+}{2\,d^3f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f), -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f)]
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.255.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.78

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{(2Ba^2c^2-2Aa^2cd-4Ba^2cd+4Aa^2d^2+3Ba^2d^2)(fx+e)}{d^3} - \frac{4\,(Ba^2c^3-Aa^2c^2d-2Ba^2c^2d+2Aa^2cd^2+Ba^2cd^2-Aa^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\right.}{\sqrt{c^2-d^2}d^3}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)
*(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^
2 + B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3)
+ 2*(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2
*A*a^2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2
*d*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x
 + 1/2*e)^2 + 1)^2*d^2))/f
```

3.255.9 Mupad [B] (verification not implemented)

Time = 20.39 (sec) , antiderivative size = 7371, normalized size of antiderivative = 43.11

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)
```

output
```
(atan(((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 +
 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*
c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32
*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 +
 (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*
B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^
2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*t
an(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*
B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i +
(a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e
/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^
7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^
4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 +
28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d
^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B
*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A
*c + 4*B*c)*1i)/2)*1i)/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5
 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4
*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 4
4*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8...
```

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

3.256.1 Optimal result 2081
3.256.2 Mathematica [A] (verified) 2082
3.256.3 Rubi [A] (verified) 2082
3.256.4 Maple [A] (verified) 2086
3.256.5 Fricas [A] (verification not implemented) 2087
3.256.6 Sympy [F(-1)] 2088
3.256.7 Maxima [F(-2)] 2088
3.256.8 Giac [B] (verification not implemented) 2089
3.256.9 Mupad [B] (verification not implemented) 2090

3.256.1 Optimal result

Integrand size = 35, antiderivative size = 198

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} \\
&\quad - \frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}f} \\
&\quad + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-a^2*(-A*d+2*B*c-2*B*d)*x/d^3+a^2*(A*d-B*(2*c+d))*cos(f*x+e)/d^2/(c+d)/f+(
-A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))-2*a^2
*(c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a^2(1+\sin(e+fx))^2\left((-2Bc+Ad+2Bd)(e+fx)+\frac{2(c-d)(-Ad(c+2d)+B(2c^2+2cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right.}{d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(
-(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2]
)/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c
+ d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])))/(d^3*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

3.256.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3454, 25, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3447$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(Ad-B(2c+d))\sin^2(e+fx)a^2+(B(c-d)-2Ad)a^2+\left((B(c-d)-2Ad)a^2+(Ad-B(2c+d))a^2\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(Ad-B(2c+d))\sin(e+fx)^2a^2+(B(c-d)-2Ad)a^2+\left((B(c-d)-2Ad)a^2+(Ad-B(2c+d))a^2\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3502$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{d(B(c-d)-2Ad)a^2+(c+d)(2B(c-d)-Ad)\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{d(B(c-d)-2Ad)a^2+(c+d)(2B(c-d)-Ad)\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$$\downarrow 3214$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{\frac{a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d} - \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{\frac{a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d} - \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$$\downarrow 3139$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d} - \frac{a^2(Ad-B(2c+d))\cos(e}{df}}{d(c+d)}$$

$$\downarrow 1083$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d} - \frac{4a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d} - \frac{a^2(Ad-B(2c+d}{df}}{d(c+d)}$$

$$\downarrow 217$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d} - \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-((((a^2*(c + d)*(2*B*c - A*d - 2*B*d)*x)/d + (2*a^2*(c - d)*(A*d*(c + 2*d
) - B*(2*c^2 + 2*c*d - d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c
^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a^2*(A*d - B*(2*c + d))*Cos[e + f
*x])/(d*f))/(d*(c + d))) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*
x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

3.256.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.256.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{2a^2\left(-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-2Bc+2dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-Ad^2-Bc^2+cdB\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-Ad^2-Bc\right.}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}}{f}$
default	$\frac{2a^2\left(-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-2Bc+2dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-Ad^2-Bc^2+cdB\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-Ad^2-Bc\right.}{c+d}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}}{f}$
risch	$\frac{xa^2A}{d^2}-\frac{2xa^2Bc}{d^3}+\frac{2xa^2B}{d^2}-\frac{Ba^2e^{i(fx+e)}}{2d^2f}-\frac{Ba^2e^{-i(fx+e)}}{2d^2f}+\frac{2ia^2\left(-Acd+Ad^2+Bc^2-cdB\right)\left(id+ce^{i(fx+e)}\right)}{d^3(c+d)f\left(-ie^{2i(fx+e)}d+id+2ce^{i(fx+e)}\right)}$

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2*(1/d^3*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-2*B*c+2*B*d)*arctan(tan
(1/2*f*x+1/2*e)))-1/d^3*((-d^2*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d)/c*tan(1/2*f
*x+1/2*e)-d*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*t
an(1/2*f*x+1/2*e)+c)+(A*c^2*d+A*c*d^2-2*A*d^3-2*B*c^3+3*B*c*d^2-B*d^3)/(c+
d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.69

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \left[-\frac{2\,(2\,Ba^2c^3 - Aa^2c^2d - (A+2\,B)a^2cd^2)fx + (2\,Ba^2c^3 - (A-2\,B)a^2c^2d - (2\,A+B)a^2cd^2 + (2\,Ba^2c^2}{} \right.$$

$$\left. -\frac{(2\,Ba^2c^3 - Aa^2c^2d - (A+2\,B)a^2cd^2)fx + (2\,Ba^2c^3 - (A-2\,B)a^2c^2d - (2\,A+B)a^2cd^2 + (2\,Ba^2c^2d}{} \right.$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*
c^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*
B)*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log
(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2
+ c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)
/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B
*a^2*c^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A
*a^2*c*d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -(
(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A
- 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*
d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*si
n(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d
^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.256.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(194) = 388.

Time = 0.30 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.42

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(2\,Ba^2c^3-Aa^2c^2d-Aa^2cd^2-3\,Ba^2cd^2+2\,Aa^2d^3+Ba^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^3+d^4)\sqrt{c^2-d^2}} - \frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
(2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3
+ B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*
f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B
*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B
*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B
*a^2*c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a
^2*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a
^2*c*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d
^3*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan
(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e
)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^
2*d - 2*B*a^2*d)*(f*x + e)/d^3)/f
```

3.256.9 Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 8706, normalized size of antiderivative = 43.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x
)
```

output
```
- ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 +
(f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e
/2 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c +
 d)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*
c*d))/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/
2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^
2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A
^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2
 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c
^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((
((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan
(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)
)/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*
(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3
*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*
(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A
*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a
^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A
^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4
 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*...
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

3.257.1 Optimal result 2091
3.257.2 Mathematica [A] (verified) 2092
3.257.3 Rubi [A] (verified) 2092
3.257.4 Maple [B] (verified) 2096
3.257.5 Fricas [B] (verification not implemented) 2097
3.257.6 Sympy [F(-1)] 2098
3.257.7 Maxima [F(-2)] 2099
3.257.8 Giac [B] (verification not implemented) 2099
3.257.9 Mupad [B] (verification not implemented) 2100

3.257.1 Optimal result

Integrand size = 35, antiderivative size = 215

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx \\
&= \frac{a^2Bx}{d^3} + \frac{a^2(3Ad^3 - B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}f} \\
&\quad + \frac{(Bc-Ad)\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2d(c+d)f(c+d\sin(e+fx))^2} \\
&\quad - \frac{a^2(3Ad^2 - B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

output

```
a^2*B*x/d^3+1/2*(-A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*
sin(f*x+e))^2-1/2*a^2*(3*A*d^2-B*(2*c^2+3*c*d-2*d^2))*cos(f*x+e)/d^2/(c+d)
^2/f/(c+d*sin(f*x+e))+a^2*(3*A*d^3-B*(2*c^3+4*c^2*d+c*d^2-4*d^3))*arctan((
d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{a^2(1+\sin(e+fx))^2\left(2B(e+fx) - \frac{2\left(-3Ad^3+B\left(2c^3+4c^2d+cd^2-4d^3\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} - \frac{d(-c+d)(-Bc+Ad)\cos}{(c+d)(c+d\sin(e+}\right.}{2d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2
*d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c
 + d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c +
d)*(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2
))*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4)
```

3.257.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3454, 25, 3042, 3447, 3042, 3500, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)(a(Bc-3Ad-2Bd)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int\frac{(\sin(e+fx)a+a)(a(B(c-2d)-3Ad)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int\frac{(\sin(e+fx)a+a)(a(B(c-2d)-3Ad)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3447$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int\frac{-2B(c+d)\sin^2(e+fx)a^2+(B(c-2d)-3Ad)a^2+(a^2(B(c-2d)-3Ad)-2a^2B(c+d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int\frac{-2B(c+d)\sin(e+fx)^2a^2+(B(c-2d)-3Ad)a^2+(a^2(B(c-2d)-3Ad)-2a^2B(c+d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3500$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(c-d)d(3Ad+B(c+4d))a^2+2B(c-d)(c+d)^2\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(c-d)d(3Ad+B(c+4d))a^2+2B(c-d)(c+d)^2\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 3214$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2Bx(c-d)(c+d)^2}{d} - \frac{a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2Bx(c-d)(c+d)^2}{d} - \frac{a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 3139$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2Bx(c-d)(c+d)^2}{d} - \frac{2a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan}{df}}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 1083$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{4a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d(c^2-d^2)}}{2d(c+d)}$$

$$\downarrow 217$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2Bx(c-d)(c+d)^2}{d} - \frac{2a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}}{d(c^2-d^2)}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

3.257. $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

output
```
((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*
Sin[e + f*x])^2) - (-(((2*a^2*B*(c - d)*(c + d)^2*x)/d - (2*a^2*(c - d)*(2
*B*c*(c + d)^2 - d^2*(3*A*d + B*(c + 4*d)))*ArcTan[(2*d + 2*c*Tan[(e + f*x
)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/(d*(c^2 - d^2))) + (a^2
*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f*x])/(d*(c + d)*f*(c + d*S
in[e + f*x])))/(2*d*(c + d))
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

3.257.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(206) = 412$.

Time = 1.02 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.00

method	result
derivativedivides	$2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(c^2dA-4d^2cA-2Ad^3+Bc^3+4c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4Ac^3d^2+Ac^2d^3+8Acd^4+2Ad^5-2B\right.}{}$
default	$2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(c^2dA-4d^2cA-2Ad^3+Bc^3+4c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4Ac^3d^2+Ac^2d^3+8Acd^4+2Ad^5-2B\right.}{}$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2*(B/d^3*arctan(tan(1/2*f*x+1/2*e))+1/d^3*((1/2*d^2*(A*c^2*d-4*A*c*d
^2-2*A*d^3+B*c^3+4*B*c^2*d)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(
4*A*c^3*d^2+A*c^2*d^3+8*A*c*d^4+2*A*d^5-2*B*c^5-4*B*c^4*d-3*B*c^3*d^2-8*B*
c^2*d^3+2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(A*c^2
*d+12*A*c*d^2+2*A*d^3-7*B*c^3-12*B*c^2*d+4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(
1/2*f*x+1/2*e)-1/2*d*(4*A*c*d^2+A*d^3-2*B*c^3-4*B*c^2*d+B*c*d^2)/(c^2+2*c*
d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(3*A*d^3-2
*B*c^3-4*B*c^2*d-B*c*d^2+4*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1
/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(206) = 412$.

Time $= 0.33$ (sec) , antiderivative size $= 1483$, normalized size of antiderivative $= 6.90$

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
[1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*
cos(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*
d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*
a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a
^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x
+ e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*
a^2*c*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*co
s(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2)) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3
- (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) -
2*(4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x +
 (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A -
 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d
^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5
- 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2
*d^7 - 2*c*d^8 - d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^
2*c*d^5 - B*a^2*d^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B
*a^2*c^4*d^2 - B*a^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c
^5 + 4*B*a^2*c^4*d + 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - ...
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(206) = 412$.

Time $= 0.33$ (sec) , antiderivative size $= 678$, normalized size of antiderivative $= 3.15$

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{(fx+e)Ba^2}{d^3} - \frac{(2\,Ba^2c^3+4\,Ba^2c^2d+Ba^2cd^2-3\,Aa^2d^3-4\,Ba^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3+2\,cd^4+d^5)\sqrt{c^2-d^2}} + \frac{Ba^2c^4d\tan(\frac{1}{2}\,fx+}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^
2*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*
tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqr
t(c^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/
2*f*x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^5*tan(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^
2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2
- A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2
*e)^2 - 8*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x +
 1/2*e)^2 - 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x
 + 1/2*e) - A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*
f*x + 1/2*e) - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B
*a^2*c^4*d - 4*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 +
2*c^3*d^3 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)
+ c)^2))/f
```

3.257.9 Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 8632, normalized size of antiderivative = 40.15

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x
)
```

output

```
(2*B*a^2*atan(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2
*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6
*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^
7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*
a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*
B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8
))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2
 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*
a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*
B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7
+ c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A
*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a
^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((
8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*
d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c
*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9
 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3
)*1i)/d^3))/d^3 + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*
B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9
+ 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*...
```

3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

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3.258.1 Optimal result

Integrand size = 35, antiderivative size = 604

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+90c^2d+78cd^2+23d^3\right)\right)x \\
&\quad -\frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-9}{420d^3f} \\
&\quad -\frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4-}{1680d^2f} \\
&\quad -\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+fx)(c-}{840d^3f} \\
&\quad -\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{840d^3f} \\
&\quad -\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f} \\
&\quad -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df} \\
&\quad +\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f}
\end{aligned}
$$

output
```
1/16*a^3*(3*B*(10*c^3+26*c^2*d+23*c*d^2+7*d^3)+A*(40*c^3+90*c^2*d+78*c*d^2
+23*d^3))*x-1/420*a^3*(7*A*d*(2*c^5-18*c^4*d+107*c^3*d^2+472*c^2*d^3+456*c
*d^4+136*d^5)-3*B*(2*c^6-14*c^5*d+51*c^4*d^2-189*c^3*d^3-920*c^2*d^4-952*c
*d^5-288*d^6))*cos(f*x+e)/d^3/f-1/1680*a^3*(7*A*d*(4*c^4-36*c^3*d+216*c^2*
d^2+626*c*d^3+345*d^4)-3*B*(4*c^5-28*c^4*d+104*c^3*d^2-392*c^2*d^3-1263*c*
d^4-735*d^5))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/840*a^3*(7*A*d*(2*c^3-18*c^2*d
+111*c*d^2+136*d^3)-B*(6*c^4-42*c^3*d+165*c^2*d^2-651*c*d^3-864*d^4))*cos(
f*x+e)*(c+d*sin(f*x+e))^2/d^3/f-1/840*a^3*(7*A*d*(2*c^2-18*c*d+115*d^2)-B*
(6*c^3-42*c^2*d+177*c*d^2-735*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/
210*a^3*(-14*A*c*d+91*A*d^2+6*B*c^2-27*B*c*d+87*B*d^2)*cos(f*x+e)*(c+d*sin
(f*x+e))^4/d^3/f-1/7*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4/
d/f+1/42*(3*B*(c-3*d)-7*A*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+
e))^4/d^2/f
```

3.258.2 Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.87

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx =$$

$$-\frac{a^3\cos(e+fx)\left(420(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))\arcsin\left(\frac{\sqrt{1-s}\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^3,x]
```

output

```
-1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3)
 + A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]
]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2
*d + 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*
d^3 - (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*
c^2*d + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) +
B*(14*c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)]
 + 5040*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e
 + f*x] + 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*
B*c*d^2*Sin[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] -
 210*B*c^3*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*
Sin[3*(e + f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e +
 f*x)] - 980*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*
c*d^2*Sin[5*(e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e +
f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

3.258.3 Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3455, 3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$

$$\downarrow 3455$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3(a(7Ad+2B(c+2d))-a(3B(c-3d)-7Ad)\sin(e+fx))dx}{7d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3(a(7Ad+2B(c+2d))-a(3B(c-3d)-7Ad)\sin(e+fx))dx}{7d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3455$$

$$\frac{\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3\left((7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right))a^2+\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)a^2\right)dx}{6d} + \frac{(3B(c-}{7d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3\left((7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right))a^2+\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)a^2\right)dx}{6d} + \frac{(3B(c-}{7d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}}$$

$$\downarrow 3447$$

$$\frac{\frac{\int (c+d\sin(e+fx))^3\left(\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin^2(e+fx)a^3+(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right))a^3+\left(\left(6Bc^2-14Adc-27Bdc+91A\right.\right.\right.}{6d}}{7d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (c+d\sin(e+fx))^3\left(\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)^2a^3+(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right))a^3+\left(\left(6Bc^2-14Adc-27Bdc+91A\right.\right.\right.}{6d}}{7d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}}$$

$$\downarrow 3502$$

$$\frac{\frac{\frac{\int -(c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d} - \frac{a^3\left(-14Acd+91A\right.}{6d}}{7d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}}$$

$$\downarrow 25$$

$$-\frac{\frac{\int (c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d}-\frac{a^3\left(-14Acd+91A\right.}{6d}}{7d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3042$$

$$-\frac{\frac{\int (c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d}-\frac{a^3\left(-14Acd+91A\right.}{6d}}{7d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3232$$

$$-\frac{\frac{1}{4}\int 3(c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-651\right.\right.}{5d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 27$$

$$-\frac{\frac{3}{4}\int (c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-651d\right.\right.}{5d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3042$$

$$-\frac{\frac{3}{4}\int (c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-651d\right.\right.}{5d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3232$$

$$-\frac{\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(7Ad\left(2c^3-318dc^2-567d^2c-272d^3\right)-3B\left(2c^4-14dc^3+577d^2c^2+1169d^3c+576d^4\right)\right)-a^3\left(7Ad\left(4c^4-36dc^3+216d^2c^2+626d^3c+345d^4\right.\right.\right.}{\qquad}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3042$$

$$-\frac{\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(7Ad\left(2c^3-318dc^2-567d^2c-272d^3\right)-3B\left(2c^4-14dc^3+577d^2c^2+1169d^3c+576d^4\right)\right)-a^3\left(7Ad\left(4c^4-36dc^3+216d^2c^2+626d^3c+345d^4\right.\right.\right.}{\qquad}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\downarrow 3213$$

$$-\frac{a^3\left(-14Acd+91Ad^2+6Bc^2-27Bcd+87Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{a^3\left(7Ad\left(2c^2-18cd+115d^2\right)-B\left(6c^3-42c^2d+177cd^2-735d^3\right)\right)\cos(e+fx)(c+d\sin(e+}{4f}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/7*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(d*f
) + (((3*B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d
*Sin[e + f*x])^4)/(6*d*f) + (-1/5*(a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 91
*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(d*f) - ((a^3*(7*A
*d*(2*c^2 - 18*c*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3)
)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f) + (3*((a^3*(7*A*d*(2*c^3 - 18
*c^2*d + 111*c*d^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*
d^3 - 864*d^4))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((-105*a^3*d^
3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 78*
c*d^2 + 23*d^3))*x)/2 + (2*a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 18
9*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/f + (a^3*d*(
7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4*c^5
- 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e + f*
x]*Sin[e + f*x])/(2*f))/3))/4)/(5*d))/(6*d))/(7*d)
```

3.258. $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

3.258.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.258.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{\left(\left(\frac{\left(19A+\frac{81B}{4}\right)d^3}{4}+\frac{51c\left(A+\frac{19B}{17}\right)d^2}{4}+9\left(\frac{17B}{12}+A\right)c^2d+c^3(A+3B)\right)\cos(3fx+3e)+3\left(\frac{(-63A-61B)d^3}{16}-12c\left(A+\frac{63B}{64}\right)d^2\right.\right.$
parts	$\frac{(Aa^3d^3+3Ba^3d^2c+3Ba^3d^3)\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}-\frac{(3Aa^3c^3+3A$
risch	$-\frac{189\sin(2fx+2e)Ba^3d^2c}{64f}-\frac{15a^3\cos(fx+e)Ac^3}{4f}-\frac{21a^3\cos(fx+e)Ad^3}{8f}-\frac{13a^3\cos(fx+e)Bc^3}{4f}-\frac{155a^3\cos(f$
derivativedivides	Expression too large to display
default	Expression too large to display
norman	Expression too large to display

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output

```
1/12*((1/4*(19*A+81/4*B)*d^3+51/4*c*(A+19/17*B)*d^2+9*(17/12*B+A)*c^2*d+c^
3*(A+3*B))*cos(3*f*x+3*e)+3*(1/16*(-63*A-61*B)*d^3-12*c*(A+63/64*B)*d^2-12
*c^2*d*(A+B)-3*(A+4/3*B)*c^3)*sin(2*f*x+2*e)+3/8*(1/2*(11*B+9*A)*d^3+9*(3/
2*B+A)*c*d^2+3*c^2*(A+3*B)*d+B*c^3)*sin(4*f*x+4*e)-9/20*d*((A+19/12*B)*d^2
+c*(A+3*B)*d+B*c^2)*cos(5*f*x+5*e)-1/16*((A+3*B)*d+3*B*c)*d^2*sin(6*f*x+6*
e)+3/112*B*d^3*cos(7*f*x+7*e)+3*(1/2*(-155/8*B-21*A)*d^3-69/2*(21/23*B+A)*
c*d^2-39*c^2*(23/26*B+A)*d-15*c^3*(A+13/15*B))*cos(f*x+e)+(-864/35*B+69/4*
f*x*A+63/4*f*x*B-136/5*A)*d^3+117/2*c*(f*x*A+23/26*f*x*B-304/195*A-272/195
*B)*d^2+135/2*c^2*(f*x*A+13/15*f*x*B-8/5*A-304/225*B)*d+30*c^3*(f*x*A+3/4*
f*x*B-22/15*A-6/5*B))*a^3/f
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{240\,Ba^3d^3\cos(fx+e)^7 - 1008\,(Ba^3c^2d+(A+3\,B)a^3cd^2+(A+2\,B)a^3d^3)\cos(fx+e)^5 + 560\,((A+3$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c
*d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A
 + 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x
 + e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*
A + 23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 +
 3*(A + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e)
 - 35*(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c
^3 + 18*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^
3*d^3)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c
^2*d + 3*(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*si
n(f*x + e))/f
```

3.258.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. 2(598) = 1196.

Time = 0.72 (sec) , antiderivative size = 2878, normalized size of antiderivative = 4.76

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output

```
Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x
)**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*
a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/
(3*f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8
 + 9*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x
*sin(e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d
*x*cos(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f)
 - 9*A*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e
+ f*x)*cos(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(
2*f) - 6*A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f
+ 27*A*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d*
*2*x*cos(e + f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*
a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 8*A*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f
 + 5*A*a**3*d**3*x*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*c
os(e + f*x)**2/16 + 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*...
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1056, normalized size of antiderivative = 1.75

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output

```
1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2
*e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*
x + e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*
e) - 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(
2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*
cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3
*c*d^2 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*
x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*
x + 2*e - sin(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(
f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a^3*c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4
*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin
(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 -
 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3
*cos(f*x + e))*A*a^3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9...
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.93

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{Ba^3d^3\cos(7fx+7e)}{448f}$$
$$+\frac{1}{16}\left(40Aa^3c^3+30Ba^3c^3+90Aa^3c^2d+78Ba^3c^2d+78Aa^3cd^2+69Ba^3cd^2+23Aa^3d^3+21Ba^3d^3\right)x$$
$$-\frac{(12Ba^3c^2d+12Aa^3cd^2+36Ba^3cd^2+12Aa^3d^3+19Ba^3d^3)\cos(5fx+5e)}{320f}$$
$$+\frac{(16Aa^3c^3+48Ba^3c^3+144Aa^3c^2d+204Ba^3c^2d+204Aa^3cd^2+228Ba^3cd^2+76Aa^3d^3+81Ba^3d^3)}{192f}$$
$$-\frac{(240Aa^3c^3+208Ba^3c^3+624Aa^3c^2d+552Ba^3c^2d+552Aa^3cd^2+504Ba^3cd^2+168Aa^3d^3+155Ba}{64f}$$
$$-\frac{(3Ba^3cd^2+Aa^3d^3+3Ba^3d^3)\sin(6fx+6e)}{192f}$$
$$+\frac{(2Ba^3c^3+6Aa^3c^2d+18Ba^3c^2d+18Aa^3cd^2+27Ba^3cd^2+9Aa^3d^3+11Ba^3d^3)\sin(4fx+4e)}{64f}$$
$$-\frac{(48Aa^3c^3+64Ba^3c^3+192Aa^3c^2d+192Ba^3c^2d+192Aa^3cd^2+189Ba^3cd^2+63Aa^3d^3+61Ba^3d^3)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 9
0*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^
3*d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^
3*c*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^
3*c^3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2
 + 228*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/6
4*(240*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552
*A*a^3*c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x +
e)/f - 1/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f
+ 1/64*(2*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27
*B*a^3*c*d^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A
*a^3*c^3 + 64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*
d^2 + 189*B*a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

3.258.9 Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 1395, normalized size of antiderivative = 2.31

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

output

```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d
^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 +
 (23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)
/4 + (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A
*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d
^2 + 78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3
)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3
*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^
10*(40*A*a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*
c^2*d + 12*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*
c^3 + (23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c
*d^2)/4 + (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) +
tan(e/2 + (f*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (3
5*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 5
7*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 +
 17*B*a^3*c^3 + (35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*
B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3
 + (148*A*a^3*d^3)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246
*A*a^3*c^2*d + 148*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*(
(448*A*a^3*c^3)/3 + (328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 +...
```

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

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3.259.1 Optimal result

Integrand size = 35, antiderivative size = 463

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26d^2\right)\right)x \\
&\quad -\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4-136d^5}{60d^3f} \\
&\quad -\frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\cos(e+fx)\sin(}{240d^2f} \\
&\quad -\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f} \\
&\quad +\frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f} \\
&\quad -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} \\
&\quad +\frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f}
\end{aligned}$$

output
```
1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2
*A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3
*d^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*
c^3-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345
*d^4))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-
B*(2*c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1
/40*a^3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+
e))^3/d^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1
/30*(-6*A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^
3/d^2/f
```

3.259.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{a^3\cos(e+fx)\left(60(B(30c^2+52cd+23d^2)+A(40c^2+60cd+26d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
-1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 6
0*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x
]^2]*(1840*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 146
8*B*d^2 - 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*
Cos[2*(e + f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2
*Sin[e + f*x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*
c*d*Sin[e + f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*
B*c^2*Sin[3*(e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e +
f*x)] - 90*A*d^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*S
in[5*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

3.259.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 25, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$

$$\downarrow 3455$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2(a(2Bc+6Ad+3Bd)-a(3Bc-6Ad-8Bd)\sin(e+fx))dx}{6d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2(a(2Bc+6Ad+3Bd)-a(3Bc-6Ad-8Bd)\sin(e+fx))dx}{6d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\downarrow 3455$$

$$\frac{\frac{\int 3(\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d} + \frac{(-6Ad+3B}{6d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}}$$

$$\downarrow 27$$

$$\frac{\frac{3\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d} + \frac{(-6Ad+3B}{6d}}{\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}}$$

$$\downarrow 3042$$

$$\frac{3\int(\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d}+\frac{(-6Ad+3B}{}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 3447$$

$$\frac{3\int(c+d\sin(e+fx))^2\left(-\left(\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin^2(e+fx)a^3\right)+\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)a^3+\left(a^3\left(2Ad(c+8d)-B\left(c^2-3dc-\right.\right.\right.\right.}{5d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 3042$$

$$\frac{3\int(c+d\sin(e+fx))^2\left(-\left(\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)^2a^3\right)+\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)a^3+\left(a^3\left(2Ad(c+8d)-B\left(c^2-3dc-\right.\right.\right.\right.}{5d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 3502$$

$$3\left(\frac{\int-(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+76d^2\right)-B\left(2c^3-12dc^2+41d^2c-136d^3\right)\right)\sin(e+fx)\right)dx}{4d}+\frac{a^3\left(2Ad(2c-11\right.}{5d}\right)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 25$$

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+76\right.\right.\right.}{4d}\right)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 3042$$

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+76\right.\right.\right.}{4d}\right)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow 3232$$

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(2Ad\left(2c^2-165dc-152d^2\right)-B\left(2c^3-12dc^2+263d^2c+272d^3\right)\right)-\right.}{}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\downarrow 3042$$

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(2Ad\left(2c^2-165dc-152d^2\right)-B\left(2c^3-12dc^2+263d^2c+272d^3\right)\right)-\right.}{}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\downarrow 3213$$

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{a^3\left(2Ad\left(2c^2-15cd+76d^2\right)-B\left(2c^3-12c^2d+41cd^2-136d^3\right)\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3f}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/6*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3)/(d*f
) + (((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d
*Sin[e + f*x])^3)/(5*d*f) + (3*((a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*
d + 21*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f) - ((a^3*(2*A*d*(
2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2*d + 41*c*d^2 - 136*d^3))*Cos[
e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((-15*a^3*d^3*(B*(30*c^2 + 52*c*d
 + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/2 + (2*a^3*(2*A*d*(2*c^4 - 1
5*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5 - 12*c^4*d + 37*c^3*
d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x])/f + (a^3*d*(2*A*d*
(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*c^4 - 24*c^3*d + 76*c^2*d^
2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/(4*d)))/(5*
d))/(6*d)
```

3.259. $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

3.259.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.259.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^3\left(\left(\frac{(17A+19B)d^2}{4}+6\left(\frac{17B}{12}+A\right)cd+c^2(A+3B)\right)\cos(3fx+3e)+3\left(\left(-\frac{63B}{16}-4A\right)d^2-8dc(A+B)-3\left(A+\frac{4B}{3}\right)c^2\right)\sin(2f$
parts	$-\frac{(Aa^3d^2+2Ba^3cd+3Ba^3d^2)\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5f}-\frac{(3Aa^3c^2+2Aa^3cd+Ba^3c^2)\cos(fx+}{f}$
risch	$\frac{13Ba^3cdx}{4}-\frac{15a^3\cos(fx+e)Ac^2}{4f}-\frac{23a^3\cos(fx+e)Ad^2}{8f}-\frac{13a^3\cos(fx+e)Bc^2}{4f}-\frac{21a^3\cos(fx+e)d^2B}{8f}-\frac{Ba^3}{}$
derivativedivides	$-\frac{Aa^3c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2Aa^3cd\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{Aa^3d^2\left(\frac{8}{3}+\sin^4(fx+e)\right.}{}$
default	$-\frac{Aa^3c^2\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+2Aa^3cd\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{Aa^3d^2\left(\frac{8}{3}+\sin^4(fx+e)\right.}{}$
norman	Expression too large to display

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/12*a^3*((1/4*(17*A+19*B)*d^2+6*(17/12*B+A)*c*d+c^2*(A+3*B))*cos(3*f*x+3*
e)+3*((-63/16*B-4*A)*d^2-8*d*c*(A+B)-3*(A+4/3*B)*c^2)*sin(2*f*x+2*e)+3/4*(
3/2*(3/2*B+A)*d^2+c*(A+3*B)*d+1/2*B*c^2)*sin(4*f*x+4*e)-3/20*((A+3*B)*d+2*
B*c)*d*cos(5*f*x+5*e)-1/16*B*d^2*sin(6*f*x+6*e)+3*(1/2*(-23*A-21*B)*d^2-26
*c*(23/26*B+A)*d-15*c^2*(A+13/15*B))*cos(f*x+e)+(-136/5*B+39/2*f*x*A+69/4*
f*x*B-152/5*A)*d^2+45*c*(f*x*A+13/15*f*x*B-8/5*A-304/225*B)*d+30*c^2*(f*x*
A+3/4*f*x*B-22/15*A-6/5*B))/f
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.65

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx =$$
$$-\frac{48\,(2\,Ba^3cd + (A+3\,B)a^3d^2)\cos(fx+e)^5 - 80\,((A+3\,B)a^3c^2 + 2\,(3\,A+5\,B)a^3cd + (5\,A+7\,B)a^3d}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B
)*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 -
15*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d
^2)*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(
f*x + e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a
^3*c*d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^
2 + 4*(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x
 + e))/f
```

3.259.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(449) = 898$.

Time = 0.50 (sec) , antiderivative size = 1804, normalized size of antiderivative = 3.90

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output

```
Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x
)**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*
a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/
(3*f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 +
3*A*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e +
f*x)**2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**
2 - 5*A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e +
 f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f)
- 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/
f - 2*A*a**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)
**2/2 + 9*A*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/
2 - A*a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(
3*f) - 3*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 8*A*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f
+ 3*B*a**3*c**2*x*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 3*B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e
+ f*x)**4/8 + 3*B*a**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e +...
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.52

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx$$

$$= \frac{320\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^3c^2 + 720\,(2\,fx + 2\,e - \sin(2\,fx+2\,e))Aa^3c^2 + 960\,(fx+e)Aa^3c}{}$$

input

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e
- sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*
c^2 + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e
 + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e -
sin(2*f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*
d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5
- 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3
*cos(f*x + e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 1
92*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 32
0*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 +
60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*
a^3*c^2*cos(f*x + e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x
 + e))/f
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{Ba^3d^2\sin(6fx+6e)}{192f}$$
$$+\frac{1}{16}\left(40Aa^3c^2+30Ba^3c^2+60Aa^3cd+52Ba^3cd+26Aa^3d^2+23Ba^3d^2\right)x$$
$$-\frac{(2Ba^3cd+Aa^3d^2+3Ba^3d^2)\cos(5fx+5e)}{80f}$$
$$+\frac{(4Aa^3c^2+12Ba^3c^2+24Aa^3cd+34Ba^3cd+17Aa^3d^2+19Ba^3d^2)\cos(3fx+3e)}{48f}$$
$$-\frac{(30Aa^3c^2+26Ba^3c^2+52Aa^3cd+46Ba^3cd+23Aa^3d^2+21Ba^3d^2)\cos(fx+e)}{8f}$$
$$+\frac{(2Ba^3c^2+4Aa^3cd+12Ba^3cd+6Aa^3d^2+9Ba^3d^2)\sin(4fx+4e)}{64f}$$
$$-\frac{(48Aa^3c^2+64Ba^3c^2+128Aa^3cd+128Ba^3cd+64Aa^3d^2+63Ba^3d^2)\sin(2fx+2e)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 +
60*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a
^3*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 +
 12*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46
*B*a^3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c
^2 + 4*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4
*e)/f - 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d
+ 64*A*a^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

3.259.9 Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.11

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

output
```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d
^2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*
c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))*(40*A*c
^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e
/2 + (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*
x)/2)*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/
8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^11*(3*A*a^3*
c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c
*d)/2 + (13*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^8*(34*A*a^3*c^2 + 12*A*a^3*
d^2 + 22*B*a^3*c^2 + 4*B*a^3*d^2 + 44*A*a^3*c*d + 24*B*a^3*c*d) + tan(e/2
+ (f*x)/2)^5*(6*A*a^3*c^2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^
3*d^2)/4 + 19*A*a^3*c*d + 25*B*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*A*a^3*c^
2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 + 19*A*a^3*c*d
+ 25*B*a^3*c*d) + tan(e/2 + (f*x)/2)^4*(76*A*a^3*c^2 + 64*A*a^3*d^2 + 68*B
*a^3*c^2 + 64*B*a^3*d^2 + 136*A*a^3*c*d + 128*B*a^3*c*d) + tan(e/2 + (f*x)
/2)^3*(9*A*a^3*c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)
/24 + (53*A*a^3*c*d)/2 + (63*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^9*(9*A*a^3
*c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^
3*c*d)/2 + (63*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^2*(38*A*a^3*c^2 + (152*A
*a^3*d^2)/5 + 34*B*a^3*c^2 + (136*B*a^3*d^2)/5 + 68*A*a^3*c*d + (304*B*...
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

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3.260.1 Optimal result

Integrand size = 33, antiderivative size = 201

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx \\
&= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{5f} \\
&\quad + \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos^3(e+fx)}{60f} \\
&\quad - \frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sin(e+fx)}{40f} \\
&\quad - \frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} \\
&\quad - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af}
\end{aligned}$$

output

```
1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B
*d)*cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)^3/f-3/4
0*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/20*(5*A*d+5*
B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^3/f-1/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e
))^4/a/f
```

3.260.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$= \frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Bc+5Ad-Bd)(1+\sin(e+fx))^3 - Bd(a+a\sin(e+fx))^4 - \frac{a^4(20Ac+15Bc+15Ad+1}{}\right.}{5af}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
(Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d
*(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*Arc
Sin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e +
 f*x] + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

3.260.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\downarrow 3447$$

$$\int (a\sin(e+fx)+a)^3\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (\sin(e+fx)a+a)^3(a(5Ac+4Bd)+a(5Bc+5Ad-Bd)\sin(e+fx))dx}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^3(a(5Ac+4Bd)+a(5Bc+5Ad-Bd)\sin(e+fx))dx}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$$\downarrow 3230$$

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^3dx - \frac{a(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^3dx - \frac{a(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$$\downarrow 3124$$

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int \left(\sin^3(e+fx)a^3+3\sin^2(e+fx)a^3+3\sin(e+fx)a^3+a^3\right)dx - \frac{a(5Ad+5Bc-}{}}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\left(\frac{a^3\cos^3(e+fx)}{3f} - \frac{4a^3\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{5a^3x}{2}\right) - \frac{a(5Ad+5Bc-Bd)\cos}{}}{5a} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/5*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(a*f) + (-1/4*(a*(5*B*c + 5
*A*d - B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/f + (a*(20*A*c + 15*B*c +
 15*A*d + 13*B*d)*((5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x
]^3)/(3*f) - (3*a^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)/(5*a)
```

3.260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.260.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(\left(3c+\frac{17d}{4}\right)B+A(c+3d)\right)\cos(3fx+3e)+3\left(4(-c-d)B-3\left(c+\frac{4d}{3}\right)A\right)\sin(2fx+2e)+\frac{3(B(c+3d)+dA)\sin(4fx+4e)}{8}-\frac{3Bd\,c}{}\right.}{}$
parts	$\frac{(A\,a^3d+B\,a^3c+3a^3dB)\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(3A\,a^3c+A\,a^3d+B\,a^3c)\cos(fx+e)}{f}-$
risch	$\frac{5A\,a^3cx}{2}+\frac{15A\,a^3dx}{8}+\frac{15B\,a^3cx}{8}+\frac{13B\,a^3dx}{8}-\frac{15a^3\cos(fx+e)Ac}{4f}-\frac{13a^3\cos(fx+e)dA}{4f}-\frac{13a^3\cos(fx+e)B}{4f}$
derivativedivides	$-\frac{A\,a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A\,a^3d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B\,a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\,s}{}\right.}{}\right.$
default	$-\frac{A\,a^3c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+A\,a^3d\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B\,a^3c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\,s}{}\right.}{}\right.$
norman	$\frac{\left(\frac{5}{2}A\,a^3c+\frac{15}{8}A\,a^3d+\frac{15}{8}B\,a^3c+\frac{13}{8}a^3dB\right)x+\left(25A\,a^3c+\frac{75}{4}A\,a^3d+\frac{75}{4}B\,a^3c+\frac{65}{4}a^3dB\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(25A\,a^3c+\frac{75}{4}A\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
1/12*(((3*c+17/4*d)*B+A*(c+3*d))*cos(3*f*x+3*e)+3*(4*(-c-d)*B-3*(c+4/3*d)*
A)*sin(2*f*x+2*e)+3/8*(B*(c+3*d)+d*A)*sin(4*f*x+4*e)-3/20*B*d*cos(5*f*x+5*
e)+3*((-13*c-23/2*d)*B-15*(c+13/15*d)*A)*cos(f*x+e)+(45/2*c*f*x+39/2*d*f*x
-36*c-152/5*d)*B+30*(3*(-2/5+1/4*f*x)*d+c*(f*x-22/15))*A)*a^3/f
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$

$$-\frac{24\,Ba^3d\cos(fx+e)^5-40\,((A+3\,B)a^3c+(3\,A+5\,B)a^3d)\cos(fx+e)^3-15\,(5\,(4\,A+3\,B)a^3c+(15}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*
d)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 4
80*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*
B)*a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos
(f*x + e))*sin(f*x + e))/f
```

3.260.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(201) = 402$.

Time = 0.33 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.78

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
 + A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c
*cos(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*
x)**2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*co
s(e + f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)
**3*cos(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a
**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e +
 f*x)/(2*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B
*a**3*c*x*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2
/4 + 3*B*a**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B
*a**3*c*x*cos(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 3*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*co
s(e + f*x)**3/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**
3*c*cos(e + f*x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x
)**4/8 + 9*B*a**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e
 + f*x)**2/2 + 9*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2
/2 - B*a**3*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3
*cos(e + f*x)/(8*f) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3
*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(...
```

3.260.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(189) = 378$.

Time = 0.22 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))\,dx$$
$$= \frac{160\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^3c + 360\left(2fx + 2e - \sin(2fx+2e)\right)Aa^3c + 480(fx+e)Aa^3c +$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e) - 480*A*a^3*d*cos(f*x + e))/f
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c + d\sin(e+fx))\,dx \\
&= -\frac{Ba^3d\cos(5fx+5e)}{80f} + \frac{1}{8}\left(20Aa^3c + 15Ba^3c + 15Aa^3d + 13Ba^3d\right)x \\
&\quad + \frac{\left(4Aa^3c + 12Ba^3c + 12Aa^3d + 17Ba^3d\right)\cos(3fx+3e)}{48f} \\
&\quad - \frac{\left(30Aa^3c + 26Ba^3c + 26Aa^3d + 23Ba^3d\right)\cos(fx+e)}{8f} \\
&\quad + \frac{\left(Ba^3c + Aa^3d + 3Ba^3d\right)\sin(4fx+4e)}{32f} \\
&\quad - \frac{\left(3Aa^3c + 4Ba^3c + 4Aa^3d + 4Ba^3d\right)\sin(2fx+2e)}{4f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
-1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d)*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*e)/f - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e)/f
```

3.260.9 Mupad [B] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$= \frac{a^3\,\mathrm{atan}\left(\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(20Ac+15Ad+15Bc+13Bd)}{4\left(5Aa^3c+\frac{15Aa^3d}{4}+\frac{15Ba^3c}{4}+\frac{13Ba^3d}{4}\right)}\right)(20Ac+15Ad+15Bc+13Bd)}{4f}$$

$$- \frac{a^3\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(20Ac+15Ad+15Bc+13Bd)}{4f}$$

$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(6Aa^3c+\frac{19Aa^3d}{2}+\frac{19Ba^3c}{2}+\frac{25Ba^3d}{2}\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9\left(3Aa^3c+\frac{15Aa^3d}{4}+\frac{15Ba^3c}{4}+\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

output

```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*
(5*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4)))*(20*A*c +
 15*A*d + 15*B*c + 13*B*d))/(4*f) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(6*
A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/2) - tan(e/2 + (f
*x)/2)^9*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4) -
tan(e/2 + (f*x)/2)^7*(6*A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*
a^3*d)/2) + tan(e/2 + (f*x)/2)^6*(28*A*a^3*c + 20*A*a^3*d + 20*B*a^3*c + 1
2*B*a^3*d) + tan(e/2 + (f*x)/2)^2*((92*A*a^3*c)/3 + 28*A*a^3*d + 28*B*a^3*
c + (76*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^4*((136*A*a^3*c)/3 + 40*A*a^3*d +
 40*B*a^3*c + (116*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^8*(6*A*a^3*c + 2*A*a^3
*d + 2*B*a^3*c) + tan(e/2 + (f*x)/2)*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a
^3*c)/4 + (13*B*a^3*d)/4) + (22*A*a^3*c)/3 + 6*A*a^3*d + 6*B*a^3*c + (76*B
*a^3*d)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(
e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

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3.261.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \frac{5}{8}a^3(4A+3B)x - \frac{5a^3(4A+3B)\cos(e+fx)}{6f} - \frac{5a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{24f}$$
$$- \frac{a(4A+3B)\cos(e+fx)(a+a\sin(e+fx))^2}{12f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}$$

output
```
5/8*a^3*(4*A+3*B)*x-5/6*a^3*(4*A+3*B)*cos(f*x+e)/f-5/24*a^3*(4*A+3*B)*cos(
f*x+e)*sin(f*x+e)/f-1/12*a*(4*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B
*cos(f*x+e)*(a+a*sin(f*x+e))^3/f
```

3.261.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx =$$
$$-\frac{a^3\cos(e+fx)\left(30(4A+3B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(88A+72B+9(4A+5B)\sin(e+\right.}{24f\sqrt{\cos^2(e+fx)}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output
```
-1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt
[2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*
(A + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

3.261.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{1}{4}(4A+3B)\int (\sin(e+fx)a+a)^3dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\downarrow 3042$$

$$\frac{1}{4}(4A+3B)\int (\sin(e+fx)a+a)^3dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\downarrow 3124$$

$$\frac{1}{4}(4A+3B)\int \left(\sin^3(e+fx)a^3+3\sin^2(e+fx)a^3+3\sin(e+fx)a^3+a^3\right)dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\downarrow 2009$$

$$\frac{1}{4}(4A+3B)\left(\frac{a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{3a^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{5a^3x}{2}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/f + ((4*A + 3*B)*((5*a^3*x)/2
 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*Cos[e + f*
x]*Sin[e + f*x])/(2*f)))/4
```

3.261.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.261.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{a^3(8(A+3B)\cos(3fx+3e)+24(-3A-4B)\sin(2fx+2e)+3B\sin(4fx+4e)+24(-15A-13B)\cos(fx+e)+240fxA+180f}{96f}$
risch	$\frac{5a^3Ax}{2}+\frac{15a^3Bx}{8}-\frac{15a^3\cos(fx+e)A}{4f}-\frac{13a^3\cos(fx+e)B}{4f}+\frac{Ba^3\sin(4fx+4e)}{32f}+\frac{a^3\cos(3fx+3e)A}{12f}+\frac{a^3\cos}{}$
parts	$a^3Ax-\frac{(Aa^3+3Ba^3)(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{(3Aa^3+Ba^3)\cos(fx+e)}{f}+\frac{(3Aa^3+3Ba^3)\left(-\frac{\cos(fx+e)\sin}{2}\right.}{f}$
derivativedivides	$-\frac{Aa^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Ba^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3Aa^3\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.$
default	$-\frac{Aa^3\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+Ba^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3Aa^3\left(-\frac{\cos(fx+e)\sin(fx+e}{2}\right.$
norman	$\frac{(\frac{5}{2}Aa^3+\frac{15}{8}Ba^3)x+(10Aa^3+\frac{15}{2}Ba^3)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10Aa^3+\frac{15}{2}Ba^3)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(15Aa^3+\frac{45}{4}Ba^3)x\left(t\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/96*a^3*(8*(A+3*B)*cos(3*f*x+3*e)+24*(-3*A-4*B)*sin(2*f*x+2*e)+3*B*sin(4*
f*x+4*e)+24*(-15*A-13*B)*cos(f*x+e)+240*f*x*A+180*f*x*B-352*A-288*B)/f
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\frac{8\,(A+3\,B)a^3\cos\left(fx+e\right)^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos\left(fx+e\right)+3\,\left(2\,Ba^3\cos\left(fx+e\right)^3-\right.}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)
*a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x
+ e))*sin(f*x + e))/f
```

3.261.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(119) = 238$.

Time $= 0.20$ (sec) , antiderivative size $= 371$, normalized size of antiderivative $= 2.92$

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\, dx$$

$$= \begin{cases} \frac{3Aa^3 x \sin^2(e+fx)}{2} + \frac{3Aa^3 x \cos^2(e+fx)}{2} + Aa^3 x - \frac{Aa^3 \sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Aa^3 \sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^3 \cos^3(e+fx)}{3f} \\ x(A + B\sin(e))\,(a\sin(e) + a)^3 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A
*a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*co
s(e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/
f + 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)
**2/4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*
a**3*x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*B*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f
*x)**3/f - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
 a)**3, True))
```

3.261.7 Maxima [A] (verification not implemented)

Time $= 0.29$ (sec) , antiderivative size $= 171$, normalized size of antiderivative $= 1.35$

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\, dx$$

$$= \frac{32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^3 + 72\,(2fx + 2e - \sin(2fx + 2e))Aa^3 + 96\,(fx+e)Aa^3 + 96\,(c}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) -
96*B*a^3*cos(f*x + e))/f
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int (a + a\sin(e+fx))^3(A + B\sin(e+fx))\,dx$$
$$= \frac{Ba^3\sin(4fx+4e)}{32f} + \frac{5}{8}\left(4Aa^3+3Ba^3\right)x + \frac{\left(Aa^3+3Ba^3\right)\cos(3fx+3e)}{12f}$$
$$- \frac{\left(15Aa^3+13Ba^3\right)\cos(fx+e)}{4f} - \frac{\left(3Aa^3+4Ba^3\right)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 +
3*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1
/4*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

3.261.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.60

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \frac{5a^3\,\mathrm{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A+3B)}{4\left(5Aa^3+\frac{15Ba^3}{4}\right)}\right)(4A+3B)}{4f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3Aa^3+\frac{15Ba^3}{4}\right)+\frac{22Aa^3}{3}+6Ba^3+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(6Aa^3+2Ba^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3A\right.}{f\left(\tan\left(\frac{e}{2}+\right.\right.}$$
$$- \frac{5a^3(4A+3B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

output

```
(5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*A + 3*B))/(4*(5*A*a^3 + (15*B*a^3
)/4)))*(4*A + 3*B))/(4*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3 + (15*B*a^3)/4) +
 (22*A*a^3)/3 + 6*B*a^3 + tan(e/2 + (f*x)/2)^6*(6*A*a^3 + 2*B*a^3) - tan(e
/2 + (f*x)/2)^7*(3*A*a^3 + (15*B*a^3)/4) + tan(e/2 + (f*x)/2)^3*(3*A*a^3 +
 (23*B*a^3)/4) - tan(e/2 + (f*x)/2)^5*(3*A*a^3 + (23*B*a^3)/4) + tan(e/2 +
 (f*x)/2)^4*(22*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^2*((70*A*a^3)/3 + 2
2*B*a^3))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2
+ (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (5*a^3*(4*A + 3*B)*(atan(tan(e
/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

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3.262.1 Optimal result

Integrand size = 35, antiderivative size = 246

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx \\
&= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} \\
&\quad + \frac{2a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}f} \\
&\quad + \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} \\
&\quad - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} \\
&\quad + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{6d^2f}
\end{aligned}$$

output

```
1/2*a^3*(A*d*(2*c^2-6*c*d+7*d^2)-B*(2*c^3-6*c^2*d+7*c*d^2-5*d^3))*x/d^4+1/
2*a^3*(A*(2*c-5*d)*d-B*(2*c^2-5*c*d+5*d^2))*cos(f*x+e)/d^3/f-1/3*a*B*cos(f
*x+e)*(a+a*sin(f*x+e))^2/d/f+1/6*(-3*A*d+3*B*c-5*B*d)*cos(f*x+e)*(a^3+a^3*
sin(f*x+e))/d^2/f+2*a^3*(c-d)^3*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))
/(c^2-d^2)^(1/2))/d^4/f/(c^2-d^2)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{a^3(1+\sin(e+fx))^3\left(6(Ad(2c^2-6cd+7d^2)+B(-2c^3+6c^2d-7cd^2+5d^3))(e+fx)+\frac{24(c-d)^3(Bc-Ad)}{}\right.}{12}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*
c^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d
 + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c +
 3*d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)]
 - 3*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6)
```

3.262.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3455$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc+3Ad)-a(3Bc-3Ad-5Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc+3Ad)-a(3Bc-3Ad-5Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3455$$

$$\frac{\int -\frac{3(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d} + \frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df}}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 27$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\int \frac{(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d}}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3042$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\int \frac{(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d}}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3447$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\int \frac{\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin^2(e+fx)a^3+(Bc(c-3d)-Ad(c+2d))a^3+\left((Bc(c-3d)-Ad(c+\right.}{c+d\sin(e+fx)}}{2d}}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3042$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\int \frac{\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)^2a^3+(Bc(c-3d)-Ad(c+2d))a^3+\left((Bc(c-3d)-Ad(c+\right.}{c+d\sin(e+fx)}}{2d}}{3d} - \frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3502$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df} - \frac{3\left(\int \frac{a^3 d(Bc(c-3d)-Ad(c+2d))-a^3\left(Ad\left(2c^2-6dc+7d^2\right)-B\left(2c^3-6dc^2+7d^2c-5d^3\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} - \cdots}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3042$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df} - \frac{3\left(\int \frac{a^3 d(Bc(c-3d)-Ad(c+2d))-a^3\left(Ad\left(2c^2-6dc+7d^2\right)-B\left(2c^3-6dc^2+7d^2c-5d^3\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} - \cdots}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3214$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df} - \frac{3\left(-\frac{2a^3(c-d)^3(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)}{d} - \cdots}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3042$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df} - \frac{3\left(-\frac{2a^3(c-d)^3(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)}{d} - \cdots}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 3139$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df} - \frac{3\left(-\frac{4a^3(c-d)^3(Bc-Ad)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df} - \frac{a^3x\left(Ad\left(2c^2-\cdots\right)\right.}{d} - \cdots}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 1083$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\left(\frac{8a^3(c-d)^3(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df} - \frac{a^3x\left(A\right.}{d}\right.}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\downarrow 217$$

$$\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df} - \frac{3\left(-\frac{4a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}} - \frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+\right.\right.}{d}\right)}{2d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
-1/3*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*f) + ((-3*((-((a^3*(A*d*
(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*x)/d) - (
4*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^
2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a^3*(A*(2*c - 5*d)*d - B*(2*c^2 -
5*c*d + 5*d^2))*Cos[e + f*x])/(d*f)))/(2*d) + ((3*B*c - 3*A*d - 5*B*d)*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f))/(3*d)
```

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.262.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54

method	result
derivativedivides	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}d^2cB+\frac{3}{2}d^3B\right)$
default	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}d^2cB+\frac{3}{2}d^3B\right)$
risch	Expression too large to display

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
2/f*a^3*((-A*c^3*d+3*A*c^2*d^2-3*A*c*d^3+A*d^4+B*c^4-3*B*c^3*d+3*B*c^2*d^2
-B*c*d^3)/d^4/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))+1/d^4*(((1/2*A*d^3-1/2*d^2*c*B+3/2*d^3*B)*tan(1/2*f*x+1/2*e)^
5+(A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^2-3*B*d^3)*tan(1/2*f*x+1/2*e)^4+(2*A*c*
d^2-6*A*d^3-2*B*c^2*d+6*B*c*d^2-8*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3+
1/2*d^2*c*B-3/2*d^3*B)*tan(1/2*f*x+1/2*e)+d^2*c*A-3*A*d^3-c^2*d*B+3*d^2*c*
B-11/3*d^3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(2*A*c^2*d-6*A*c*d^2+7*A*d^3-
2*B*c^3+6*B*c^2*d-7*B*c*d^2+5*B*d^3)*arctan(tan(1/2*f*x+1/2*e))))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[\frac{2\,Ba^3d^3\cos(fx+e)^3 - 3\,(2\,Ba^3c^3 - 2\,(A+3\,B)a^3c^2d + (6\,A+7\,B)a^3cd^2 - (7\,A+5\,B)a^3d^3)fx + 3\,(B}{}\right.$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

output
```
[1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d - (A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

3.262.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.262.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more de
```

3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(233) = 466$.

Time $= 0.31$ (sec) , antiderivative size $= 597$, normalized size of antiderivative $= 2.43$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx =$$

$$\frac{3\,(2\,Ba^3c^3-2\,Aa^3c^2d-6\,Ba^3c^2d+6\,Aa^3cd^2+7\,Ba^3cd^2-7\,Aa^3d^3-5\,Ba^3d^3)(fx+e)}{d^4} - \frac{12\,(Ba^3c^4-Aa^3c^3d-3\,Ba^3c^3d+3\,Aa^3c^2d^2+3\,Ba^3c^2$$

$$-$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
-1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B
*a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*
a^3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*
d^3 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) +
arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4
) + 2*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*
e)^5 - 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*
e)^4 - 6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2
*e)^4 + 18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1
/2*e)^4 + 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x +
 1/2*e)^2 - 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x
 + 1/2*e)^2 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*
x + 1/2*e) + 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x +
1/2*e) + 6*B*a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^
3*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

3.262.9 Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 10256, normalized size of antiderivative = 41.69

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x)),x)
```

output

```
- ((2*(9*A*a^3*d^2 + 3*B*a^3*c^2 + 11*B*a^3*d^2 - 3*A*a^3*c*d - 9*B*a^3*c*
d))/(3*d^3) - (tan(e/2 + (f*x)/2)^5*(A*a^3*d - B*a^3*c + 3*B*a^3*d))/d^2 +
 (4*tan(e/2 + (f*x)/2)^2*(3*A*a^3*d^2 + B*a^3*c^2 + 4*B*a^3*d^2 - A*a^3*c*
d - 3*B*a^3*c*d))/d^3 + (2*tan(e/2 + (f*x)/2)^4*(3*A*a^3*d^2 + B*a^3*c^2 +
 3*B*a^3*d^2 - A*a^3*c*d - 3*B*a^3*c*d))/d^3 + (tan(e/2 + (f*x)/2)*(A*a^3*
d - B*a^3*c + 3*B*a^3*d))/d^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f
*x)/2)^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(49*A^2*a^6*c^2*d^9 -
84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c
^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 1
04*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c
^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 -
128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*ta
n(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*
c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 1
40*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a
^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^
4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^
6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^
7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*
B*a^6*c*d^11))/d^9 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 ...
```

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

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3.263.1 Optimal result

Integrand size = 35, antiderivative size = 283

$$
\begin{aligned}
&\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4} \\
&\quad + \frac{2a^3(c-d)^2\left(Ad(2c+3d)-B(3c^2+3cd-d^2)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)\sqrt{c^2-d^2}f} \\
&\quad - \frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f} \\
&\quad + \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} \\
&\quad + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))}
\end{aligned}
$$

output

```
-1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4-1/2*a^3*(4*A*c*d-B
*(6*c^2-3*c*d-5*d^2))*cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*cos(f*x
+e)*(a^3+a^3*sin(f*x+e))/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+
e))^2/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c
*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)/f/(c^2
-d^2)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 5.95 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a^3(1+\sin(e+fx))^3\left(2(2Ad(-2c+3d)+B(6c^2-12cd+7d^2))(e+fx)-\frac{8(c-d)^2(-Ad(2c+3d)+B(3c^2+3cd-d}{(c+d)\sqrt{c^2}}\right.}{4d^4f\left(\cos\left(\frac{1}{2}(e+f\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d
^2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2
))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d
^2]) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d
)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/
(4*d^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

3.263.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3454, 25, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^2(a(B(2c-d)-3Ad)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)^2(a(2Bc-3Ad-Bd)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)^2(a(2Bc-3Ad-Bd)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\downarrow 3455$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)\left(a^2\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^2\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d}-\frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)\left(a^2\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^2\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d}-\frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}}{d(c+d)}$$

$$\downarrow 3447$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{-\left(\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin^2(e+fx)a^3\right)+\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+\left(a^3\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^3\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\right)\sin}{c+d\sin(e+fx)}}{2d}}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\int \frac{-\left(\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)^2a^3\right)+\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+\left(a^3\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^3\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\right)\sin}{c+d\sin(e+fx)}}{2d}$$
$$d(c+d)$$

$$\downarrow 3502$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\int \frac{d\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+(c+d)\left(2A(2c-3d)d-B\left(6c^2-12dc+7d^2\right)\right)\sin(e+fx)a^3}{c+d\sin(e+fx)}dx}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d} - \frac{(2Ad-B(3c+}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\int \frac{d\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+(c+d)\left(2A(2c-3d)d-B\left(6c^2-12dc+7d^2\right)\right)\sin(e+fx)a^3}{c+d\sin(e+fx)}dx}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d} - \frac{(2Ad-B(3c+}{d(c+d)}$$

$$\downarrow 3214$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d} - \frac{(}{d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d} - \frac{(}{d(c+d)}$$

$$\downarrow 3139$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} -$$
$$\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{4a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}}{d}+\frac{a^3\left(4Ac}{2d}$$
$$d(c+d)$$

$$\downarrow 1083$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} - \frac{8a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df} + \frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d} + \frac{\frac{\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(}{df}}{2d}}{d(c+d)}$$

$$\downarrow 217$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d} - \frac{4a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}}{d} + \frac{\frac{\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(}{df}}{2d}}{d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Si
n[e + f*x])) - ((((a^3*(c + d)*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*
d^2))*x)/d - (4*a^3*(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*
ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^
2]*f))/d + (a^3*(4*A*c*d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(d*f))
/(2*d) - ((2*A*d - B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*
d*f))/(d*(c + d))
```

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.263.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.43

method	result
derivativedivides	$2a^3\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-2cdB+3d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-2cdB+3d^2B+\frac{\left(4Acd-6A\,d^2-6\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}}{d^4}\right.$
default	$2a^3\left(-\frac{-\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(A\,d^2-2cdB+3d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}+A\,d^2-2cdB+3d^2B+\frac{\left(4Acd-6A\,d^2-6\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}}{d^4}\right.$
risch	Expression too large to display

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output

```
2/f*a^3*(-1/d^4*((-1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(A*d^2-2*B*c*d+3*B*d^2)*
tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)*d^2+A*d^2-2*c*d*B+3*d^2*B)/(
1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(4*A*c*d-6*A*d^2-6*B*c^2+12*B*c*d-7*B*d^2)*a
rctan(tan(1/2*f*x+1/2*e)))+1/d^4*((-d^2*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B
*c^2*d-B*c*d^2)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^
3+2*B*c^2*d-B*c*d^2)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)
+c)+(2*A*c^3*d-A*c^2*d^2-4*A*c*d^3+3*A*d^4-3*B*c^4+3*B*c^3*d+4*B*c^2*d^2-5
*B*c*d^3+B*d^4)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2))))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1027, normalized size of antiderivative = 3.63

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3
*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*
B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 +
(3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4
)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e)
 + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3
*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a
^3*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d
^4)*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 +
 c*d^5)*f), 1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 -
 2*(2*A + 3*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3
)*f*x + 2*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)
*a^3*c*d^3 + (3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A
 + B)*a^3*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e)
 + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(
2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*
x + e) + ((6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*...
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.263.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.263.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(271) = 542$.

Time $= 0.31$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 2.02$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx =$$

$$\frac{4\left(3Ba^3c^4-2Aa^3c^3d-3Ba^3c^3d+Aa^3c^2d^2-4Ba^3c^2d^2+4Aa^3cd^3+5Ba^3cd^3-3Aa^3d^4-Ba^3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{\sqrt{c^2-d^2}}\right.\right.}{(cd^4+d^5)\sqrt{c^2-d^2}}$$

$$-$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B
*a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(p
i*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) +
d)/sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(
1/2*f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*ta
n(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(
1/2*f*x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*
d - 2*B*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d
^3 + c*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (
6*B*a^3*c^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x
 + e)/d^4 - 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/
2*e)^2 - 2*A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)
^2 - B*a^3*d*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((t
an(1/2*f*x + 1/2*e)^2 + 1)^2*d^3))/f
```

3.263.9 Mupad [B] (verification not implemented)

Time = 24.89 (sec) , antiderivative size = 11993, normalized size of antiderivative = 42.38

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x
)
```

output

```
- ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d
^2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 -
3*B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*
a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3
*c^3 + B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3
*c^2*d))/(d^3*(c + d)) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3
- A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c
+ d)) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 +
 2*A*a^3*c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/
2 + (f*x)/2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 +
 9*B*a^3*c^2*d))/(c*d^2*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 3*c*tan
(e/2 + (f*x)/2)^2 + 3*c*tan(e/2 + (f*x)/2)^4 + c*tan(e/2 + (f*x)/2)^6 + 4*
d*tan(e/2 + (f*x)/2)^3 + 2*d*tan(e/2 + (f*x)/2)^5)) - (atan(((((8*(36*A^2*
a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6
 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a
^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4
 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*
a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4
))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^1
0 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 3...
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

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3.264.1 Optimal result

Integrand size = 35, antiderivative size = 305

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx &= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4}\\ &-\frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)^2\sqrt{c^2-d^2}f}\\ &-\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f}\\ &+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2}\\ &-\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))}\end{aligned}$$

output

```
-a^3*(-A*d+3*B*c-3*B*d)*x/d^4-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*cos(
f*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d
)/f/(c+d*sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*cos(f*x+e)*
(a^3+a^3*sin(f*x+e))/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3*(c-d)*(A*d*(2*c^2+
6*c*d+7*d^2)-3*B*(2*c^3+4*c^2*d+c*d^2-2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*
e))/(c^2-d^2)^(1/2))/d^4/(c+d)^2/f/(c^2-d^2)^(1/2)
```

3.264.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 830 vs. $2(305) = 610$.

Time = 7.18 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.72

$$\int \frac{(a + a\sin(e+fx))^3(A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx$$

$$= a^3(1+\sin(e+fx))^3\left(\frac{4(c-d)\left(-Ad\left(2c^2+6cd+7d^2\right)+3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bc^5e+4Ac^4de-1}{}\right.$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*
B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e +
 8*A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4
*e + 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x -
12*B*c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B
*c^2*d^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x -
 d*(2*A*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^
2*d^2 + 4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B
*d)*(e + f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Co
s[3*(e + f*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*
c^3*d^2*e*Sin[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[
e + f*x] + 24*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c
*d^4*e*Sin[e + f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e
+ f*x] - 24*B*c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 2
4*B*c^2*d^3*f*x*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x
*Sin[e + f*x] - 9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)
] - 9*B*c^2*d^3*Sin[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*
Sin[2*(e + f*x)] - 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c
 + d*Sin[e + f*x])^2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + ...
```

3.264.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3454, 25, 3042, 3454, 25, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\downarrow 3454$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int -\frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{d(c+d)} + \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(}{df(c+d)(c+d\sin(e+fx))}$$

$$2d(c+d)$$

$$\downarrow 25$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(e+\right.}{c+d\sin(e+fx)}}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(e+\right.}{c+d\sin(e+fx)}}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3447$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(3Bc(2c+3d)-Ad(2c+5d))\sin^2(e+fx)a^3+\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^3+\left((3Bc(\right.}{c+d\sin(e+fx)}}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(3Bc(2c+3d)-Ad(2c+5d))\sin(e+fx)^2a^3+\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^3+\left((3Bc(\right.}{c+d\sin(e+fx)}}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3502$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{a^3d\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)-2a^3(c+d)^2(3B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} - \frac{a^3(3B}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{a^3d\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)-2a^3(c+d)^2(3B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d} - \frac{a^3(3B}{d(c+d)}$$

$$2d(c+d)$$

$$\downarrow 3214$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{2a^3x}{d}}{d(c+d)}}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d} - \frac{2a^3x}{d}}{d(c+d)}}{2d(c+d)}$$

$$\downarrow 3139$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{-\frac{2a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\ta}}{df}}{d}}{2d(c+d)}$$

$$\downarrow 1083$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{4a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}}{df}}{d}}{2d(c+d)}$$

$$\downarrow 217$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)(a^3\sin(e+fx)+a^3)}{df(c+d)(c+d\sin(e+fx))} - \frac{-\frac{2a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)}{2\sqrt{c^2-d^2}}\right.}{df\sqrt{c^2-d^2}}}{d}}{d(c+d)}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*
Sin[e + f*x])^2) - (-((((-2*a^3*(c + d)^2*(3*B*c - A*d - 3*B*d)*x)/d - (2*
a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 -
2*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[
c^2 - d^2]*f))/d - (a^3*(3*B*c*(2*c + 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x]
)/(d*f))/(d*(c + d))) + ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e
 + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x])))/(2*d
*(c + d))
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.264.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.92

method	result
derivativedivides	$2a^3\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-3Bc+3dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(Ac^3d+5Ac^2d^2-4Acd^3-2Ad^4-3Bc^4-3Bc^3d+6Bc^2d^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
default	$2a^3\left(\frac{-\frac{dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(dA-3Bc+3dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(Ac^3d+5Ac^2d^2-4Acd^3-2Ad^4-3Bc^4-3Bc^3d+6Bc^2d^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3*(1/d^4*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-3*B*c+3*B*d)*arctan(tan
(1/2*f*x+1/2*e)))-1/d^4*((-1/2*d^2*(A*c^3*d+5*A*c^2*d^2-4*A*c*d^3-2*A*d^4-
3*B*c^4-3*B*c^3*d+6*B*c^2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*
d*(2*A*c^5*d+4*A*c^4*d^2-A*c^3*d^3+7*A*c^2*d^4-10*A*c*d^5-2*A*d^6-4*B*c^6-
2*B*c^5*d-B*c^4*d^2-5*B*c^3*d^3+14*B*c^2*d^4-2*B*c*d^5)/(c^2+2*c*d+d^2)/c^
2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(7*A*c^3*d+11*A*c^2*d^2-16*A*c*d^3-2*A*d^4-
13*B*c^4-5*B*c^3*d+22*B*c^2*d^2-4*B*c*d^3)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1
/2*e)-1/2*d*(2*A*c^3*d+4*A*c^2*d^2-5*A*c*d^3-A*d^4-4*B*c^4-2*B*c^3*d+7*B*c
^2*d^2-B*c*d^3)/(c^2+2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1
/2*e)+c)^2+1/2*(2*A*c^3*d+4*A*c^2*d^2+A*c*d^3-7*A*d^4-6*B*c^4-6*B*c^3*d+9*
B*c^2*d^2+9*B*c*d^3-6*B*d^4)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(294) = 588$.

Time $= 0.34$ (sec) , antiderivative size $= 1670$, normalized size of antiderivative $= 5.48$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
[-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4
 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4
  + B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*
a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)
*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 -
3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 -
(6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A
+ 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^
2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(
c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x
+ e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2)) - 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*
c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*
B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*
a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*
a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2
*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x
 + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B
*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B...
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.264.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.264.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(294) = 588$.

Time $= 0.34$ (sec) , antiderivative size $= 953$, normalized size of antiderivative $= 3.12$

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*
c^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*flo
or(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sq
rt(c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((ta
n(1/2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A
*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)
^3 - 5*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x
 + 1/2*e)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/
2*f*x + 1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(
1/2*f*x + 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^
2*tan(1/2*f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^
3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*
A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*
e)^2 + 10*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x +
 1/2*e)^2 + 2*A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*
x + 1/2*e) - 7*A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/
2*f*x + 1/2*e) - 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*
tan(1/2*f*x + 1/2*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2
*d^4*tan(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c
^6 - 2*A*a^3*c^5*d + 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 ...
```

3.264.9 Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 13891, normalized size of antiderivative = 45.54

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x
)
```

output

```
- ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3
+ 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*
tan(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^
3*d + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c
*d^2*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a
^3*c*d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2
))/(c*d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 +
5*A*a^3*c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*
d^4 + 3*A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d
^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)^4*(2*A*a^
3*d^6 + 6*B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B
*a^3*c^5*d - 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*
c^2*d^4 + 5*B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e
/2 + (f*x)/2)*(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d
 + 4*B*a^3*c*d^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/
(c*d^2*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + tan(e/2 + (f
*x)/2)^4*(3*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*tan(e/2
+ (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)^5 + 4*c*d*tan(e/2 + (f*x)/2))) - (
atan((((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a
^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^...
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

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3.265.1 Optimal result

Integrand size = 35, antiderivative size = 220

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx \\
&= \frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a} \\
&\quad + \frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af} \\
&\quad + \frac{d^2(6Ac-11Bc-9Ad+9Bd)\cos(e+fx)\sin(e+fx)}{6af} \\
&\quad + \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))}
\end{aligned}$$

output

```
1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3
*A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*
A*d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+
d*sin(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
```

3.265.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 788 vs. $2(220) = 440$.

Time = 7.02 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.58

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) + \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(4Ad(6c^2(e+fx) - 3cd(1+2e+2fx) + d^2(1+3e+3fx)) + B(\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*
(1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d
*(1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x))
)*Cos[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos
[(3*(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f
*x))/2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A
*c^3*Sin[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f
*x)/2] + 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180
*B*c*d^2*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e +
f*x)/2] + 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72
*B*c^2*d*e*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*
e*Sin[(e + f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x
)/2] + 24*B*c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 7
2*B*c^2*d*f*x*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c
*d^2*f*x*Sin[(e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*S
in[(e + f*x)/2] - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e +
 f*x))/2] + 27*B*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2]
 - 18*B*d^3*Sin[(3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^
3*Sin[(5*(e + f*x))/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e +
f*x))/2]))/(24*a*f*(1 + Sin[e + f*x]))
```

3.265.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3456, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a\sin(e+fx)+a}dx$$

$$\downarrow 3456$$

$$\frac{\int (c+d\sin(e+fx))^2(a(B(c-3d)+3Ad)-a(3A-4B)d\sin(e+fx))dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))^2(a(B(c-3d)+3Ad)-a(3A-4B)d\sin(e+fx))dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3232$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(a\left(3A(3c-2d)d+B\left(3c^2-9dc+8d^2\right)\right)-ad(6Ac-11Bc-9Ad+9Bd)\sin(e+fx)\right)dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(a\left(3A(3c-2d)d+B\left(3c^2-9dc+8d^2\right)\right)-ad(6Ac-11Bc-9Ad+9Bd)\sin(e+fx)\right)dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3213$$

$$\frac{\frac{1}{3}\left(\frac{2ad\left(3A\left(c^2-3cd+d^2\right)-B\left(7c^2-9cd+4d^2\right)\right)\cos(e+fx)}{f}+\frac{3}{2}ax\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(2c^3-6c^2d+9cd^2-3d^3\right)\right)+\frac{ad^2(6}{a^2}}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

output
```
-(((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])))
+ ((a*(3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((3*a*(3*
A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/2 +
(2*a*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f*x])
/f + (a*d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f*x])/(2
*f))/3)/a^2
```

3.265.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3456

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.265.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{2\left(\left(\frac{1}{2}Ad^3+\frac{3}{2}d^2cB-\frac{1}{2}d^3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-3d^2cA+Ad^3-3c^2dB+3d^2cB-d^3B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-6d^2cA+2Ad^3-6c^2dB+6\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$
default	$\frac{2\left(\left(\frac{1}{2}Ad^3+\frac{3}{2}d^2cB-\frac{1}{2}d^3B\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-3d^2cA+Ad^3-3c^2dB+3d^2cB-d^3B\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-6d^2cA+2Ad^3-6c^2dB+6\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$
parallelrisch	$\frac{\left((36fxA-36fxB+84A-109B)d^3-72c\left(\frac{(-3fx-7)B}{2}+A\left(fx+\frac{7}{2}\right)\right)d^2+72\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)c^2d-48\left(\left(-\frac{fx}{2}-1\right)B\right.\right.}{}$
risch	$-\frac{d^3\sin(2fx+2e)A}{4af}+\frac{3xAd^3}{2a}+\frac{xBc^3}{a}-\frac{3xd^3B}{2a}+\frac{3xc^2dA}{a}-\frac{3xd^2cA}{a}-\frac{3xc^2dB}{a}+\frac{9xd^2cB}{2a}+\frac{Bd^3\cos(}{12}$
norman	Expression too large to display

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
2/f/a*(((1/2*A*d^3+3/2*d^2*c*B-1/2*d^3*B)*tan(1/2*f*x+1/2*e)^5+(-3*A*c*d^2
+A*d^3-3*B*c^2*d+3*B*c*d^2-B*d^3)*tan(1/2*f*x+1/2*e)^4+(-6*A*c*d^2+2*A*d^3
-6*B*c^2*d+6*B*c*d^2-4*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3-3/2*d^2*c*B
+1/2*d^3*B)*tan(1/2*f*x+1/2*e)-3*d^2*c*A+A*d^3-3*c^2*d*B+3*d^2*c*B-5/3*d^3
*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(6*A*c^2*d-6*A*c*d^2+3*A*d^3+2*B*c^3-6*
B*c^2*d+9*B*c*d^2-3*B*d^3)*arctan(tan(1/2*f*x+1/2*e))-(A*c^3-3*A*c^2*d+3*A
*c*d^2-A*d^3-B*c^3+3*B*c^2*d-3*B*c*d^2+B*d^3)/(tan(1/2*f*x+1/2*e)+1))
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(212) = 424$.

Time = 0.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.14

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{2\,Bd^3\cos(fx+e)^4 - 6\,(A-B)c^3 + 18\,(A-B)c^2d - 18\,(A-B)cd^2 + 6\,(A-B)d^3 + (9\,Bcd^2 + (3\,A -$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B
)*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(
2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(
3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)
*c^3 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^
3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x +
e) + (2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A -
B)*c*d^2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*
d^2 + 3*(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*
(6*B*c^2*d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e)
)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.265.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14644 vs. $2(204) = 408$.

Time = 3.87 (sec) , antiderivative size = 14644, normalized size of antiderivative = 66.56

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

output

```
Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x
/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*
x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*A*c**3/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)*
*5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/
2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 1
8*A*c**2*d*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(
e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 ...
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(212) = 424$.

Time $= 0.32$ (sec) , antiderivative size $= 1124$, normalized size of antiderivative $= 5.11$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output

```
-1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a)
- 9*B*c*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x +
e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(
a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) +...
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(212) = 424$.

Time $= 0.29$ (sec) , antiderivative size $= 460$, normalized size of antiderivative $= 2.09$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx$$

$$= \frac{3\,(2\,Bc^3+6\,Ac^2d-6\,Bc^2d-6\,Acd^2+9\,Bcd^2+3\,Ad^3-3\,Bd^3)(fx+e)}{a} - \frac{12\,(Ac^3-Bc^3-3\,Ac^2d+3\,Bc^2d+3\,Acd^2-3\,Bcd^2-Ad^3+Bd^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\left(9\,Bc$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3
- 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c
*d^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*
c*d^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*ta
n(1/2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(
1/2*f*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f
*x + 1/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x +
1/2*e)^2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/
2*e)^2 + 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2
 - 9*B*c*d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3
*tan(1/2*f*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 1
0*B*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

3.265.9 Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.81

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

output

```
-(12*A*c^3*cos(e/2 + (f*x)/2) - 18*A*d^3*cos(e/2 + (f*x)/2) - 12*B*c^3*cos
(e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2) + 6*A*d^3*cos(e/2 + (f*x)/2)
^3 - 12*A*d^3*cos(e/2 + (f*x)/2)^5 - 6*B*d^3*cos(e/2 + (f*x)/2)^3 + 36*B*d
^3*cos(e/2 + (f*x)/2)^5 - 16*B*d^3*cos(e/2 + (f*x)/2)^7 - 9*A*d^3*cos(e/2
+ (f*x)/2)*(e + f*x) - 6*B*c^3*cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*cos(
e/2 + (f*x)/2)*(e + f*x) - 9*A*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*
sin(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A
*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*A*d^3*cos(e/2 + (f*x)/2)
^4*sin(e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) +
 12*B*d^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) - 16*B*d^3*cos(e/2 + (f*
x)/2)^6*sin(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2) - 36*A*c^2*d*co
s(e/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f
*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*cos(e/2 + (f*x)/2)^3
 + 36*B*c^2*d*cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*cos(e/2 + (f*x)/2)^5 + 18*
A*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*cos(e/2 + (f*x)/2)*(e +
f*x) - 27*B*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*cos(e/2 + (f*x
)/2)*(e + f*x) + 18*A*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*sin(
e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*
c^2*d*sin(e/2 + (f*x)/2)*(e + f*x) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e
/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36...
```

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

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3.266.1 Optimal result

Integrand size = 35, antiderivative size = 143

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx\\&=\frac{(2A(2c-d)d+B(2c^2-4cd+3d^2))\,x}{2a}+\frac{2(A(c-d)-B(2c-d))d\cos(e+fx)}{af}\\&\quad+\frac{(2A-3B)d^2\cos(e+fx)\sin(e+fx)}{2af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}\end{aligned}$$

output
```
1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(
f*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c
+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))
```

3.266.2 Mathematica [A] (verified)

Time = 6.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.40

$$\begin{aligned}&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx\\&=\frac{\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\left(\tfrac{1}{2}(e+fx)\right)+2(2A(2c-d)d+B(2c^2-4cd+\right.\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

3.266.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a\sin(e+fx)+a}dx$$

$$\downarrow 3456$$

$$\frac{\int (c+d\sin(e+fx))(a(B(c-2d)+2Ad)-a(2A-3B)d\sin(e+fx))dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{\int (c+d\sin(e+fx))(a(B(c-2d)+2Ad)-a(2A-3B)d\sin(e+fx))dx}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

$$\downarrow 3213$$

$$\frac{\frac{1}{2}ax\left(-\left(d^2(2A-3B)\right)+4Acd+2Bc(c-2d)\right)+\frac{2ad(Ac-Ad-2Bc+Bd)\cos(e+fx)}{f}+\frac{ad^2(2A-3B)\sin(e+fx)\cos(e+fx)}{2f}}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

output
```
-(((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])))
+ ((a*(2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/2 + (2*a*d*(A*c - 2
*B*c - A*d + B*d)*Cos[e + f*x])/f + (a*(2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e
+ f*x])/(2*f))/a^2
```

3.266.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.266.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.35

method	result
derivativedivides	$-\frac{2\left(A\,c^2-2Acd+A\,d^2-B\,c^2+2cdB-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(-A\,d^2-2cdB+d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}-$ $\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2$ af
default	$-\frac{2\left(A\,c^2-2Acd+A\,d^2-B\,c^2+2cdB-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{2}+\left(-A\,d^2-2cdB+d^2B\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{2}-$ $\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2$ af
parallelrisch	$\frac{\left((-2fxA+3fxB-7A+7B)d^2+4c\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)d-4\left(\left(-\frac{fx}{2}-1\right)B+A\right)c^2\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+((-2fxA+3fxB-$
risch	$\frac{2xAcd}{a}-\frac{xA\,d^2}{a}+\frac{xB\,c^2}{a}-\frac{2xcdB}{a}+\frac{3x\,d^2B}{2a}-\frac{d^2\mathrm{e}^{i(fx+e)}A}{2af}-\frac{d\,\mathrm{e}^{i(fx+e)}Bc}{af}+\frac{d^2\mathrm{e}^{i(fx+e)}B}{2af}-\frac{d^2\mathrm{e}^{-i(fx+e}}{2af}$
norman	$\frac{\left(2A\,c^2-4Acd-2B\,c^2-2d^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(2A\,c^2-4Acd+2A\,d^2-2B\,c^2+4cdB-3d^2B\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(6A\,c^2-12Acd+2A\,d^2-$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-(A*c^2-2*A*c*d+A*d^2-B*c^2+2*B*c*d-B*d^2)/(tan(1/2*f*x+1/2*e)+1)+(
1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(-A*d^2-2*B*c*d+B*d^2)*tan(1/2*f*x+1/2*e)^2
-1/2*B*tan(1/2*f*x+1/2*e)*d^2-A*d^2-2*c*d*B+d^2*B)/(1+tan(1/2*f*x+1/2*e)^2
)^2+1/2*(4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+3*B*d^2)*arctan(tan(1/2*f*x+1/2*e
)))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(139) = 278$.

Time $= 0.26$ (sec) , antiderivative size $= 303$, normalized size of antiderivative $= 2.12$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{Bd^2\cos(fx+e)^3-2(A-B)c^2+4(A-B)cd-2(A-B)d^2+(2Bc^2+4(A-B)cd-(2A-3B)d^2)}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2
+ (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d
^2)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 -
(2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos
(f*x + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4
*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x +
e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.266.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5763 vs. $2(117) = 234$.

Time = 2.04 (sec) , antiderivative size = 5763, normalized size of antiderivative = 40.30

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-4*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*
f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c**2*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c*
*2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
+ 4*A*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e
/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*
x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/
2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 +
f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(
e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a
*f) + 4*A*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**...
```

3.266.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.24

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$

$$= \frac{Bd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-4Bcd\left(\frac{}{a+\frac{a s}{\cos}}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
(B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x
 + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a) - 2*A*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) - 2*A*c^2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.50

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$

$$= \frac{\frac{(2Bc^2+4Acd-4Bcd-2Ad^2+3Bd^2)(fx+e)}{a}-\frac{4(Ac^2-Bc^2-2Acd+2Bcd+Ad^2-Bd^2)}{a(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)}+\frac{2\left(Bd^2\tan(\frac{1}{2}fx+\frac{1}{2}e)^3-4Bcd\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-\right.}{}}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*
c^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e)
+ 1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 -
 2*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*t
an(1/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^
2 + 1)^2*a))/f
```

3.266.9 Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{x\,(2\,B\,c^2-2\,A\,d^2+3\,B\,d^2+4\,A\,c\,d-4\,B\,c\,d)}{2\,a}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,d^2-3\,B\,d^2+4\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,A\,c^2+2\,A\,d^2-2\,B\,c^2-3\,B\,d^2-4\,A\,c\,d+}{f\,\left(a\tan\left(\frac{e}{2}+\right.\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

output
```
(x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (tan(e/2 + (
f*x)/2)^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 +
2*A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + tan(e/2 + (f*x)/2)^2*(4
*A*c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A
*d^2 - 2*B*c^2 - 4*B*d^2 + tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d)
- 4*A*c*d + 8*B*c*d)/(f*(a + a*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)
^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)
/2)^5))
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

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3.267.1 Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(B(c-d)+Ad)x}{a} - \frac{Bd\cos(e+fx)}{af} - \frac{(A-B)(c-d)\cos(e+fx)}{af(1+\sin(e+fx))}$$

output
```
(B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x
+e))
```

3.267.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)((B(c-d)+Ad)(e+fx)-Bd\cos(e+fx))+(2A\right.}{af(1+\sin(e+fx))}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d
)*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*
(-2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*
x]))
```

3.267.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{a\sin(e+fx)+a}\,dx$$

$$\downarrow 3502$$

$$\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\downarrow 3042$$

$$\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\downarrow 3214$$

$$\frac{a(A-B)(c-d)\int \frac{1}{\sin(e+fx)a+a}\,dx + x(Ad+B(c-d))}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\downarrow 3042$$

$$\frac{a(A-B)(c-d)\int \frac{1}{\sin(e+fx)a+a}dx + x(Ad+B(c-d))}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\downarrow 3127$$

$$\frac{x(Ad+B(c-d)) - \frac{a(A-B)(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a} - \frac{Bd\cos(e+fx)}{af}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
-((B*d*Cos[e + f*x])/(a*f)) + ((B*(c - d) + A*d)*x - (a*(A - B)*(c - d)*Co
s[e + f*x])/(f*(a + a*Sin[e + f*x])))/a
```

3.267.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
 + 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.267.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(Ac-dA-Bc+dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(dA+Bc-dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
default	$\frac{-\frac{2(Ac-dA-Bc+dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2dB}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+2(dA+Bc-dB)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
parallelrisch	$\frac{\left((2cfx-2dfx-4c+3d)B+4\left(\left(\frac{fx}{2}-1\right)d+c\right)A\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+((2cfx-2dfx-3d)B+2Adfx)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-dB\left(\cos\left(\frac{3fx}{2}-$
risch	$\frac{xdA}{a}+\frac{xBc}{a}-\frac{xdB}{a}-\frac{Bd\,e^{i(fx+e)}}{2af}-\frac{Bd\,e^{-i(fx+e)}}{2af}-\frac{2Ac}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2dA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2Bc}{fa\left(e^{i(fx+e)}+i\right)}$
norman	$\frac{(dA+Bc-dB)x}{a}+\frac{(dA+Bc-dB)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{(dA+Bc-dB)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(dA+Bc-dB)x\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{2Ac-2dA-2}{af}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/a*(-(A*c-A*d-B*c+B*d)/(tan(1/2*f*x+1/2*e)+1)-d*B/(1+tan(1/2*f*x+1/2*e)
^2)+(A*d+B*c-B*d)*arctan(tan(1/2*f*x+1/2*e)))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{Bd\cos(fx+e)^2-(Bc+(A-B)d)fx+(A-B)c-(A-B)d-((Bc+(A-B)d)fx-(A-B)c-}{af\cos(fx+e)+}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B
*c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A -
 B)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*
cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.267.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1307 vs. $2(49) = 98$.

Time = 1.04 (sec) , antiderivative size = 1307, normalized size of antiderivative = 19.51

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(
e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*t
an(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(
e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(
e/2 + f*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x
/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(
e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
2*A*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*
tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/
2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan
(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*
x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*ta
n(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f
*tan(e/2 + f*x/2) + a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(...
```

3.267.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{2\left(Bd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+}{\cos(fx+e)}}\right.\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/
(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.267.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(67) = 134$.

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$=\frac{\frac{(Bc+Ad-Bd)(fx+e)}{a}-\frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac-Bc\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1
/2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^
2 + B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/
2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

3.267.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{x\,(A\,d+B\,c-B\,d)}{a} - \frac{(2\,A\,c-2\,A\,d-2\,B\,c+2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,B\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,c-2\,A\,d-2\,B\,c+4\,B\,d}{f\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
(x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + tan(e/2 + (f*x)
/2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*tan(e/2 + (f*x)/2))/(f*(a +
a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

3.268.1 Optimal result 2200
3.268.2 Mathematica [B] (verified) 2200
3.268.3 Rubi [A] (verified) 2201
3.268.4 Maple [A] (verified) 2202
3.268.5 Fricas [A] (verification not implemented) 2202
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3.268.7 Maxima [B] (verification not implemented) 2203
3.268.8 Giac [A] (verification not implemented) 2204
3.268.9 Mupad [B] (verification not implemented) 2204

3.268.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{Bx}{a} - \frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

output
```
B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

3.268.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2A+B(-2+e+fx))\sin\left(\frac{1}{2}(e+fx\right.\right.}{af(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*
A + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

3.268.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{a\sin(e+fx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{A + B\sin(e+fx)}{a\sin(e+fx)+a}dx$$
$$\downarrow 3214$$
$$(A-B)\int \frac{1}{\sin(e+fx)a+a}dx + \frac{Bx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int \frac{1}{\sin(e+fx)a+a}dx + \frac{Bx}{a}$$
$$\downarrow 3127$$
$$\frac{Bx}{a} - \frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

output
```
(B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

3.268.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.268.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	si
derivativedivides	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
default	$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{af}$	42
parallelrisch	$\frac{fxB+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(fxB+2A-2B)}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	47
risch	$\frac{Bx}{a}-\frac{2A}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2B}{fa\left(e^{i(fx+e)}+i\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{Bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{Bx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}+\frac{(-2B+2A)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{(-2B+2A)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	13

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f/a*(B*arctan(tan(1/2*f*x+1/2*e))-(A-B)/(tan(1/2*f*x+1/2*e)+1))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{Bfx+(Bfx-A+B)\cos(fx+e)+(Bfx+A-B)\sin(fx+e)-A+B}{af\cos(fx+e)+af\sin(fx+e)+af}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
(B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
 B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.268.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{A + B\sin(e+fx)}{a + a\sin(e+fx)}\,dx = \begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f
*tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*t
an(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True)
)
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(35) = 70$.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \frac{A + B\sin(e+fx)}{a + a\sin(e+fx)}\,dx = \frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right) - \frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{A + B\sin(e+fx)}{a + a\sin(e+fx)}\,dx = \frac{\frac{(fx+e)B}{a} - \frac{2(A-B)}{a(\tan(\frac{1}{2}fx+\frac{1}{2}e)+1)}}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.268.9 Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sin(e+fx)}{a + a\sin(e+fx)}\,dx = \frac{B\,x}{a} - \frac{2\,A - 2\,B}{a\,f\,\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) + 1\right)}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

output
```
(B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

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3.269.2 Mathematica [A] (verified) . . . 2205
3.269.3 Rubi [A] (verified) . . . 2206
3.269.4 Maple [A] (verified) . . . 2208
3.269.5 Fricas [B] (verification not implemented) . . . 2209
3.269.6 Sympy [F(-1)] . . . 2209
3.269.7 Maxima [F(-2)] . . . 2210
3.269.8 Giac [A] (verification not implemented) . . . 2210
3.269.9 Mupad [B] (verification not implemented) . . . 2211

3.269.1 Optimal result

Integrand size = 35, antiderivative size = 101

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

output
```
-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))+2*(-A*d+B*c)*arctan((d+c*tan(1/
2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e-\right.\right.\right.}{\sqrt{c^2-d^2}}$$

$$a(c-d)\sqrt{c^2-d^2}f(1+\sin(e+fx))$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
,x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
 f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Si
n[e + f*x]))
```

3.269.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3457, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 27$$

$$\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{a(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{a(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 3139$$

$$\frac{2(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{af(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 1083$$

$$-\frac{4(Bc-Ad)\int \frac{1}{-(2d+2c\tan(\frac{1}{2}(e+fx)))^2-4(c^2-d^2)}d(2d+2c\tan(\frac{1}{2}(e+fx)))}{af(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\downarrow 217$$

$$\frac{2(Bc-Ad)\arctan\left(\frac{2c\tan(\frac{1}{2}(e+fx))+2d}{2\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

output
```
(2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(
a*(c - d)*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Si
n[e + f*x]))
```

3.269.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.269.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\dfrac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-dA+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
default	$\dfrac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-dA+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
risch	$-\frac{2A}{f(c-d)a\left(e^{i(fx+e)}+i\right)}+\frac{2B}{f(c-d)a\left(e^{i(fx+e)}+i\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)dA}{\sqrt{-c^2+d^2}\,(c-d)fa}+\frac{\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2}}{\sqrt{-}}\right.}{\sqrt{-c^2+d^2}\,(}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/a*(-(A-B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)+1/(c-d)*(-A*d+B*c)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(96) = 192$.

Time = 0.28 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.89

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))}\,dx$$

$$= \left[-\frac{2\,(A-B)c^2 - 2\,(A-B)d^2 + (Bc - Ad + (Bc - Ad)\cos(fx+e) + (Bc - Ad)\sin(fx+e))\sqrt{-c^2 + \right.$$

$$\left. -\frac{(A-B)c^2 - (A-B)d^2 + (Bc - Ad + (Bc - Ad)\cos(fx+e) + (Bc - Ad)\sin(fx+e))\sqrt{c^2 - d^2}\,\text{arc}}{(ac^3 - ac^2d - acd^2 + ad^3)f\cos(fx+e) + (ac^3 - ac^2d} \right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x +
e) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
 + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e)
+ d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)
*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*c
os(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3
- a*c^2*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d
+ (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 -
 (A - B)*d^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a
*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*
d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}} - \frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A -
 B)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.269.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\frac{\frac{(A\,d-B\,c)\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,(c-d)^{3/2}} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d-B\,c)\,(a\,c-a\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}}{2\,A\,d-2\,B\,c}\right)(A\,d-B\,c)}{a\,f\,\sqrt{c+d}\,(c-d)^{3/2}} - \frac{2\,(A-B)}{f\,\left(a+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(c-d)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

output
```
(2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2))
 - (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c -
d)^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
 - (2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

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3.270.1 Optimal result

Integrand size = 35, antiderivative size = 181

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{3/2}f} \\
&\quad + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)
^(1/2))/a/(c-d)/(c^2-d^2)^(3/2)/f+d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-
d)^2/(c+d)/f/(c+d*sin(f*x+e))-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c
+d*sin(f*x+e))
```

3.270.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2\left(-Ad(2c+d)+B\left(c^2+cd+d^2\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2-d}}\right.}{(c+d)\sqrt{c^2-d}}\right.}{a(c-d)^2 f(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-
(A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d
^2]) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/
((c + d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

3.270.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3457, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 3233$$

$$-\frac{-\frac{\int -\frac{a\left(Ad(2c+d)-B\left(c^2+dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{a\left(Ad(2c+d)-B\left(c^2+dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$-\frac{\frac{a(Ad(2c+d)-B(c^2+cd+d^2))\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\frac{a(Ad(2c+d)-B(c^2+cd+d^2))\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 3139$$

$$-\frac{\frac{2a(Ad(2c+d)-B(c^2+cd+d^2))\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c^2-d^2)}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\downarrow 1083$$

$$-\frac{\frac{4a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{\frac{a^2(c-d)}{(A-B)\cos(e+fx)}}-$$

$$\frac{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}{\downarrow\ 217}$$

$$-\frac{\frac{2a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{\frac{a^2(c-d)}{(A-B)\cos(e+fx)}}-$$

$$f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

output
```
-(((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*
x]))) - ((2*a*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(2*d + 2*c*Tan[
(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*d*(B*(2*c +
 d) - A*(c + 2*d))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(a^
2*(c - d))
```

3.270.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.270.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(2Acd+Ad^2-Bc^2-cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{(c-d)^2}}{af}$
default	$-\frac{2(A-B)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(2Acd+Ad^2-Bc^2-cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}{2\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{(c-d)^2}}{af}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-(A-B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)-1/(c-d)^2*((d^2*(A*d-B*c)/(c+
d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan
(1/2*f*x+1/2*e)+c)+(2*A*c*d+A*d^2-B*c^2-B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)
*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(176) = 352$.

Time $= 0.31$ (sec) , antiderivative size $= 1538$, normalized size of antiderivative $= 8.50$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output

```
[1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3
*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 +
 (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (
2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d -
(A - B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2
 - (A - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*
sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*((A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*c
os(f*x + e) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B
)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e
))*sin(f*x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d
^5 - a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^
3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d
^4 - a*d^6)*f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^
5 - a*d^6)*f*cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)
*sin(f*x + e)), ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B
)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e
)^2 + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c...
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output

```
Timed out
```

3.270.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(176) = 352.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.35

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$
$$= 2\left(\frac{(Bc^2-2\,Acd+Bcd-Ad^2+Bd^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}} - \frac{Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+Ac^2}{}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^
3 - a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2
*e)^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*
c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(
1/2*f*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*
x + 1/2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*
e) + A*d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*
c*d^2)/((a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3
+ c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x +
1/2*e) + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.270.9 Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{2\,\mathrm{atan}\left(\frac{\frac{\left(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4\right)\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,(c+d)^{3/2}\,(c-d)^{5/2}}+\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3\right)\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\right.}{a\,(c+d)^{3/2}\,(c-d)^{5/2}}}{2\,B\,c^2-2\,A\,d^2+2\,B\,d^2-4\,A\,c\,d+2\,B\,c\,d}\right.}{a\,f\,(c+d)^{3/2}\,(c-d)^{5/2}}$$

$$- \frac{\frac{2\,(A\,c^2+A\,d^2-B\,c^2+A\,c\,d-2\,B\,c\,d)}{(c+d)\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d^2+2\,A\,c\,d-3\,B\,c\,d)}{c\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(A\,c^3+A\,d^3-B\,c^3+A\,c^2\,d-B\,c\,d^2-B\,c^2\,d}{c\,(c+d)\,(c-d)^2}}{f\,\left(a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

output
```
(2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
 B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2
+ (f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*
A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*
d^2 - 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*
(c + d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*
d))/((c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d
))/(c*(c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*
d - B*c*d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2
)^2*(a*c + 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)
/2)^3))
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

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3.271.1 Optimal result

Integrand size = 35, antiderivative size = 283

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{5/2}f} \\
&\quad -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} \\
&\quad -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-(3*A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan
(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*A*c+3
*A*d-3*B*c-2*B*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2-(A-B)*co
s(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*
d+4*A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin
(f*x+e))
```

3.271.2 Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2\left(-3Ad\left(2c^2+2cd+d^2\right)+B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)}{(c+}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-
3*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTa
n[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) +
 (d*(-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c
- d)^3*f*(1 + Sin[e + f*x]))
```

3.271.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3457, 3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$-\frac{\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}-\frac{\int -\frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{\int \frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$-\frac{\frac{-\frac{a\left(3Ad\left(2c^2+2dc+d^2\right)-B\left(2c^3+4dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+}{2f(c^2-d^2)(c+d\sin(e+fx))^2}$$

$$\frac{a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$-\frac{\frac{\frac{a\left(3Ad\left(2c^2+2dc+d^2\right)-B\left(2c^3+4dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2B}{2f(c^2-d^2)(c+d\sin(}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{\frac{a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2B}{2f(c^2-d^2)(c+d\sin(}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 3139$$

$$-\frac{\frac{\frac{2a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2B}{2f(c^2-d^2)(c+d\sin(}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 1083$$

$$-\frac{\frac{\frac{4a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2B}{2f(c^2-d^2)(c+d\sin(}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\downarrow 217$$

$$-\frac{\frac{2a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3A}{2f(c^2-}$$
$$\frac{a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

output
```
-(((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2)) - ((a*d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*(c^2 - d^
2)*f*(c + d*Sin[e + f*x])^2) + ((2*a*(3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c
^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sq
rt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*d*(B*(5*c^2 + 6*c*d + 4*d^2) -
 A*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*
x])))/(2*(c^2 - d^2)))/(a^2*(c - d))
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.271.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{2\left(\frac{d^2\left(7c^2dA+2d^2cA-2Ad^3-5Bc^3-2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6Ac^4d+2Ac^3d^2+11Ac^2d^3+4Acd^4-2Ad^5-4Bc^5-2Bc^4d-9\right.}{2\left(c^2+2cd+d^2\right)c^2}\right.}{((}$
default	$-\frac{2\left(\frac{d^2\left(7c^2dA+2d^2cA-2Ad^3-5Bc^3-2c^2dB\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6Ac^4d+2Ac^3d^2+11Ac^2d^3+4Acd^4-2Ad^5-4Bc^5-2Bc^4d-9\right.}{2\left(c^2+2cd+d^2\right)c^2}\right.}{((}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-1/(c-d)^3*((1/2*d^2*(7*A*c^2*d+2*A*c*d^2-2*A*d^3-5*B*c^3-2*B*c^2*d
)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*A*c^4*d+2*A*c^3*d^2+11*A
*c^2*d^3+4*A*c*d^4-2*A*d^5-4*B*c^5-2*B*c^4*d-9*B*c^3*d^2-4*B*c^2*d^3-2*B*c
*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(17*A*c^2*d+6*A*c*d
^2-2*A*d^3-11*B*c^3-6*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)+1/2*d*(6*A*c^2*d+2*A*c*d^2-A*d^3-4*B*c^3-2*B*c^2*d-B*c*d^2)/(c^2+2*c*d
+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(6*A*c^2*d+
6*A*c*d^2+3*A*d^3-2*B*c^3-4*B*c^2*d-7*B*c*d^2-2*B*d^3)/(c^2+2*c*d+d^2)/(c^
2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-(A-
B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1))
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(274) = 548$.

Time $= 0.37$ (sec) , antiderivative size $= 3303$, normalized size of antiderivative $= 11.67$

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output

```
[1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*
d^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 -
 3*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*
d + 4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 -
(2*A - B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*
B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c
*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B
)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3
*d^2 - 18*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x
 + e)^2 + (2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A
- 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*
c^5 - 2*(3*A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3
- (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d
- 2*(3*A - 2*B)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x
 + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f
*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
- d^2)) + 2*(2*(A - B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*
A + B)*c^3*d^3 - (7*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)...
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output

```
Timed out
```

3.271.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(274) = 548.

Time = 0.38 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.57

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{(2Bc^3-6Ac^2d+4Bc^2d-6Acd^2+7Bcd^2-3Ad^3+2Bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5)\sqrt{c^2-d^2}} - \frac{2(A-B)}{(ac^3-3ac^2d+3acd^2-ad^3)(t}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output

```
((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*
d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/
2*e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3
+ a*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*
c*d^2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/
2*e)^3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/
2*e)^3 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)
^2 + 2*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)
^2 + 9*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e
)^2 + 4*B*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^
2 + 2*B*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11
*B*c^4*d^2*tan(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*
c^3*d^3*tan(1/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*
d^4*tan(1/2*f*x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*
A*c^4*d^2 + 2*B*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a
*c^6*d - 2*a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.271.9 Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 1076, normalized size of antiderivative = 3.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{A\,d^4-2\,A\,c^4+2\,B\,c^4-8\,A\,c^2\,d^2+4\,B\,c^2\,d^2-2\,A\,c\,d^3-4\,A\,c^3\,d+B\,c\,d^3+8\,B\,c^3\,d}{(c+d)\,(c^2-d^2)\,(c^2-2\,c\,d+d^2)} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,d^6-13\,A\,c^2\,d^4-17\,A\,c^3\,d^3-22\,A\,c^4\,d^2+4\,B}{c^2\,(c^2-2\,c\,d+d^2)}$$

$$-\operatorname{atan}\left(\frac{\frac{\left(-2\,a\,c^5\,d+2\,a\,c^4\,d^2+4\,a\,c^3\,d^3-4\,a\,c^2\,d^4-2\,a\,c\,d^5+2\,a\,d^6\right)\left(2\,B\,c^3-3\,A\,d^3+2\,B\,d^3-6\,A\,c\,d^2-6\,A\,c^2\,d+7\,B\,c\,d^2+4\,B\,c^2\,d\right)}{2\,a\,(c+d)^{5/2}\,(c-d)^{7/2}} - \frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a\,c^5-a\right.}{2\,B\,c^3-3\,A\,d^3+2\,B\,d^3-6\,A\,c\,d^2-6\,A\,c^2\,d+7\,B\,c\,d^2}\right.$$

input

```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

output

```
((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*
c^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (
tan(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2
 + 4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B
*c*d^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)
) + (tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14
*A*c^3*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d
^4 + 8*B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/
2)^4*(2*A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^
3 - 7*B*c^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^
2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*
d^3 - 22*A*c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d +
 4*B*c*d^4 + 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*
(tan(e/2 + (f*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3
*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c
*d) + tan(e/2 + (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)
) - (atan((((2*a*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5
 - 2*a*c^5*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c
*d^2 + 4*B*c^2*d))/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/
2)*(a*c^5 - a*d^5 + 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B...
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

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3.272.1 Optimal result

Integrand size = 35, antiderivative size = 228

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2} \\
&\quad + \frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f} \\
&\quad + \frac{d^2(B(4c-21d)+2A(c+6d))\cos(e+fx)\sin(e+fx)}{6a^2f} \\
&\quad - \frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2}
\end{aligned}$$

output

```
1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5
*d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(
c+6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)
*(c+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x
+e))^3/f/(a+a*sin(f*x+e))^2
```

3.272.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 547 vs. $2(228) = 456$.

Time = 2.41 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.40

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) + \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(8Ad(6c^2+d^2(5-6e-6fx)+3cd(-4+3e+3fx))+B(16c^3+$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6
*f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x
) - 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*
x)/2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 +
 3*e + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41
+ 12*e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(
d^2*(12*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x)
)/2] + 2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B
*c*d^2 + 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e
- 32*A*d^3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f
*x - 32*A*d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e +
f*x)) + B*(24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f
*x)))*Cos[e + f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d
^3*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

3.272.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^2}\,dx$$
$$\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^2}dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3Ad-3Bd)-a(2A-5B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3Ad-3Bd)-a(2A-5B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3456$$

$$\frac{\frac{\int (c+d\sin(e+fx))\left(a^2d(9Bc+10Ad-16Bd)-a^2d(B(4c-21d)+2A(c+6d))\sin(e+fx)\right)dx}{a^2} - \frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{f(\sin(e+fx)+1)}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\frac{\int (c+d\sin(e+fx))\left(a^2d(9Bc+10Ad-16Bd)-a^2d(B(4c-21d)+2A(c+6d))\sin(e+fx)\right)dx}{a^2} - \frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{f(\sin(e+fx)+1)}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3213$$

$$\frac{\frac{2a^2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{f}+\frac{3}{2}a^2dx(2Ad(3c-2d)+B(6c^2-12cd+7d^2))+\frac{a^2d^2(2A(c+6d)+B(4c-21d))\sin(e+fx)\cos(e+fx)}{2f}}{a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x
]
```

3.272. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}dx$

output
```
-1/3*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^2) + (-(((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^
2)/(f*(1 + Sin[e + f*x]))) + ((3*a^2*d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*
c*d + 7*d^2))*x)/2 + (2*a^2*d*(A*(c^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d
 + 8*d^2))*Cos[e + f*x])/f + (a^2*d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos
[e + f*x]*Sin[e + f*x])/(2*f))/a^2)/(3*a^2)
```

3.272.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.272.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.49

method	result
derivativedivides	$-\frac{2\left(A\,c^3-3d^2cA+2A\,d^3-3c^2dB+6d^2cB-3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^3+6c^2dA-6d^2cA+2A\,d^3+2B\,c^3-6c^2dB+6d^2cB-2d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A\,c^3-6c^2d$
default	$-\frac{2\left(A\,c^3-3d^2cA+2A\,d^3-3c^2dB+6d^2cB-3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A\,c^3+6c^2dA-6d^2cA+2A\,d^3+2B\,c^3-6c^2dB+6d^2cB-2d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A\,c^3-6c^2d$
parallelrisch	$\left((720fxA-1260fxB+696A-993B)d^3-1080\left(fxA-2fxB+\frac{7}{15}A-\frac{29}{15}B\right)c\,d^2-72c^2(15fxB+A+7B)d+216\left(A-\frac{B}{9}\right)c^3\right)$
risch	$\frac{3x\,d^2Ac}{a^2}-\frac{2x\,d^3A}{a^2}+\frac{3xdB\,c^2}{a^2}-\frac{6x\,d^2cB}{a^2}+\frac{7x\,d^3B}{2a^2}+\frac{id^3B\,\mathrm{e}^{2i(fx+e)}}{8a^2f}-\frac{d^3\mathrm{e}^{i(fx+e)}A}{2a^2f}-\frac{3d^2\mathrm{e}^{i(fx+e)}Bc}{2a^2f}+$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/f/a^2*(-(A*c^3-3*A*c*d^2+2*A*d^3-3*B*c^2*d+6*B*c*d^2-3*B*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c^3+6*A*c^2*d-6*A*c*d^2+2*A*d^3+2*B*c^3-6*B*c^2*d+6*B*c*d^2-2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c^3-6*A*c^2*d+6*A*c*d^2-2*A*d^3-2*B*c^3+6*B*c^2*d-6*B*c*d^2+2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^3+d*((1/2*B*tan(1/2*f*x+1/2*e)^3*d^2+(-A*d^2-3*B*c*d+2*B*d^2)*tan(1/2*f*x+1/2*e)^2-1/2*B*tan(1/2*f*x+1/2*e)*d^2-A*d^2-3*c*d*B+2*d^2*B)/(1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(6*A*c*d-4*A*d^2+6*B*c^2-12*B*c*d+7*B*d^2)*arctan(tan(1/2*f*x+1/2*e))))
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(218) = 436.

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{3\,Bd^3\cos(fx+e)^4-2\,(A-B)c^3+6\,(A-B)c^2d-6\,(A-B)cd^2+2\,(A-B)d^3+6\,(3\,Bcd^2+(A-$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)
*c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6
*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6
*(2*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2
*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A +
B)*c^3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 -
3*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3
*B*d^3*cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2
- 2*(A - B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x
- 3*(6*B*c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d
 + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(
a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e)
+ 2*a^2*f)*sin(f*x + e))
```

3.272.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14612 vs. $2(216) = 432$.

Time = 7.60 (sec) , antiderivative size = 14612, normalized size of antiderivative = 64.09

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f
*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*t
an(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) - 32*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/
2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42
*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*A*c**3*tan
(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2
)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*
a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) - 28*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/
2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)
**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*
c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)*...
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(218) = 436$.

Time $= 0.36$ (sec) , antiderivative size $= 1382$, normalized size of antiderivative $= 6.06$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output

```
1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1
) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x
 + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1)
+ 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + ...
```

3.272.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(218) = 436$.

Time $= 0.32$ (sec) , antiderivative size $= 472$, normalized size of antiderivative $= 2.07$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{3\,(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3)(fx+e)}{a^2} + \frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2 a^2}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/
a^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 -
 2*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*t
an(1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e
)^2 + 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*
x + 1/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x +
 1/2*e)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^
2 + 3*A*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*
d*tan(1/2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(
1/2*f*x + 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x
+ 1/2*e) - 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 1
2*B*c^2*d - 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*
x + 1/2*e) + 1)^3))/f
```

3.272.9 Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.91

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(6Bc^2-4Ad^2+7Bd^2+6Acd-12Bcd)}{7Bd^3-4Ad^3+6Acd^2-12Bcd^2+6Bc^2d}\right)\,(6Bc^2-4Ad^2+7Bd^2+6Acd-12Bcd)}{a^2 f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2Ac^3+16Ad^3+2Bc^3-25Bd^3-18Acd^2+6Ac^2d+48Bcd^2-18Bc^2d)+\frac{4Ac^3}{3}}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x
)
```

output

```
(d*atan((d*tan(e/2 + (f*x)/2)*(6*B*c^2 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*
B*c*d))/(7*B*d^3 - 4*A*d^3 + 6*A*c*d^2 - 12*B*c*d^2 + 6*B*c^2*d))*(6*B*c^2
 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B*c*d))/(a^2*f) - (tan(e/2 + (f*x)/2)*
(2*A*c^3 + 16*A*d^3 + 2*B*c^3 - 25*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 48*B*c
*d^2 - 18*B*c^2*d) + (4*A*c^3)/3 + (20*A*d^3)/3 + (2*B*c^3)/3 - (32*B*d^3)
/3 + tan(e/2 + (f*x)/2)^6*(2*A*c^3 + 4*A*d^3 - 7*B*d^3 - 6*A*c*d^2 + 12*B*
c*d^2 - 6*B*c^2*d) + tan(e/2 + (f*x)/2)^5*(2*A*c^3 + 12*A*d^3 + 2*B*c^3 -
21*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 36*B*c*d^2 - 18*B*c^2*d) + tan(e/2 + (
f*x)/2)^3*(4*A*c^3 + 28*A*d^3 + 4*B*c^3 - 42*B*d^3 - 36*A*c*d^2 + 12*A*c^2
*d + 84*B*c*d^2 - 36*B*c^2*d) + tan(e/2 + (f*x)/2)^4*((16*A*c^3)/3 + (56*A
*d^3)/3 + (2*B*c^3)/3 - (98*B*d^3)/3 - 20*A*c*d^2 + 2*A*c^2*d + 56*B*c*d^2
 - 20*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((14*A*c^3)/3 + (64*A*d^3)/3 + (4*B*
c^3)/3 - (97*B*d^3)/3 - 22*A*c*d^2 + 4*A*c^2*d + 64*B*c*d^2 - 22*B*c^2*d)
- 8*A*c*d^2 + 2*A*c^2*d + 20*B*c*d^2 - 8*B*c^2*d)/(f*(5*a^2*tan(e/2 + (f*x
)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*t
an(e/2 + (f*x)/2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^
7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

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3.273.1 Optimal result

Integrand size = 35, antiderivative size = 132

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2 f} \\
&\quad - \frac{(c-d)(2B(c-3d)+A(c+3d))\cos(e+fx)}{3a^2 f(1+\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}
\end{aligned}$$

output
```
d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c
-3*d)+A*(c+3*d))*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d
*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^2
```

3.273.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(132) = 264$.

Time = 1.33 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.56

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(Ad(4c+d(-4+3e+3fx))+B(2c^2+d^2(5-6e-6fx)+2cd(-\right.}{}
\end{aligned}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x)
) + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e +
 f*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x
)) + 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] +
 3*B*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d
- 6*A*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*
A*d^2*f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1
 + e + f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12
*a^2*f*(1 + Sin[e + f*x])^2)
```

3.273.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3456, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c+2d))-a(A-4B)d\sin(e+fx))}{\sin(e+fx)a+a}\,dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c+2d))-a(A-4B)d\sin(e+fx))}{\sin(e+fx)a+a}\,dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-a(A-4B)d^2\sin^2(e+fx)+(ad(2B(c-d)+A(c+2d))-a(A-4B)cd)\sin(e+fx)+ac(2B(c-d)+A(c+2d))}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-a(A-4B)d^2\sin(e+fx)^2+(ad(2B(c-d)+A(c+2d))-a(A-4B)cd)\sin(e+fx)+ac(2B(c-d)+A(c+2d))}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3502$$

$$\frac{\frac{\int \frac{c(2B(c-d)+A(c+2d))a^2+3d(2B(c-d)+Ad)\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{c(2B(c-d)+A(c+2d))a^2+3d(2B(c-d)+Ad)\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3214$$

$$\frac{\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\int \frac{1}{\sin(e+fx)a+a}dx+3adx(Ad+2B(c-d))}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\int \frac{1}{\sin(e+fx)a+a}dx+3adx(Ad+2B(c-d))}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3127$$

$$\frac{\frac{3adx(Ad+2B(c-d))-\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^2) + (((A - 4*B)*d^2*Cos[e + f*x])/f + (3*a*d*(2*B*(c - d) + A*d)*x - (a^
2*(c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(f*(a + a*Sin[e + f*
x])))/a)/(3*a^2)
```

3.273.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.273.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{2\left(A c^2-A d^2-2cdB+2d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^2+4Acd-2A d^2+2B c^2-4cdB+2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^2-4Acd+2A d^2-2B c^2+4cdB-2d^2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d\left($
default	$\frac{-\frac{2\left(A c^2-A d^2-2cdB+2d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^2+4Acd-2A d^2+2B c^2-4cdB+2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^2-4Acd+2A d^2-2B c^2+4cdB-2d^2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2d\left($
risch	$\frac{x d^2 A}{a^2}+\frac{2xdBc}{a^2}-\frac{2x d^2 B}{a^2}-\frac{B d^2 e^{i(fx+e)}}{2a^2 f}-\frac{B d^2 e^{-i(fx+e)}}{2a^2 f}-\frac{2\left(10cdB+3iA c^2 e^{i(fx+e)}-6A d^2 e^{2i(fx+e)}+9B c$
parallelrisch	$\frac{\left((-18fxA+36fxB-30A+78B)d^2+12((-3fx-5)B+A)cd+18\left(A+\frac{B}{3}\right)c^2\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+((6fxA-12fxB-2A+5B)d$
norman	$\frac{d(dA+2Bc-2dB)x}{a}+\frac{d(dA+2Bc-2dB)x\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4A c^2+4Acd-8A d^2+2B c^2-16cdB+20d^2B}{3af}-\frac{\left(2A c^2-2A d^2-4cdB+4d^2$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output

```
2/f/a^2*(-(A*c^2-A*d^2-2*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c
^2+4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(
2*A*c^2-4*A*c*d+2*A*d^2-2*B*c^2+4*B*c*d-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^3+
d*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+2*B*c-2*B*d)*arctan(tan(1/2*f*x+1/2*
e))))
```

3.273.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(126) = 252$.

Time $= 0.27$ (sec) , antiderivative size $= 375$, normalized size of antiderivative $= 2.84$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx =$$
$$-\frac{3\,Bd^2\cos(fx + e)^3 - (A - B)c^2 + 2\,(A - B)cd - (A - B)d^2 + 6\,(2\,Bcd + (A - 2\,B)d^2)fx - ((A + 2$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
 6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5
*A - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A +
 B)*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)
*f*x)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d
 - (A - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A
 - 5*B)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x
+ e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f -
 (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5358 vs. $2(121) = 242$.

Time $= 4.01$ (sec) , antiderivative size $= 5358$, normalized size of antiderivative $= 40.59$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

output

```
Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9
*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e
/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*
tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) - 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*
f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*
x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) - 12*A*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**
2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)*...
```

3.273.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(126) = 252$.

Time $= 0.32$ (sec) , antiderivative size $= 831$, normalized size of antiderivative $= 6.30$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output

```
-2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
+ 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2
) - 2*B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a
^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)...
```

3.273.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.

Time $= 0.31$ (sec) , antiderivative size $= 264$, normalized size of antiderivative $= 2.00$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx$$

$$= \frac{\frac{3\,(2\,Bcd + Ad^2 - 2\,Bd^2)(fx+e)}{a^2} - \frac{6\,Bd^2}{\left(\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 + 1\right)a^2} - \frac{2\left(3\,Ac^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - 6\,Bcd\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - 3\,Ad^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 + 6\,B\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
 1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*
f*x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/
2*e)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A
*c*d*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/
2*f*x + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d
 - 8*B*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.273.9 Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{2\,d\,\mathrm{atan}\left(\frac{2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d+2\,B\,c-2\,B\,d)}{2\,A\,d^2-4\,B\,d^2+4\,B\,c\,d}\right)\,(A\,d+2\,B\,c-2\,B\,d)}{a^2\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+12\,B\,d^2+4\,A\,c\,d-12\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{10\,A\,c^2}{3}-\frac{14\,A\,d^2}{3}\right.}{f\,(}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^
2 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*
c^2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)
/2)^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d
)/3 - (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)
/3 + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/
2 + (f*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d)
 + (4*A*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(
e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 +
 a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

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3.274.1 Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{B dx}{a^2} - \frac{(Ac+2Bc+2Ad-5Bd)\cos(e+fx)}{3a^2 f(1+\sin(e+fx))} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

output
```
B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*
(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2
```

3.274.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.

Time = 1.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c
+ 2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
+ 3*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 +
Sin[e + f*x])^2)
```

3.274.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3498$$

$$-\frac{\int -\frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}\,dx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3214$$

$$\frac{a(Ac+2Ad+2Bc-5Bd)\int \frac{1}{\sin(e+fx)a+a}dx+3Bdx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{a(Ac+2Ad+2Bc-5Bd)\int \frac{1}{\sin(e+fx)a+a}dx+3Bdx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3127$$

$$\frac{3Bdx - \frac{a(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) + (3*B*d*x
- (a*(A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))
/(3*a^2)
```

3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.274.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{2(Ac-dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2dA+2Bc-2dB}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2dA-2Bc+2dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2 f}$
default	$\frac{-\frac{2(Ac-dB)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2dA+2Bc-2dB}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2dA-2Bc+2dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2 f}$
parallelrisch	$\frac{3Bx\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)df+((9fx+6)dB-6Ac)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+((9dfx-6c+18d)B-6A(c+d))\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+(3dfx-2c+$ $3f a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3$
risch	$\frac{Bdx}{a^2}-\frac{2\left(-Ac-2dA+3iAce^{i(fx+e)}+3ide^{i(fx+e)}A-2Bc+5dB+3iBce^{i(fx+e)}-9iBde^{i(fx+e)}+3Ade^{2i(fx+e)}+3Bce\right.}{3f a^2\left(e^{i(fx+e)}+i\right)^3}$
norman	$\frac{xdB}{a}+\frac{xdB\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{4Ac+2dA+2Bc-8dB}{3af}-\frac{(2Ac-2dB)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(16Ac+2dA+2Bc-20dB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(1}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^2*(-(A*c-B*d)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c+2*A*d+2*B*c-2*B*d)/
(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c-2*A*d-2*B*c+2*B*d)/(tan(1/2*f*x+1/2*e)
+1)^3+d*B*arctan(tan(1/2*f*x+1/2*e)))
```

3.274.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^2}\,dx =$$
$$-\frac{6\,Bdfx - (3\,Bdfx + (A + 2\,B)c + (2\,A - 5\,B)d)\cos(fx + e)^2 - (A - B)c + (A - B)d + (3\,Bdfx - (2}{3\left(a^2 f\cos(fx + e)^2 - a^2 f\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
 - (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x
 + e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2
*A - 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos
(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.274.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(83) = 166$.

Time = 2.11 (sec) , antiderivative size = 1062, normalized size of antiderivative = 12.49

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output

```
Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)*
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*
tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*
x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 +
9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B
*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f
*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
+ 9*B*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**
2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
+ f*x/2) + 3*a**2*f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2
)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**...
```

3.274.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(81) = 162$.

Time $= 0.31$ (sec) , antiderivative size $= 454$, normalized size of antiderivative $= 5.34$

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))}{(a + a\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(Bd\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 4}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right) - \frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(f}{(\cos(fx+}}}{3f}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*
sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 - A*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(
cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3))/f
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(fx+e)Bd}{a^2} - \frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Bd\tan\left(\frac{1}{2}\,\right.\right.}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2
*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e)
+ 3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c +
A*d - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.274.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{B\,d\,x}{a^2}$$
$$-\frac{(2\,A\,c-2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+(2\,A\,c+2\,A\,d+2\,B\,c-6\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,A\,c}{3}+\frac{2\,A\,d}{3}+\frac{2\,B\,c}{3}-\frac{8\,B\,d}{3}}{a^2\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
(B*d*x)/a^2 - ((4*A*c)/3 + (2*A*d)/3 + (2*B*c)/3 - (8*B*d)/3 + tan(e/2 + (
f*x)/2)*(2*A*c + 2*A*d + 2*B*c - 6*B*d) + tan(e/2 + (f*x)/2)^2*(2*A*c - 2*
B*d))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

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3.275.1 Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(A+2B)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

output
```
-1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a
^2*sin(f*x+e))
```

3.275.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)(2A+B+(A+2B)\sin(e+fx))}{3a^2f(1+\sin(e+fx))^2}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
 f*x])^2)
```

3.275.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2}\,dx$$

$$\downarrow 3229$$

$$\frac{(A+2B)\int \frac{1}{\sin(e+fx)a+a}dx}{3a} - \frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{(A+2B)\int \frac{1}{\sin(e+fx)a+a}dx}{3a} - \frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\downarrow 3127$$

$$-\frac{(A+2B)\cos(e+fx)}{3af(a\sin(e+fx)+a)} - \frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e
+ f*x])/(3*a*f*(a + a*Sin[e + f*x]))
```

3.275.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.275.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{-6A\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-6A-6B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-2B}{3fa^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$	60
risch	$-\frac{2\left(-A+3iA\,e^{i(fx+e)}+3iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2B\right)}{3fa^2\left(e^{i(fx+e)}+i\right)^3}$	68
derivativedivides	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(-2B+2A)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2f}$	70
default	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(-2B+2A)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^2f}$	70
norman	$\frac{-\frac{4A+2B}{3af}-\frac{2A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2(5A+B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{(2A+2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2(A+B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{a\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$	139

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-6*A*tan(1/2*f*x+1/2*e)^2+(-6*A-6*B)*tan(1/2*f*x+1/2*e)-4*A-2*B)/f/a^
2/(tan(1/2*f*x+1/2*e)+1)^3
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{(A+2B)\cos(fx+e)^2+(2A+B)\cos(fx+e)+((A+2B)\cos(fx+e)-A+B)\sin(fx+e)+A-}{3\left(a^2f\cos(fx+e)^2-a^2f\cos(fx+e)-2a^2f-(a^2f\cos(fx+e)+2a^2f)\sin(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*
x + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*
x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(56) = 112.

Time = 1.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.72

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e
/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
6*B*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*
x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 +
f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

3.275.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(61) = 122$.

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
B*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3))/f
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2\left(3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 2A + B\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
-2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*
f*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{5A}{2}+\frac{B}{2}-\frac{A\cos(e+fx)}{2}+\frac{B\cos(e+fx)}{2}+\frac{3A\sin(e+fx)}{2}+\frac{3B\sin(e+fx)}{2}\right)}{3a^2 f\left(\frac{3\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}-\frac{\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{2}\right)}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*
x))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)
*cos(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/
2))
```

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

3.276.1 Optimal result . . . 2265
3.276.2 Mathematica [A] (verified) . . . 2265
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3.276.8 Giac [A] (verification not implemented) . . . 2271
3.276.9 Mupad [B] (verification not implemented) . . . 2272

3.276.1 Optimal result

Integrand size = 35, antiderivative size = 152

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2d(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

output
```
-1/3*(A*(c-4*d)+B*(2*c+d))*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*(A-
B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2-2*d*(-A*d+B*c)*arctan((d+c*tan(1/
2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.51

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d)
 + B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 (6*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*
(1 + Sin[e + f*x])^2)
```

3.276.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3457, 25, 3042, 3457, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}\,dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}\,dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}\,dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int \frac{3a^2d(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 27$$

$$\frac{-\frac{3d(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{c-d} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{3d(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{c-d} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 3139$$

$$\frac{-\frac{6d(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c-d)} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 1083$$

$$\frac{\frac{12d(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c-d)} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\downarrow 217$$

$$\frac{-\frac{6d(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f(c-d)\sqrt{c^2-d^2}} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2) + ((-6*d*(B
*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c -
d)*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/((c - d
)*f*(1 + Sin[e + f*x])))/(3*a^2*(c - d))
```

3.276.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.276.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2d(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(-2B+2A)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-2dA+dB)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ $\over a^2 f$
default	$\frac{2d(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^2\sqrt{c^2-d^2}}-\frac{2(-2B+2A)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-2dA+dB)}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$ $\over a^2 f$
risch	$\frac{\frac{2Ac}{3}-\frac{8dA}{3}-2iAc\,e^{i(fx+e)}+6id\,e^{i(fx+e)}A+2Ad\,e^{2i(fx+e)}+\frac{4Bc}{3}+\frac{2dB}{3}-2iBc\,e^{i(fx+e)}-2iBd\,e^{i(fx+e)}-2Bc\,e^{2i(fx+e)}}{\left(e^{i(fx+e)}+i\right)^3(c-d)^2 f\,a^2}$ −

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^2*(d*(A*d-B*c)/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))-1/3*(-2*B+2*A)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3-
1/2*(2*B-2*A)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-2*A*d+B*d)/(c-d)^2/(tan(
1/2*f*x+1/2*e)+1))
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(143) = 286$.

Time = 0.30 (sec) , antiderivative size = 1285, normalized size of antiderivative = 8.45

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))^2(c + d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 +
2*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(
f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*
d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e
))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + 2*((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*
d^3)*cos(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A -
B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2
*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f
*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^
2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 -
 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 -
 (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*
c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A
*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*
B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d
^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*...
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.276.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more de
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2\left(\frac{3\,(Bcd-Ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^2-2a^2cd+a^2d^2)\sqrt{c^2-d^2}}+\frac{3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
-2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x + 1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.276.9 Mupad [B] (verification not implemented)

Time = 14.74 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$$

$$= \frac{2\,d\,\operatorname{atan}\left(\frac{\frac{d\,(A\,d-B\,c)\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}+\frac{2\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d-B\,c)\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}}{2\,A\,d^2-2\,B\,c\,d}\right)(A\,d-B\,c)}{a^2\,f\,\sqrt{c+d}\,(c-d)^{5/2}}$$

$$-\frac{\frac{2\,(2\,A\,c-5\,A\,d+B\,c+2\,B\,d)}{3\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,c-3\,A\,d+B\,c+B\,d)}{(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(A\,c-2\,A\,d+B\,d)}{(c-d)^2}}{f\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

output
```
(2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
 + d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^
2 + a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*
B*c*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*
A*d + B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B
*c + B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d
)^2)/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a
^2*tan(e/2 + (f*x)/2)))
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

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3.277.1 Optimal result

Integrand size = 35, antiderivative size = 275

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx \\
&= \frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}f} \\
&\quad - \frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} \\
&\quad - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-1/3*d*(A*(c^2-6*c*d-10*d^2)+B*(2*c^2+9*c*d+4*d^2))*cos(f*x+e)/a^2/(c-d)^3
/(c+d)/f/(c+d*sin(f*x+e))-1/3*(A*c-6*A*d+2*B*c+3*B*d)*cos(f*x+e)/a^2/(c-d)
^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin
(f*x+e))^2/(c+d*sin(f*x+e))+2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*arctan
((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(
1/2)
```

3.277.2 Mathematica [A] (verified)

Time = 7.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.14

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d)
 + 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
 - (6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(
e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
 + d)*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x])))/(3*a^2*(c - d)
^3*f*(1 + Sin[e + f*x])^2)
```

3.277.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3457, 25, 3042, 3457, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int \frac{2a^2d(3Bc-5Ad+2Bd)-a^2d(Ac+2Bc-6Ad+3Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{-\frac{\int \frac{2a^2d(3Bc-5Ad+2Bd)-a^2d(Ac+2Bc-6Ad+3Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 3233$$

$$\frac{-\frac{a^2d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{\int \frac{3a^2d\left(Ad(3c+2d)-B\left(2c^2+2dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{-\frac{a^2d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{3a^2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+f}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{3a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+f}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\Bigg/3a^2(c-d)$$

$$\downarrow 3139$$

$$-\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{6a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}}{a^2(c-d)}-$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\Bigg/3a^2(c-d)$$

$$\downarrow 1083$$

$$-\frac{\frac{12a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}+\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\Bigg/3a^2(c-d)$$

$$\downarrow 217$$

$$-\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{6a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3B}{f(c-d)(}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\Bigg/3a^2(c-d)$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) + (-(((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos[e + f*x])/((c - d)*f*(1
+ Sin[e + f*x])*(c + d*Sin[e + f*x]))) - ((-6*a^2*d*(A*d*(3*c + 2*d) - B*(
2*c^2 + 2*c*d + d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^
2])])/((c^2 - d^2)^(3/2)*f) + (a^2*d*(A*(c^2 - 6*c*d - 10*d^2) + B*(2*c^2
+ 9*c*d + 4*d^2))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(a^2
*(c - d)))/(3*a^2*(c - d))
```

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.277.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{2d\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(3Acd+2Ad^2-2Bc^2-2cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}}{(c-d)^3}-\frac{2(-2B+}{3(c-d)^2\left(\tan\left(\frac{f}{}\right.\right.}}{a^2f}$
default	$\frac{2d\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(3Acd+2Ad^2-2Bc^2-2cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}}{(c-d)^3}-\frac{2(-2B+}{3(c-d)^2\left(\tan\left(\frac{f}{}\right.\right.}}{a^2f}$
risch	Expression too large to display

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(1/(c-d)^3*d*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c
)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(3*A*c*d+2*A*d^
2-2*B*c^2-2*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/3*(-2*B+2*A)/(c-d)^2/(tan(1/2*f*x+1/2*e)+
1)^3-1/2*(2*B-2*A)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-3*A*d+2*B*d)/(c-d
)^3/(tan(1/2*f*x+1/2*e)+1))
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(264) = 528$.

Time = 0.36 (sec) , antiderivative size = 3123, normalized size of antiderivative = 11.36

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*
d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)
*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)
*cos(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2
 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3
*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^
4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*
B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f
*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A -
B)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*
c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4
)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 -
(2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 -
d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*s
in(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(
A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*
d^5)*cos(f*x + e) - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2
*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A
 - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A -...
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.277.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx =$$

$$-2\left(\frac{3\left(2\,Bc^2d-3\,Acd^2+2\,Bcd^2-2\,Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ad\right.}{\left(a^2c^5-2\,a^2c^4d+2\,a^2c^2d^3-a^2cd^4\right)\left(c\right.}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^
2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2
)) + 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*
d^2 - A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan
(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x
+ 1/2*e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 +
 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*
f*x + 1/2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/(
(a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)
^3))/f
```

3.277.9 Mupad [B] (verification not implemented)

Time = 16.73 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.07

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{2\,d\,\operatorname{atan}\left(\frac{\frac{d\left(-2a^2c^4d+4a^2c^3d^2-4a^2cd^4+2a^2d^5\right)\left(2Bc^2-2Ad^2+Bd^2-3Acd+2Bcd\right)}{a^2(c+d)^{3/2}(c-d)^{7/2}}-\frac{2cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4\right)\left(2B\right.}{a^2(c+d)^{3/2}(c-d)^{7/2}}}{2Bd^3-4Ad^3-6Acd^2+4Bcd^2+4Bc^2d}\right.}{a^2f(c+d)^{3/2}(c-d)^{7/2}}$$

$$-\frac{\frac{2\left(2Ac^3-3Ad^3+Bc^3-8Acd^2-6Ac^2d+8Bcd^2+6Bc^2d\right)}{3(c+d)(c-d)(c^2-2cd+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(5Ac^3-9Ad^3+Bc^3-30Acd^2-11Ac^2d+27Bcd^2+17B\right.}{3c(c-d)(c^2-2cd+d^2)}}{f\left(a^2c+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x
)
```

output

```
(2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B
*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7
/2)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*
c^3*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)
*(c - d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*
(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c -
 d)^(7/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*
c*d^2 + 6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 +
 (f*x)/2)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*
d^2 + 17*B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/
2)^3*(A*c^4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*
A*c^3*d + 7*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2))
+ (2*tan(e/2 + (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B
*c^2*d^2 - 25*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)
*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3
*A*c^2*d^2 + 2*B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c
 - d)*(c^2 - 2*c*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^
2*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*
a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/
2 + (f*x)/2)^5))
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

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3.278.1 Optimal result

Integrand size = 35, antiderivative size = 386

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx \\
&= \frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}f} \\
&\quad - \frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} \\
&\quad - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} \\
&\quad - \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*cos(f*x+e)/a^2/(c
-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*cos(f*x+e)/a^
2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*(A-B)*cos(f*x+e)/(c-d)/f
/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2-3
2*d^3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/
f/(c+d*sin(f*x+e))+d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2
+4*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d
)^2/f/(c^2-d^2)^(1/2)
```

3.278.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1257 vs. $2(386) = 772$.

Time = 10.90 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.26

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)^3),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-48*d*(-(A*d*(12*c^2 + 16*c*d + 7
*d^2)) + B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2
 - d^2] + ((-(A*d*(96*c^4 + 524*c^3*d + 776*c^2*d^2 + 487*c*d^3 + 112*d^4)
) + B*(48*c^5 + 240*c^4*d + 536*c^3*d^2 + 701*c^2*d^3 + 400*c*d^4 + 70*d^5
))*Cos[(e + f*x)/2] - (A*(16*c^5 - 80*c^4*d - 536*c^3*d^2 - 1028*c^2*d^3 -
 695*c*d^4 - 134*d^5) + B*(32*c^5 + 224*c^4*d + 728*c^3*d^2 + 893*c^2*d^3
+ 482*c*d^4 + 98*d^5))*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x
))/2] - 12*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))
/2] - 15*A*c*d^4*Cos[(5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] +
6*A*d^5*Cos[(5*(e + f*x))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*
Cos[(7*(e + f*x))/2] + 8*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos
[(7*(e + f*x))/2] + 59*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*
(e + f*x))/2] + 76*B*c*d^4*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x
))/2] + 34*B*d^5*Cos[(7*(e + f*x))/2] + 48*A*c^5*Sin[(e + f*x)/2] + 48*B*c
^5*Sin[(e + f*x)/2] - 224*A*c^4*d*Sin[(e + f*x)/2] + 416*B*c^4*d*Sin[(e +
f*x)/2] - 872*A*c^3*d^2*Sin[(e + f*x)/2] + 992*B*c^3*d^2*Sin[(e + f*x)/2]
- 1144*A*c^2*d^3*Sin[(e + f*x)/2] + 967*B*c^2*d^3*Sin[(e + f*x)/2] - 685*A
*c*d^4*Sin[(e + f*x)/2] + 496*B*c*d^4*Sin[(e + f*x)/2] - 168*A*d^5*Sin[(e
+ f*x)/2] + 126*B*d^5*Sin[(e + f*x)/2] + 48*B*c^4*d*Sin[(3*(e + f*x))/2...
```

3.278.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3457, 25, 3042, 3457, 3042, 3233, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int \frac{3a^2d(3Bc-7Ad+4Bd)-2a^2d(Ac+2Bc-8Ad+5Bd)\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))^2}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{-\frac{\int \frac{3a^2d(3Bc-7Ad+4Bd)-2a^2d(Ac+2Bc-8Ad+5Bd)\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))^2}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$\frac{-\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2} - \frac{\int \frac{2d\left(Ad(19c+16d)-B\left(9c^2+16dc+10d^2\right)\right)a^2+d\left(A\left(2c^2-16dc-21d^2\right)+B\left(4c^2+19dc+12d^2\right)\right)}{(c+d\sin(e+fx))^2}}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\int \frac{2d\left(Ad(19c+16d)-B\left(9c^2+16dc+10d^2\right)\right)a^2+d\left(A\left(2c^2-16dc-21d^2\right)+B\left(4c^2+19dc+12d^2\right)\right)}{(c+d\sin(e+fx))^2}}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{-\int -\frac{3a^2 d\left(Ad\left(12c^2+16dc+7d^2\right)-B\left(6c^3+12dc^2+13d^2c+4d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2 d\left(A\left(2c^3-16c^2d-59\right.\right.}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{3a^2 d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2 d\left(A\left(2c^3\right.\right.}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{3a^2 d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2 d\left(A\left(2c^3\right.\right.}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 3139$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{6a^2 d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e-\right.}{f\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 1083$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{-\frac{12a^2 d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2 d+13cd^2+4d^3\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-}}{f\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\downarrow 217$$

$$-\frac{\frac{a^2 d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{6a^2 d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2 d+13cd^2+4d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}}{2\left(c^2-d^2\right)}}{a^2(c-d)}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])^2) + (-(((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/((c - d)*f*(
1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2)) - ((a^2*d*(3*d*(3*B*c - 7*A*d +
 4*B*d) + 2*c*(A*c + 2*B*c - 8*A*d + 5*B*d))*Cos[e + f*x])/(2*(c^2 - d^2)*
f*(c + d*Sin[e + f*x])^2) - ((6*a^2*d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(
6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/
(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a^2*d*(A*(2*c^3 - 16*c^2*d
- 59*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f
*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(2*(c^2 - d^2)))/(a^2*(c - d)))
/(3*a^2*(c - d))
```

3.278.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.278.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-\frac{2(-2B+2A)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-4dA+3dB)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+2d\left(\frac{d^2\left(9c^2dA+4d^2cA-2Ad^3-7Bc^3-4c\right.}{2c\left(c^2+2cd+d^2\right.}\right.$
default	$-\frac{2(-2B+2A)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2B-2A}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(Ac-4dA+3dB)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+2d\left(\frac{d^2\left(9c^2dA+4d^2cA-2Ad^3-7Bc^3-4c\right.}{2c\left(c^2+2cd+d^2\right.}\right.$
risch	Expression too large to display

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(-1/3*(-2*B+2*A)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(2*B-2*A)/(c
-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-(A*c-4*A*d+3*B*d)/(c-d)^4/(tan(1/2*f*x+1/2*
e)+1)+d/(c-d)^4*((1/2*d^2*(9*A*c^2*d+4*A*c*d^2-2*A*d^3-7*B*c^3-4*B*c^2*d)/
c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(8*A*c^4*d+4*A*c^3*d^2+15*A*c
^2*d^3+8*A*c*d^4-2*A*d^5-6*B*c^5-4*B*c^4*d-13*B*c^3*d^2-8*B*c^2*d^3-2*B*c*
d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(23*A*c^2*d+12*A*c*d
^2-2*A*d^3-17*B*c^3-12*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)+1/2*d*(8*A*c^2*d+4*A*c*d^2-A*d^3-6*B*c^3-4*B*c^2*d-B*c*d^2)/(c^2+2*c*
d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(12*A*c^2*
d+16*A*c*d^2+7*A*d^3-6*B*c^3-12*B*c^2*d-13*B*c*d^2-4*B*d^3)/(c^2+2*c*d+d^2
)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
))
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2456 vs. $2(373) = 746$.

Time $= 0.45$ (sec) , antiderivative size $= 4997$, normalized size of antiderivative $= 12.95$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output

```
[-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*
c^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A
 - B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)
*c^3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7
)*cos(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A
- B)*c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 8
9*B)*c*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2
*A - 3*B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13
*A + B)*c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A -
 2*B)*d^7)*cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A
 - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A
 - 4*B)*d^6 + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7
*A - 4*B)*d^6)*cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*
(22*A - 19*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x +
e)^3 - (6*B*c^5*d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A
- 92*B)*c^2*d^4 - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2
+ (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14
*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B
*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*
c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*...
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

output

```
Timed out
```

3.278.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(373) = 746$.

Time $= 0.37$ (sec) , antiderivative size $= 911$, normalized size of antiderivative $= 2.36$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
 - 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c
*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2
*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 -
 d^2)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x +
 1/2*e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x +
 1/2*e)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1
/2*e)^2 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1
/2*e)^2 - 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x +
1/2*e)^2 - 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x +
 1/2*e)^2 - 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2
*e)^2 + 2*A*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e)
 - 23*A*c^3*d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) -
 12*A*c^2*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*
A*c*d^6*tan(1/2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4
*A*c^3*d^4 + B*c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2
+ 4*a^2*c^5*d^3 - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*
x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1
/2*e)^2 - 12*A*d*tan(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3
*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2...
```

3.278.9 Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.37

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x
)
```

output

```
(d*atan(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2
*c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 1
6*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^(5/2)*(c
 - d)^(9/2)) + (c*d*tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 +
2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^
3 + 4*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c
+ d)^(5/2)*(c - d)^(9/2)))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^
2 - 16*A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d))*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 1
6*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^(5/2)*(c
 - d)^(9/2)) - ((tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*
c^2*d^4 - 40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*
c^4*d^2 - 4*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c
^4*d - 3*c*d^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*
c^5 - 46*A*c^2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d
^4 - 14*A*c^4*d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2
- 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B
*c^6 - 177*A*c^2*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215
*B*c^3*d^3 + 150*B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^
5*d))/(3*c^2*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*
x)/2)*(6*A*c^5 + 6*A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97...
```

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

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3.279.1 Optimal result

Integrand size = 35, antiderivative size = 225

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx \\
&= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3 f} \\
&\quad - \frac{(c-d)\left(3B(c^2+6cd-15d^2)+A(2c^2+7cd+15d^2)\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)} \\
&\quad - \frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3}
\end{aligned}$$

output

```
d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*cos(f*x+e)/a^
3/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*cos(f*x+e)/
f/(a^3+a^3*sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*cos(f*x+e)*(c+d*sin(
f*x+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f
/(a+a*sin(f*x+e))^3
```

3.279.2 Mathematica [A] (verified)

Time = 6.90 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^3\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^3\left(\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)
^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*
(2*c^2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^5))/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

3.279.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3456, 3042, 3456, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^3}dx$$
$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad-3Bd)-a(A-6B)d\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad-3Bd)-a(A-6B)d\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2d(3B(c-9d)+A(2c+7d))\sin(e+fx)\right)}{\sin(e+fx)a+a}dx}{3a^2} - \frac{a(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2d(3B(c-9d)+A(2c+7d))\sin(e+fx)\right)}{\sin(e+fx)a+a}dx}{3a^2} - \frac{a(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-d^2(3B(c-9d)+A(2c+7d))\sin^2(e+fx)a^2+c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^2+\left(a^2d\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2cd(3B(c-9d)+A(2c+\right.}{\sin(e+fx)a+a}}{3a^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-d^2(3B(c-9d)+A(2c+7d))\sin(e+fx)^2a^2+c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^2+\left(a^2d\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2cd(3B(c-9d)+A(2c+\right.}{\sin(e+fx)a+a}}{3a^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3502$$

$$\frac{\frac{\int \frac{c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^3+15d^2(3B(c-d)+Ad)\sin(e+fx)a^3}{\sin(e+fx)a+a}dx}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2} - \frac{a(2A(c+2d)+3B(c-3d))\cos(}{3f(a\sin(e+f}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^{\frac{5a^2}{3}}}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^3+15d^2(3B(c-d)+Ad)\sin(e+fx)a^3}{\sin(e+fx)a+a}dx}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2} - \frac{a(2A(c+2d)+3B(c-3d))\cos(}{3f(a\sin(e+f}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^{\frac{5a^2}{3}}}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3214$$

$$\frac{\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\int \frac{1}{\sin(e+fx)a+a}dx+15a^2d^2x(Ad+3B(c-d))}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2} - \frac{a(2A(c+2d)+3B(}{3}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^{\frac{5a^2}{3}}}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\int \frac{1}{\sin(e+fx)a+a}dx+15a^2d^2x(Ad+3B(c-d))}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2} - \frac{a(2A(c+2d)+3B(}{3}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^{\frac{5a^2}{3}}}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3127$$

$$\frac{\frac{15a^2d^2x(Ad+3B(c-d))-\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2} - \frac{a(2A(c+2d)+3B(c-3d))}{3f(a\sin($$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^{\frac{5a^2}{3}}}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x
]
```

3.279. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}dx$

output
```
-1/5*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^3) + (-1/3*(a*(3*B*(c - 3*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^2)/(f*(a + a*Sin[e + f*x])^2) + ((a*d^2*(3*B*(c - 9*d) + A*(2*c + 7
*d))*Cos[e + f*x])/f + (15*a^2*d^2*(3*B*(c - d) + A*d)*x - (a^3*(c - d)*(3
*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*c*d + 15*d^2))*Cos[e + f*x])/(f*(
a + a*Sin[e + f*x])))/a)/(3*a^2))/(5*a^2)
```

3.279.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.279.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.55

method	result
derivativedivides	$-\frac{2\left(A c^3-A d^3-3d^2cB+3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A c^3+6c^2dA-2A d^3+2B c^3-6d^2cB+4d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A c^3-18c^2dA+12d^2cA-2A d^3-6B c^3+12c^2d\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
default	$-\frac{2\left(A c^3-A d^3-3d^2cB+3d^3B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A c^3+6c^2dA-2A d^3+2B c^3-6d^2cB+4d^3B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A c^3-18c^2dA+12d^2cA-2A d^3-6B c^3+12c^2d\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
parallelrisch	$\frac{\left((-300fxA+900fxB-540A+1755B)d^3+180c(-5fxB+A-9B)d^2+180c^2d(A+B)+180c^3\left(A+\frac{B}{3}\right)\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left((3\right.}{}$
risch	$\frac{x d^3A}{a^3}+\frac{3x d^2Bc}{a^3}-\frac{3x d^3B}{a^3}-\frac{B d^3e^{i(fx+e)}}{2a^3f}-\frac{B d^3e^{-i(fx+e)}}{2a^3f}-\frac{2\left(9c^2dA+21d^2cA+21c^2dB-96d^2cB+72d^3B+\right.}{}$
norman	Expression too large to display

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output

```
2/f/a^3*(-(A*c^3-A*d^3-3*B*c*d^2+3*B*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A
*c^3+6*A*c^2*d-2*A*d^3+2*B*c^3-6*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2
-1/3*(8*A*c^3-18*A*c^2*d+12*A*c*d^2-2*A*d^3-6*B*c^3+12*B*c^2*d-6*B*c*d^2)/
(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*c^3+24*A*c^2*d-24*A*c*d^2+8*A*d^3+8*B*c
^3-24*B*c^2*d+24*B*c*d^2-8*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*c^3-12
*A*c^2*d+12*A*c*d^2-4*A*d^3-4*B*c^3+12*B*c^2*d-12*B*c*d^2+4*B*d^3)/(tan(1/
2*f*x+1/2*e)+1)^5+d^2*(-d*B/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+3*B*c-3*B*d)*arc
tan(tan(1/2*f*x+1/2*e))))
```

3.279.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(217) = 434$.

Time $= 0.27$ (sec) , antiderivative size $= 649$, normalized size of antiderivative $= 2.88$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{15\,Bd^3\cos(fx+e)^4 - 3\,(A-B)c^3 + 9\,(A-B)c^2d - 9\,(A-B)cd^2 + 3\,(A-B)d^3 + ((2\,A+3\,B)c^3 +}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A -
B)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A
 - 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*
cos(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 +
 3*(6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^
2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B
)*c^2*d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B
)*d^3)*f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A
 - B)*c^2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*
d^3)*f*x - ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 -
 2*(16*A - 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2
- 3*((2*A + 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A
- 62*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x +
e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
- 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*
x + e))
```

3.279.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11456 vs. $2(206) = 412$.

Time $= 14.03$ (sec) , antiderivative size $= 11456$, normalized size of antiderivative $= 50.92$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3
*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e
/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 100*A*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a
**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*ta
n(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 +
f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 40*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 +
 f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2...
```

3.279.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(217) = 434$.

Time $= 0.34$ (sec) , antiderivative size $= 1682$, normalized size of antiderivative $= 7.48$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output

```
-2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*
x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 +
 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos
(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) ...
```

3.279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(217) = 434$.

Time $= 0.32$ (sec) , antiderivative size $= 567$, normalized size of antiderivative $= 2.52$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{30\,Bd^3}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^3} - \frac{15\left(3\,Bcd^2+Ad^3-3\,Bd^3\right)(fx+e)}{a^3} + \frac{2\left(15\,Ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45\,Bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-15\,Ad^3\tan\left(\frac{1}{2}fx+\right.\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output

```
-1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
 - 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^
2*tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(
1/2*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*
x + 1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x
 + 1/2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/
2*e)^3 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 +
 60*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 -
145*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A
*c^3*tan(1/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan
(1/2*f*x + 1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f
*x + 1/2*e) - 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/
2*e) + 240*B*d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*
B*c^2*d + 6*A*c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x
+ 1/2*e) + 1)^5))/f
```

3.279.9 Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.64

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d+3\,B\,c-3\,B\,d)}{2\,A\,d^3-6\,B\,d^3+6\,B\,c\,d^2}\right)\,(A\,d+3\,B\,c-3\,B\,d)}{a^3\,f}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(4\,A\,c^3-10\,A\,d^3+2\,B\,c^3+30\,B\,d^3+6\,A\,c^2\,d-30\,B\,c\,d^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,A\,c^3}{3}-\frac{38\,A\,d}{3}\right.}{}$$

input

```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x
)
```

output

```
(2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*
B*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5
*(4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(
e/2 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^
2 + 6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 +
(2*B*c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^
3)/3 + 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d
) + tan(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3
 + 4*A*c*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2
*((94*A*c^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*
d^2)/5 + (36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f
*x)/2)^6*(2*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*
c^2*d)/5 - (44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2
 + 15*a^3*tan(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(
e/2 + (f*x)/2)^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 +
 a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

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3.280.1 Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx \\
&= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} \\
&\quad - \frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3}
\end{aligned}$$

output

```
B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*cos(f*x+e)/a/f/(a+a*sin(f*x
+e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*cos(f*x+e)/f/(
a^3+a^3*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x
+e))^3
```

3.280.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 514 vs. $2(164) = 328$.

Time $= 0.81$ (sec) , antiderivative size $= 514$, normalized size of antiderivative $= 3.13$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) + \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30(2Ad(c+d) + B(c^2+4cd+d^2(-9+5e+5fx)))\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
 + d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2
) + B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] -
 15*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*
A*c^2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x
)/2] + 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*
Sin[(e + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f
*x)/2] + 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 9
0*B*d^2*Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*
f*x*Sin[(3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(
e + f*x))/2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/
2] - 14*A*d^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*
d^2*e*Sin[(5*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f
*(1 + Sin[e + f*x])^3)
```

3.280.3 Rubi [A] (verified)

Time $= 1.02$ (sec) , antiderivative size $= 174$, normalized size of antiderivative $= 1.06$, number of steps used $= 11$, number of rules used $= 11$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used $= \{3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A(c+d))+5aBd\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A(c+d))+5aBd\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3447$$

$$\frac{\int \frac{5aBd^2\sin^2(e+fx)+(5aBcd+a(B(3c-2d)+2A(c+d))d)\sin(e+fx)+ac(B(3c-2d)+2A(c+d))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{5aBd^2\sin(e+fx)^2+(5aBcd+a(B(3c-2d)+2A(c+d))d)\sin(e+fx)+ac(B(3c-2d)+2A(c+d))}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3498$$

$$\frac{-\frac{\int -\frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2} - \frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2} - \frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2} - \frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3214$$

$$\frac{\frac{a^2(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\int\frac{1}{\sin(e+fx)a+a}dx+15aBd^2x}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\int\frac{1}{\sin(e+fx)a+a}dx+15aBd^2x}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3127$$

$$\frac{\frac{15aBd^2x-\frac{a^2\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
-1/5*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^3) + (-1/3*(a*(c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(f*(a +
 a*Sin[e + f*x])^2) + (15*a*B*d^2*x - (a^2*(B*(3*c^2 + 14*c*d - 29*d^2) +
2*A*(c^2 + 3*c*d + 2*d^2))*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))/(3*a^2)
)/(5*a^2)
```

3.280.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
 + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.280.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.46

method	result
parallelrisch	$\frac{15Bx\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2f+((75fx+30)Bd^2-30Ac^2)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left((150fx+150)Bd^2-60Acd-60c^2\left(A+\frac{B}{2}\right)\right)(t}{}$
derivativedivides	$\frac{-\frac{2\left(Ac^2-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac^2+4Acd+2Bc^2-2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8Ac^2+16Acd-8Ad^2+8Bc^2-16cdB+8d^2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4Ac^2-8Acd+4Ad^2-4Bc^2+8\right.}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}}{a^3f}$
default	$\frac{-\frac{2\left(Ac^2-d^2B\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac^2+4Acd+2Bc^2-2d^2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-8Ac^2+16Acd-8Ad^2+8Bc^2-16cdB+8d^2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4Ac^2-8Acd+4Ad^2-4Bc^2+8\right.}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}}{a^3f}$
risch	$\frac{Bd^2x}{a^3}-\frac{2\left(14cdB+2Ac^2+3Bc^2-32d^2B+30iAd^2e^{3i(fx+e)}+15iBc^2e^{3i(fx+e)}+30Bcde^{4i(fx+e)}-135iBd^2e^{3i(fx+e)}\right.}{}$
norman	$\frac{xd^2B}{a}+\frac{xd^2B\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a}-\frac{14Ac^2+12Acd+4Ad^2+6Bc^2+8cdB-44d^2B}{15fa}-\frac{\left(2Ac^2-2d^2B\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{\left(4Ac^2+4Acd\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/15*(15*B*x*tan(1/2*f*x+1/2*e)^5*d^2*f+((75*f*x+30)*B*d^2-30*A*c^2)*tan(1
/2*f*x+1/2*e)^4+((150*f*x+150)*B*d^2-60*A*c*d-60*c^2*(A+1/2*B))*tan(1/2*f*
x+1/2*e)^3+((150*B*f*x-40*A+290*B)*d^2-60*(A+4/3*B)*c*d-80*c^2*(A+3/8*B))*
tan(1/2*f*x+1/2*e)^2+((75*B*f*x-20*A+190*B)*d^2-60*c*(A+2/3*B)*d-40*c^2*(A
+3/4*B))*tan(1/2*f*x+1/2*e)+(15*B*f*x-4*A+44*B)*d^2-12*c*(A+2/3*B)*d-14*c^
2*(A+3/7*B))/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5
```

3.280.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(158) = 316.

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{60\,Bd^2fx-(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2)\cos(fx+e)^3-3\,(A-B}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d
- (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A
- B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B
)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*
c*d - 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A
 - B)*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B
)*c*d + (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c
^2 - 2*(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^
3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(151) = 302$.

Time $= 7.85$ (sec) , antiderivative size $= 3468$, normalized size of antiderivative $= 21.15$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*
*2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)
**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a
**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 14*A*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15
*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 60*A*c*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*...
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(158) = 316$.

Time $= 0.32$ (sec) , antiderivative size $= 1132$, normalized size of antiderivative $= 6.90$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output

```
2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*
a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*...
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(158) = 316$.

Time $= 0.31$ (sec) , antiderivative size $= 362$, normalized size of antiderivative $= 2.21$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx$$

$$= \frac{\frac{15\,(fx+e)Bd^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d
^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f
*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1
/2*e)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^
2 + 20*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*ta
n(1/2*f*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x
+ 1/2*e) - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4
*B*c*d + 2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.280.9 Mupad [B] (verification not implemented)

Time = 16.84 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.74

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{B\,d^2\,x}{a^3}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{16\,A\,c^2}{3}+\frac{8\,A\,d^2}{3}+2\,B\,c^2-\frac{58\,B\,d^2}{3}+4\,A\,c\,d+\frac{16\,B\,c\,d}{3}\right)+\frac{14\,A\,c^2}{15}+\frac{4\,A\,d^2}{15}+\frac{2\,B\,c^2}{5}-\frac{44\,B\,d^2}{15}+\mathrm{t}}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x
)
```

output
```
(B*d^2*x)/a^3 - (tan(e/2 + (f*x)/2)^2*((16*A*c^2)/3 + (8*A*d^2)/3 + 2*B*c^
2 - (58*B*d^2)/3 + 4*A*c*d + (16*B*c*d)/3) + (14*A*c^2)/15 + (4*A*d^2)/15
+ (2*B*c^2)/5 - (44*B*d^2)/15 + tan(e/2 + (f*x)/2)^3*(4*A*c^2 + 2*B*c^2 -
10*B*d^2 + 4*A*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*B*d^2) + tan(e/2 +
 (f*x)/2)*((8*A*c^2)/3 + (4*A*d^2)/3 + 2*B*c^2 - (38*B*d^2)/3 + 4*A*c*d +
(8*B*c*d)/3) + (4*A*c*d)/5 + (8*B*c*d)/15)/(f*(10*a^3*tan(e/2 + (f*x)/2)^2
 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2
+ (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

3.281.1 Optimal result 2316
3.281.2 Mathematica [A] (verified) 2316
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3.281.1 Optimal result

Integrand size = 33, antiderivative size = 127

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2Ac+3Bc+3Ad+7Bd)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8
*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos
(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

3.281.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Bc+3Ad+4Bd)\right.$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*
x)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*
A*c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x
] + (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(
30*a^3*f*(1 + Sin[e + f*x])^3)
```

3.281.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^3}dx$$

$$\downarrow 3498$$

$$-\frac{\int -\frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3229$$

$$\frac{\frac{1}{3}(2Ac+3Ad+3Bc+7Bd)\int \frac{1}{\sin(e+fx)a+a}dx - \frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}(2Ac+3Ad+3Bc+7Bd)\int \frac{1}{\sin(e+fx)a+a}dx - \frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3127$$

$$\frac{-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)} - \frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/5*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) + (-1/3*(a*
(2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) -
 ((2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x]))
)/(5*a^2)
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.281.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{-30A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+((-60c-30d)A-30Bc)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left((-80c-30d)A-30B\left(c+\frac{4d}{3}\right)\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+($ $15fa^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5$
derivativedivides	$\frac{-\frac{2(8Ac-6dA-6Bc+4dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(4Ac-4dA-4Bc+4dB)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+2dA+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8Ac+8dA+8Bc-8dB}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}}{a^3f}$
default	$\frac{-\frac{2(8Ac-6dA-6Bc+4dB)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(4Ac-4dA-4Bc+4dB)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+2dA+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-8Ac+8dA+8Bc-8dB}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}}{a^3f}$
risch	$-\frac{2(3Bc+2Ac+15Bde^{4i(fx+e)}-15iBce^{i(fx+e)}+30iBde^{3i(fx+e)}+15iBce^{3i(fx+e)}-10iAce^{i(fx+e)}-20iBde^{i(fx+e)}-}{15fa^3(e^{i(f}$
norman	$-\frac{14Ac+6dA+6Bc+4dB}{15fa}-\frac{2Ac\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(2Ac+dA+Bc)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{2(34Ac+11dA+11Bc+14dB)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/15*(-30*A*tan(1/2*f*x+1/2*e)^4*c+((-60*c-30*d)*A-30*B*c)*tan(1/2*f*x+1/2
*e)^3+((-80*c-30*d)*A-30*B*(c+4/3*d))*tan(1/2*f*x+1/2*e)^2+((-40*c-30*d)*A
-30*B*(c+2/3*d))*tan(1/2*f*x+1/2*e)+(-14*c-6*d)*A-6*B*(c+2/3*d))/f/a^3/(ta
n(1/2*f*x+1/2*e)+1)^5
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(121) = 242$.

Time = 0.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.13

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))}{(a + a\sin(e+fx))^3}\,dx =$$
$$-\frac{((2A+3B)c + (3A+7B)d)\cos(fx+e)^3 - (2(2A+3B)c + (6A-B)d)\cos(fx+e)^2 - 3(A-B$$
$$15\left(a^3 f\cos(fx+e)\right.$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
 (6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*
c + (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x
 + e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*co
s(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
+ e) - 4*a^3*f)*sin(f*x + e))
```

3.281.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(121) = 242$.

Time = 4.37 (sec) , antiderivative size = 1819, normalized size of antiderivative = 14.32

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))}{(a + a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*
x/2) + 15*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*
c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 30*A*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f
*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 +
f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +...
```

3.281.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(121) = 242$.

Time $= 0.23$ (sec) , antiderivative size $= 733$, normalized size of antiderivative $= 5.77$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*
B*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/
2*f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/
2*e)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*
B*c*tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*
x + 1/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) +
 1)^5)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53Ac}{4}+3Ad+3Bc+\frac{13Bd}{4}-4Ac\cos(e+fx)+\frac{3Ad\cos(e+fx)}{2}+\frac{3Bc\cos(e+fx)}{2}+Bd\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos
(e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e +
f*x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e +
 f*x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*co
s(2*e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/
4 - (5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4))/(15*a^3*f*((
5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 +
 (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

3.282.1 Optimal result . . . 2324
3.282.2 Mathematica [A] (verified) . . . 2324
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3.282.9 Mupad [B] (verification not implemented) . . . 2330

3.282.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-1/5*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A+3*B)*cos(f*x+e)/a/f/(
a+a*sin(f*x+e))^2-1/15*(2*A+3*B)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

3.282.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\left(7A+3B+(6A+9B)\sin(e+fx)+(2A+3B)\sin^2(e+fx)\right)}{15a^3f(1+\sin(e+fx))^3}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Si
n[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

3.282.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3}\,dx$$

$$\downarrow 3229$$

$$\frac{(2A+3B)\int \frac{1}{(\sin(e+fx)a+a)^2}\,dx}{5a} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{(2A+3B)\int \frac{1}{(\sin(e+fx)a+a)^2}\,dx}{5a} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3129$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\sin(e+fx)a+a}\,dx}{3a} - \frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\sin(e+fx)a+a}\,dx}{3a} - \frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\downarrow 3127$$

$$\frac{(2A+3B)\left(-\frac{\cos(e+fx)}{3af(a\sin(e+fx)+a)} - \frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a} - \frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) + ((2*A + 3*B)*(-1/
3*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*a*f*(a + a*Sin
[e + f*x])))/(5*a)
```

3.282.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.282.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{2i\left(20iA\,e^{2i(fx+e)}+15iB\,e^{2i(fx+e)}+15B\,e^{3i(fx+e)}-2iA-10A\,e^{i(fx+e)}-3iB-15B\,e^{i(fx+e)}\right)}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$
parallelrisch	$\frac{-30A\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-60A-30B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-80A-30B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-40A-30B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}{15f\,a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
derivativedivides	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3f}$
default	$\frac{-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{a^3f}$
norman	$\frac{-\frac{14A+6B}{15fa}-\frac{2A\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{(94A+36B)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15fa}-\frac{(20A+12B)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{(22A+6B)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{(8A-}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*I*(20*I*A*exp(2*I*(f*x+e))+15*I*B*exp(2*I*(f*x+e))+15*B*exp(3*I*(f*x
+e))-2*I*A-10*A*exp(I*(f*x+e))-3*I*B-15*B*exp(I*(f*x+e)))/f/a^3/(exp(I*(f*
x+e))+I)^5
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{(2A+3B)\cos(fx+e)^3-2(2A+3B)\cos(fx+e)^2-3(3A+2B)\cos(fx+e)-((2A+3B)\cos}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3fc\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A
+ 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x
+ e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
- 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(87) = 174$.

Time = 2.37 (sec) , antiderivative size = 1015, normalized size of antiderivative = 9.95

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*tan(e/2
 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 40*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(...
```

3.282.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(96) = 192$.

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.79

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(\frac{A\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3B\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e}{(\cos(fx+e}\right.}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(}{(\cos(f}}}{15\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

output
```
-2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\right.}{15\,a^3 f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

output
```
-2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*ta
n(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/
2*e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B
)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.47

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53A}{4}+3B-4A\cos(e+fx)+\frac{3B\cos(e+fx)}{2}+\frac{25A\sin(e+fx)}{2}+\frac{15B\sin(e+fx)}{2}-\frac{9A\cos(2e+2}{4}\right.}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}\cos\left(\frac{5e}{2}-\frac{\pi}{4}+\frac{5fx}{2}\right)}{4}\right.}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((53*A)/4 + 3*B - 4*A*cos(e + f*x) + (3*B*cos(e + f*
x))/2 + (25*A*sin(e + f*x))/2 + (15*B*sin(e + f*x))/2 - (9*A*cos(2*e + 2*f
*x))/4 - (3*B*cos(2*e + 2*f*x))/2 - (5*A*sin(2*e + 2*f*x))/4))/(15*a^3*f*(
(5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4
+ (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

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3.283.1 Optimal result

Integrand size = 35, antiderivative size = 229

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx \\
&= \frac{2d^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} \\
&\quad - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} \\
&\quad - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)}
\end{aligned}$$

output

```
-1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c-7*A*d+3*B*c+2
*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/15*(B*(3*c^2-16*c*d-2*d^
2)+A*(2*c^2-9*c*d+22*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))+2*d^2
*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^3/f
/(c^2-d^2)^(1/2)
```

3.283.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 502 vs. $2(229) = 458$.

Time $= 4.75$ (sec) , antiderivative size $= 502$, normalized size of antiderivative $= 2.19$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) + \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15Bc^2\cos\left(\frac{1}{2}(e+fx)\right) - 15Acd\cos\left(\frac{1}{2}(e+fx)\right) - 75Bcd\cos\left(\frac{1}{2}(e\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c
*d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2
] - 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c
*d*Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*
(e + f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2]
+ 15*B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e
+ f*x)/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d
^2*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f
*x))/2] + 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3
*B*c^2*Sin[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[
(5*(e + f*x))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x
))/2]))/(30*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

3.283.3 Rubi [A] (verified)

Time $= 1.21$ (sec) , antiderivative size $= 258$, normalized size of antiderivative $= 1.13$, number of steps used $= 14$, number of rules used $= 13$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used $= \{3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 27, 3042, 3139, 1083, 217\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)} - \frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)} - \frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)} - \frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int -\frac{15a^3d^2(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}-\frac{5a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}}$$

$$\downarrow 27$$

$$\frac{\frac{15ad^2(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}-\frac{5a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}}$$

$$\downarrow 3042$$

$$\frac{\frac{15ad^2(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}-\frac{5a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}}$$

$$\downarrow 3139$$

$$\frac{\frac{30ad^2(Bc-Ad)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2}{3f(c-d)(a\sin(e}-\frac{5a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}}$$

$$\downarrow 1083$$

$$\frac{-\frac{60ad^2(Bc-Ad)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad}{3f(c-}-\frac{5a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}}$$

$$\downarrow 217$$

3.283. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

$$\frac{\frac{30ad^2(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f(c-d)\sqrt{c^2-d^2}}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3) + (-1/3*(a*
(2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*
x])^2) + ((30*a*d^2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqr
t[c^2 - d^2])])/((c - d)*Sqrt[c^2 - d^2]*f) - (a^2*(B*(3*c^2 - 16*c*d - 2*
d^2) + A*(2*c^2 - 9*c*d + 22*d^2))*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e +
 f*x])))/(3*a^2*(c - d)))/(5*a^2*(c - d))
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.283.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2d^2(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6dA+2Bc-4dB}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3 f}$
default	$\frac{-\frac{2d^2(dA-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6dA+2Bc-4dB}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3 f}$
risch	$-\frac{34Bcd\,e^{2i(fx+e)}}{3}+2Bcd\,e^{4i(fx+e)}+\frac{38iA\,d^2e^{i(fx+e)}}{3}+2iAcd\,e^{3i(fx+e)}-6iAcd\,e^{i(fx+e)}-10iA\,d^2e^{3i(fx+e)}+\frac{32cdB}{15}-\frac{4A}{15}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^3*(-d^2*(A*d-B*c)/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/4*(-8*A+8*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)
^4-1/5*(4*A-4*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+6*A*d+2*B*c-4*
B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-10*A*d-6*B*c+8*B*d)/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-3*A*c*d+3*A*d^2-B*d^2)/(c-d)^3/(tan(1/2
*f*x+1/2*e)+1))
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(218) = 436$.

Time $= 0.33$ (sec) , antiderivative size $= 2292$, normalized size of antiderivative $= 10.01$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
 - 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A +
16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (1
8*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A -
4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos
(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos
(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B
*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 -
 d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*
sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d +
5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) -
 2*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((
2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c
*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B
)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c
^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*
d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^
2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*...
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.283.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(218) = 436$.

Time $= 0.32$ (sec) , antiderivative size $= 553$, normalized size of antiderivative $= 2.41$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$

$$= 2\left(\frac{15\,(Bcd^2-Ad^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\sqrt{c^2-d^2}} - \frac{15Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45Ad^2\tan\left(\frac{1}{2}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e) - 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d + 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.283.9 Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.58

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$

$$= \frac{2d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,(A\,d-B\,c)\left(-2\,a^3\,c^3\,d+6\,a^3\,c^2\,d^2-6\,a^3\,c\,d^3+2\,a^3\,d^4\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}-\frac{2\,c\,d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d-B\,c)\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}}{2\,A\,d^3-2\,B\,c\,d^2}\right)(A\,d-B}{a^3\,f\,\sqrt{c+d}\,(c-d)^{7/2}}$$

$$-\frac{\frac{2\,(7\,A\,c^2+32\,A\,d^2+3\,B\,c^2-7\,B\,d^2-24\,A\,c\,d-11\,B\,c\,d)}{15\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(4\,A\,c^2+23\,A\,d^2+3\,B\,c^2-4\,B\,d^2-15\,A\,c\,d-11\,B\,c\,d)}{3\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f}{2}\right.}{f\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

output

```
(2*d^2*atan(((d^2*(A*d - B*c)*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a
^3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/
2)*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c +
d)^(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2))*(A*d - B*c))/(a^3*f*(c + d
)^(1/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*
A*c*d - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)*(4*A*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c -
d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c
^2 - 2*B*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(
e/2 + (f*x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B
*c*d))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 +
3*A*d^2 - B*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(
e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^
4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

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3.284.1 Optimal result

Integrand size = 35, antiderivative size = 381

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}f} \\
&\quad - \frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} \\
&\quad - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} \\
&\quad - \frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} \\
&\quad - \frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-1/15*d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*
d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)
/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d
)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/15*(B*(3*c^
2-23*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*si
n(f*x+e))/(c+d*sin(f*x+e))-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*arcta
n((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/f/(c^2-d^2)^
(1/2)
```

3.284.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. $2(381) = 762$.

Time = 12.28 (sec) , antiderivative size = 1253, normalized size of antiderivative = 3.29

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)^2),x]
```

output
```
(2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*
x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a +
 a*Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos
[(e + f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2]
 + 540*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430
*A*c*d^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(
e + f*x)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] -
 60*B*c^4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*
c^3*d*Cos[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c
^2*d^2*Cos[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c
*d^3*Cos[(3*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos
[(3*(e + f*x))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*
(e + f*x))/2] + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x
))/2] + 30*B*d^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6
*B*c^3*d*Cos[(7*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c
^2*d^2*Cos[(7*(e + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^
3*Cos[(7*(e + f*x))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*
(e + f*x))/2] + 80*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 34
0*A*c^3*d*Sin[(e + f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d...
```

3.284.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)} - \frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)} - \frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)} - \frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}-}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}-}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3233$$

$$\frac{-\int \frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2\right.\right.}{f(c-d)(a\sin(e+fx)+a}$$

$$a^2(c-d)$$

$$3a^2(c-d)$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{-\frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{a^2\left(A\left(2c^2-12cd+\right.\right.}{f(c-d)(}}{a^2(c-d)}$$

$$3a^2(c-d)$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

3.284. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

$$\frac{-\frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+\right.\right.}{f(c-d)(}$$

$$\frac{}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 3139$$

$$\frac{-\frac{30a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}$$

$$\frac{}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 1083$$

$$\frac{-\frac{60a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}$$

$$\frac{}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\downarrow 217$$

$$\frac{-\frac{30a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\left(\right.}{}$$

$$\frac{}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x
]
```

3.284. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e
 + f*x])) + (-1/3*(a*(2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/((c - d
)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (-((a^2*(B*(3*c^2 - 23*
c*d - 15*d^2) + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/((c - d)*f*(a +
 a*Sin[e + f*x])*(c + d*Sin[e + f*x]))) + ((-30*a^3*d^2*(A*d*(4*c + 3*d) -
 B*(3*c^2 + 3*c*d + d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2
- d^2])])/((c^2 - d^2)^(3/2)*f) - (a^3*d*(B*(3*c^3 - 23*c^2*d - 63*c*d^2 -
 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3))*Cos[e + f*x])/((c^2 -
 d^2)*f*(c + d*Sin[e + f*x])))/(a^2*(c - d)))/(3*a^2*(c - d)))/(5*a^2*(c -
 d))
```

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.284.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{2d^2\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(4Acd+3Ad^2-3Bc^2-3cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{-8}{2(c-d)^2\left(\tan\right.}$
default	$-\frac{2d^2\left(\frac{d^2(dA-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(dA-Bc)}{c+d}+\frac{\left(4Acd+3Ad^2-3Bc^2-3cdB-d^2B\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{-8}{2(c-d)^2\left(\tan\right.}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^3*(-d^2/(c-d)^4*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*
c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(4*A*c*d+3*A*d
^2-3*B*c^2-3*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/4*(-8*A+8*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)
+1)^4-1/5*(4*A-4*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+8*A*d+2*B
*c-6*B*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-12*A*d-6*B*c+10*B*d)
/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-4*A*c*d+6*A*d^2-3*B*d^2)/(c-d)^4/
(tan(1/2*f*x+1/2*e)+1))
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2201 vs. $2(368) = 736$.

Time $= 0.42$ (sec) , antiderivative size $= 4486$, normalized size of antiderivative $= 11.77$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*
c^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A +
 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c
^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2
*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*
B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17
*B)*d^6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*
(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 -
(109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*
d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B
*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^
2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x +
e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A
- B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A -
 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*
c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*
B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*
d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d
^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x +
e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - ...
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.284.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(368) = 736.

Time = 0.35 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.95

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$$

$$= 2\left(\frac{15\,(3\,Bc^2d^2-4\,Acd^3+3\,Bcd^3-3\,Ad^4+Bd^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5)\sqrt{c^2-d^2}}+\frac{15\,(Bcd^4\tan(\frac{1}{2}\,fx-}{(a^3c^6-3\,a^3c^5d+2\,a^3c^4d^2+2\,a^3c^3d^3-}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt
(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) -
 A*d^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d
 + 2*a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f
*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1
/2*e)^4 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^
4 - 45*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15
*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*
d*tan(1/2*f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*
f*x + 1/2*e)^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x
+ 1/2*e)^2 + 420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/
2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 1
10*A*c*d*tan(1/2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*
tan(1/2*f*x + 1/2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2
- 34*A*c*d - 16*B*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a
^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.284.9 Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 1349, normalized size of antiderivative = 3.54

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x
)
```

output

```
(2*d^2*atan(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(2*a^3*d
^6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4
*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(3*
B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*
d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c
- d)^(9/2)))/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3
*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^(3/2)*(c - d
)^(9/2)) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2
 + 72*A*c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)
*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*
A*d^4 + 3*B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d -
52*B*c*d^3 - 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) +
(2*tan(e/2 + (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 10
6*A*c^3*d^2 - 286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 7
9*B*c*d^4 - 59*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 -
d^3)) + (2*tan(e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3
+ 4*A*c^3*d^2 - 16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*
B*c*d^4 - 3*B*c^4*d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))
+ (2*tan(e/2 + (f*x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 +
 4*A*c^3*d^2 - 139*B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d ...
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

3.285.1 Optimal result . . . 2352
3.285.2 Mathematica [A] (verified) . . . 2353
3.285.3 Rubi [A] (verified) . . . 2354
3.285.4 Maple [A] (verified) . . . 2359
3.285.5 Fricas [B] (verification not implemented) . . . 2360
3.285.6 Sympy [F(-1)] . . . 2360
3.285.7 Maxima [F(-2)] . . . 2361
3.285.8 Giac [B] (verification not implemented) . . . 2361
3.285.9 Mupad [B] (verification not implemented) . . . 2362

3.285.1 Optimal result

Integrand size = 35, antiderivative size = 508

$$
\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f} \\
&\quad -\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} \\
&\quad -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} \\
&\quad -\frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} \\
&\quad -\frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\cos(e+}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}
$$

output
```
-1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*c*d^2+
195*d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))^2-1/5*(A-B)*cos(
f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2-1/15*(2*A*c-11*A*d+3*
B*c+6*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/
15*(3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*cos(f*x+e)/(c-d)^3/f/
(a^3+a^3*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-119*c^
2*d^2-130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4))*
cos(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^2*(A*d*(20*c^2+30*c*d+
13*d^2)-3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))
/(c^2-d^2)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2-d^2)^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 8.64 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+6(-A+B)(c-d)^2\left(\cos\left(\frac{1}{2}(e\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)^3),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)
/2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c -
d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2
- 19*c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 + (30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d +
7*c*d^2 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d
)*d^3*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((
c + d)*(c + d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 +
6*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c
+ d)^2*(c + d*Sin[e + f*x])))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

3.285.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int -\frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$\frac{-\frac{\int -\frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)} - \frac{a^3d\left(A\left(4c^{}\right.\right.}{a^2(c-d)}}{3a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

3.285. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

$$\frac{\int \frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)} - \frac{a^3d\left(A\left(4c^3-3\right.\right.}{a^2(c-d)}$$

$$\frac{3a^2(c-d)}{}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)} - \frac{a^3d\left(A\left(4c^3-3\right.\right.}{a^2(c-d)}$$

$$\frac{3a^2(c-d)}{}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3233$$

$$-\frac{\int \frac{15a^3d^2\left(Ad\left(20c^2+30dc+13d^2\right)-3B\left(4c^3+8dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2} - \frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4\right)\right)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\frac{2\left(c^2-d^2\right)}{a^2(c-d)}$$

$$\frac{3a^2(c-}{}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$-\frac{15a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2} - \frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2-1\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\frac{2\left(c^2-d^2\right)}{a^2(c-d)}$$

$$\frac{}{3}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$-\frac{15a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2-1\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\frac{2\left(c^2-d^2\right)}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 3139$$

$$-\frac{30a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd\right.\right.}{2\left(c^2-d^2\right)}$$

$$\frac{}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 1083$$

$$\frac{60a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+\right.\right.}{2\left(c^2-d^2\right)}$$

$$\frac{}{a^2(c-d}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\downarrow 217$$

$$-\frac{30a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx)}$$

$$\frac{2\left(c^2-d^2\right)}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x
]
```

3.285. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e
 + f*x])^2) + (-1/3*(a*(2*A*c + 3*B*c - 11*A*d + 6*B*d)*Cos[e + f*x])/((c
- d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (-((a^2*(3*B*(c^2
- 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))*Cos[e + f*x])/((c - d)*f
*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)) + (-1/2*(a^3*d*(3*B*(2*c^3
- 20*c^2*d - 57*c*d^2 - 30*d^3) + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^
3))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((-30*a^3*d^2*(
A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3))*
ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/
2)*f) - (a^3*d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4)
+ A*(4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/
((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(2*(c^2 - d^2)))/(a^2*(c - d)))/(3*a
^2*(c - d)))/(5*a^2*(c - d))
```

3.285.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

3.285.4 Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10dA+2Bc-8dB}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14dA-6Bc+12dB)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A\,c^2-$
default	$-\frac{-8A+8B}{2(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+10dA+2Bc-8dB}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(8Ac-14dA-6Bc+12dB)}{3(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(A\,c^2-$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^3*(-1/4*(-8*A+8*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(c
-d)^3/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+10*A*d+2*B*c-8*B*d)/(c-d)^4/(ta
n(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-14*A*d-6*B*c+12*B*d)/(c-d)^4/(tan(1/2*f*x
+1/2*e)+1)^3-(A*c^2-5*A*c*d+10*A*d^2-6*B*d^2)/(c-d)^5/(tan(1/2*f*x+1/2*e)+
1)-d^2/(c-d)^5*((1/2*d^2*(11*A*c^2*d+6*A*c*d^2-2*A*d^3-9*B*c^3-6*B*c^2*d)/
c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(10*A*c^4*d+6*A*c^3*d^2+19*A*
c^2*d^3+12*A*c*d^4-2*A*d^5-8*B*c^5-6*B*c^4*d-17*B*c^3*d^2-12*B*c^2*d^3-2*B
*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(29*A*c^2*d+18*A*
c*d^2-2*A*d^3-23*B*c^3-18*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x
+1/2*e)+1/2*d*(10*A*c^2*d+6*A*c*d^2-A*d^3-8*B*c^3-6*B*c^2*d-B*c*d^2)/(c^2+
2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(20*A*
c^2*d+30*A*c*d^2+13*A*d^3-12*B*c^3-24*B*c^2*d-21*B*c*d^2-6*B*d^3)/(c^2+2*c
*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))))
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3599 vs. $2(493) = 986$.

Time $= 0.56$ (sec) , antiderivative size $= 7283$, normalized size of antiderivative $= 14.34$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
Too large to include
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.285.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.285.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(493) = 986$.

Time $= 0.43$ (sec) , antiderivative size $= 1224$, normalized size of antiderivative $= 2.41$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c
*d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d
+ a^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6
- a^3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*
A*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*
A*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*
c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B
*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*
B*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 1
2*B*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2
*B*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^
4*d^4*tan(1/2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*
d^5*tan(1/2*f*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6
*tan(1/2*f*x + 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*
A*c^4*d^4 + 6*B*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 -
 3*a^3*c^8*d + a^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 +
 3*a^3*c^3*d^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
+ 1/2*e) + c)^2) - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f
*x + 1/2*e)^4 + 150*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x...
```

3.285.9 Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 2387, normalized size of antiderivative = 4.70

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x
)
```

output

```
((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4
*d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c
^5*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c
*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*
c^8 - 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c
^6*d^2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*
B*c^6*d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*
(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/
2 + (f*x)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3
*d^5 - 1960*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 125
7*B*c^3*d^5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^
7 + 54*A*c^7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d
^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(
60*A*d^8 - 32*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*
c^4*d^4 - 1294*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 +
 3705*B*c^4*d^4 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*
d + 60*B*c*d^7 + 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*
c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 10
8*A*c^8 - 42*B*c^8 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4
810*A*c^5*d^3 + 10*A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B...
```

3.286 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

3.286.1 Optimal result 2364
3.286.2 Mathematica [A] (verified) 2365
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3.286.5 Fricas [A] (verification not implemented) 2369
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3.286.7 Maxima [F] 2370
3.286.8 Giac [B] (verification not implemented) 2371
3.286.9 Mupad [F(-1)] 2371

3.286.1 Optimal result

Integrand size = 37, antiderivative size = 256

$$\begin{aligned}
&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{4a(c+d)(Bc-9Ad-8Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} \\
&\quad + \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} \\
&\quad + \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

output

```
4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+4/3
15*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*si
n(f*x+e))^(1/2)+2/63*a*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/
f/(a+a*sin(f*x+e))^(1/2)-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*si
n(f*x+e))^(1/2)+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin
(f*x+e))^(1/2)/f
```

3.286.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.19

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(2520Ac^3+1680Bc^3+5040Ac^2d+4788Bc^2$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^3,x]
```

output
```
-1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 41
04*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^
2 + 162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*
B*c^3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x]
 + 2016*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e
 + f*x] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3
*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]))
```

3.286.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$
$$\downarrow 3460$$

$$-\frac{(-9Ad+Bc-8Bd)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-9Ad+Bc-8Bd)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3249$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3240$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/}}{5af}\right.\right.}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)\right.}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3230$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(}{}\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(}{}\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3125$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx}{5af}\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x
]]) - ((B*c - 9*A*d - 8*B*d)*((-2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(
7*f*Sqrt[a + a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[e + f*x]*(a + a*Si
n[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*
x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*f))/(5*a)))/7))/(9*d)
```

3.286. $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.286.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(35B\left(\cos^4(fx+e)\right)d^3+\left(-45Ad^3-135d^2cB-40d^3B\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+\left(-189d^2cA-54Ad\right.}$
parts	$\frac{2Ac^3(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2c^2(3dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2d^2(dA+3Bc)(1+\sin(fx$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
2/315*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^3+(-45*A*d^3-13
5*B*c*d^2-40*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-189*A*c*d^2-54*A*d^3-189*B*c
^2*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^
3+105*B*c^3+252*B*c^2*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+315*A*c^3+630*c^
2*d*A+693*d^2*c*A+198*A*d^3+210*B*c^3+693*c^2*d*B+594*d^2*c*B+211*d^3*B)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{2\left(35\,Bd^3\cos(fx+e)^5-5\left(27\,Bcd^2+(9\,A+B)d^3\right)\cos(fx+e)^4+105\,(3\,A+B)c^3+63\,(5\,A+7\,B)c\right.}$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
 e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 +
 (81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B
)*d^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c
*d^2 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10
*A + 11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e
) - (35*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d +
9*(49*A + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d
^3)*cos(f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)
*cos(f*x + e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^
2 + 13*(9*A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.286.6 Sympy [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx$$
$$= \int \sqrt{a\left(\sin\left(e + fx\right) + 1\right)}(A + B\sin\left(e + fx\right))\left(c + d\sin\left(e + fx\right)\right)^3\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**3, x)
```

3.286.7 Maxima [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx$$
$$= \int \left(B\sin\left(fx + e\right) + A\right)\sqrt{a\sin\left(fx + e\right) + a}\left(d\sin\left(fx + e\right) + c\right)^3\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^3, x)
```

3.286.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(236) = 472.

Time = 0.40 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.15

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi +
 9/2*f*x + 9/2*e) + 630*(8*A*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B
*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 12*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*B
*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 6*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(6*B*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(6*B*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 2*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
, x)
```

3.287 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

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3.287.1 Optimal result

Integrand size = 37, antiderivative size = 192

$$\begin{aligned}
&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}} \\
&\quad + \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} \\
&\quad + \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\
&\quad - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

output

```
2/35*d*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+2/105*a*(-
7*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/
2)-2/7*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)+4/105*
(5*c-d)*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.287.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(420Ac^2+280Bc^2+560Acd+532Bcd+26$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^2,x]
```

output
```
-1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
420*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*
d*(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140
*c^2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.287.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$
$$\downarrow 3460$$
$$-\frac{(-7Ad+Bc-6Bd)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2 dx}{7d} -$$
$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$
$$\downarrow 3042$$

$$-\frac{(-7Ad+Bc-6Bd)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3240$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3230$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+}{5af}\right.}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+}{5af}\right.}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3125$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x
]]) - ((B*c - 7*A*d - 6*B*d)*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[
a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*f))/(5*a)))/(7*d)
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.287.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(-15B\left(\cos^2(fx+e)\right)\sin(fx+e)d^2+\left(-21Ad^2-42cdB-18d^2B\right)\left(\cos^2(fx+e)\right)+(70Acd+28Ad^2+35$
	$105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f$
parts	$\frac{2Ac^2(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2c(2dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2d(dA+2Bc)(1+\sin(fx+$
	15

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(
-21*A*d^2-42*B*c*d-18*B*d^2)*cos(f*x+e)^2+(70*A*c*d+28*A*d^2+35*B*c^2+56*B
*c*d+39*B*d^2)*sin(f*x+e)+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*c*d*B+
66*d^2*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{2\left(15\,Bd^2\cos(fx+e)^4+3\,(14\,Bcd+(7\,A+6\,B)d^2)\cos(fx+e)^3-35\,(3\,A+B)c^2-14\,(5\,A+7\,B)cd-\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x +
e)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c
^2 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)
*c^2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2
*cos(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^
2 - 3*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*
B)*c*d + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.287.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \int \sqrt{a\,(\sin(e+fx)+1)}(A+B\sin(e+fx))\,(c+d\sin(e+fx))^2\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**2, x)
```

3.287.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^2\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^2, x)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.81

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{\sqrt{2}\left(15\,Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+105\left(8\,Ac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+35\left(4\,Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\,Acd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+4\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+3\,Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+21\left(4\,Bcd\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2\,Ad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Bd^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\right)\sqrt{a}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi +
7/2*f*x + 7/2*e) + 105*(8*A*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*
c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 8*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2
*e) + 21*(4*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
, x)
```

3.288 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

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3.288.1 Optimal result

Integrand size = 35, antiderivative size = 118

$$\begin{aligned}
&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx \\
&= -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} \\
&\quad -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} \\
&\quad -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}
\end{aligned}$$

output

```
-2/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*A*c+5*A*d+5*B*c+
7*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*(5*A*d+5*B*c-2*B*d)*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.288.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(30Ac+20Bc+20Ad+19Bd-3Bd\cos(2($$
$$15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
]),x]
```

output
```
-1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3
0*A*c + 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A
*d + 4*B*d)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.288.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\downarrow 3447$$
$$\int \sqrt{a\sin(e+fx)+a}\big((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\big)\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{a\sin(e+fx)+a}\big((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\big)\,dx$$
$$\downarrow 3502$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\downarrow 27$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\downarrow 3230$$

$$\frac{\frac{1}{3}a(15Ac+5Ad+5Bc+7Bd)\int \sqrt{\sin(e+fx)a+a}dx - \frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}a(15Ac+5Ad+5Bc+7Bd)\int \sqrt{\sin(e+fx)a+a}dx - \frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\downarrow 3125$$

$$\frac{-\frac{2a^2(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}} - \frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*A*
c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) -
(2*a*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))
/(5*a)
```

3.288. $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.288.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(3B\left(\sin^2(fx+e)\right)d+5A\sin(fx+e)d+5B\sin(fx+e)c+4B\sin(fx+e)d+15Ac+10dA+10Bc+8dB\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2Ac(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(dA+Bc)(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2dB(1+\sin(fx+e))a(\sin(fx+$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+
5*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*d*A+10*B*c+8*d*B)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{2\left(3\,Bd\cos(fx+e)^3-(5\,Bc+(5\,A+B)d)\cos(fx+e)^2-5\,(3\,A+B)c-(5\,A+7\,B)d-(5\,(3\,A+2\,B$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
 + B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e)
 - (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A +
 4*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x +
 e) + f*sin(f*x + e) + f)
```

3.288.6 Sympy [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int \sqrt{a\left(\sin\left(e + fx\right) + 1\right)}\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x)), x)
```

3.288.7 Maxima [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (B\sin\left(fx + e\right) + A)\sqrt{a\sin\left(fx + e\right) + a}(d\sin\left(fx + e\right) + c)\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c), x)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{\sqrt{2}\left(3\,Bd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi + \frac{5}{2}\,fx + \frac{5}{2}\,e\right) + 30\left(2\,Ac\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) +\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
1/30*sqrt(2)*(3*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*
f*x + 5/2*e) + 30*(2*A*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*c*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B
*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5
*(2*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2
*f*x + 3/2*e))*sqrt(a)/f
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),
x)
```

3.289 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

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3.289.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

output
```
-2/3*a*(3*A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*B*cos(f*x+e)*(a+a*s
in(f*x+e))^(1/2)/f
```

3.289.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(3A+2B+B\sin(e+fx))}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A
+ 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.289.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{1}{3}(3A+B)\int \sqrt{\sin(e+fx)a+a}dx - \frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3A+B)\int \sqrt{\sin(e+fx)a+a}dx - \frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\downarrow 3125$$

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

3.289.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.289.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(B\sin(fx+e)+3A+2B)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	58
parts	$\frac{2A(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	96

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx =$$
$$-\frac{2\left(B\cos(fx+e)^2+(3A+2B)\cos(fx+e)+(B\cos(fx+e)-3A-B)\sin(fx+e)+3A+B\right)\sqrt{}}{3\left(f\cos(fx+e)+f\sin(fx+e)+f\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A
- B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*
sin(f*x + e) + f)
```

3.289.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int \sqrt{a\left(\sin(e+fx)+1\right)}(A+B\sin(e+fx))\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

3.289.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \frac{\sqrt{2}\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\left(2A\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\mathrm{sg}\right.\right.}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x +
 3/2*e) + 3*(2*A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e))*sqrt(a)/f
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

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3.290.1 Optimal result

Integrand size = 37, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f} - \frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

output

```
2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e
))^(1/2))*a^(1/2)/d^(3/2)/f/(c+d)^(1/2)-2*a*B*cos(f*x+e)/d/f/(a+a*sin(f*x+
e))^(1/2)
```

3.290.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.75 (sec) , antiderivative size = 903, normalized size of antiderivative = 9.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{(2-2i)B\sqrt{d}\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}{f}+\frac{(-Bc+Ad)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^2\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x]),x]
```

output

```
((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f
+ ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d
 + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e
)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*
Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((
1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]
*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqr
t[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1
^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I
)*x*Sin[e]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e
]]) + ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*
x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1
 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f
*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^(
(I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c
*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^
(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^
3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin
[e]))/(4*f)))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[
e]]) + ((2 - 2*I)*B*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt...
```

3.290.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\downarrow 3460$$

$$-\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$-\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3252$$

$$\frac{2a(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 221$$

$$\frac{2\sqrt{a}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x
]
```

output
```
(2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]
*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x]
)/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

3.290.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)  Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.290.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(A\arctanh\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ad-B\arctanh\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)ac+\sqrt{-a(\sin(fx+$
	$d\sqrt{a(c+d)d}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f$

input

```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output

```
-2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(A*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a*d-B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a*c+(-a*(sin(f*x+e)-1))^(1/2)*B*(a*(c+d)*d)^(1/2))/d/(a*(c+
d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 651, normalized size of antiderivative = 6.51

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[-\frac{(Bc-Ad+(Bc-Ad)\cos(fx+e)+(Bc-Ad)\sin(fx+e))\sqrt{\frac{a}{cd+d^2}}\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2acd-ad^2-}{}\right.\right.$$

input

```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (
6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 +
d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*
c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
(a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)
^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e)
 - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*s
in(f*x + e) + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*s
in(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f
*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e)
+ d*f*sin(f*x + e) + d*f)]
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.290.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c), x)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{2\,B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d}+\frac{\sqrt{2}\left(Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\right.\right.}{\sqrt{-cd-d^2}d}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
sqrt(2)*(2*B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/d + sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e
)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d))*sqrt(a)/f
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
, x)
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

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3.291.2 Mathematica [C] (verified) . . . 2400
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3.291.1 Optimal result

Integrand size = 37, antiderivative size = 126

$$\begin{aligned}\int &\frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx \\ &= -\frac{\sqrt{a}(Ad+B(c+2d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} \\ &\quad + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

output
```
-(A*d+B*(c+2*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f
*x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d
)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.291.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.02 (sec) , antiderivative size = 901, normalized size of antiderivative = 7.15

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(1+\sin(e+fx))}\left(\frac{(Ad+B(c+2d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\text{RootSum}\left[-d+2ice^{ie}\#1^2+de^{2ie}\#1^4\&,\frac{(1+\right.\right.\right.$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x])^2,x]
```

output

```
((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] +
I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^
((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((
-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I
)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt
[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E
^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[
e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(C
os[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(
Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d
+ (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)
]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqr
t[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - (
(1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1
]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt
[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^
2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^
(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqr
t[d]*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(...
```

3.291.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3042, 3459, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3459$$

$$\frac{(Ad+B(c+2d))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}\,dx}{2d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{(Ad+B(c+2d))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3252$$

$$\frac{\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a(Ad+B(c+2d))\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df(c+d)}}{}$$

$$\downarrow 221$$

$$\frac{\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}(Ad+B(c+2d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}}{}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2
,x]
```

output
```
-((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c
- A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

3.291.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.291.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(110) = 220$.

Time = 0.84 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.17

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)ad(dA+Bc+2dB)+A\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a$ $d(c+d)(c+d\sin(fx+e))$

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)
*d/(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+
a*d^2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d-B*(a-a*si
n(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^
(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(110) = 220.

Time = 0.85 (sec) , antiderivative size = 1012, normalized size of antiderivative = 8.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*co
s(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c
*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (
6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 +
d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*
c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
(a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)
^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B
*c - A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2
*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 +
 d^3)*f)*sin(f*x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c
*d + (A + 2*B)*d^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e)
+ (B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x
 + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) +
 a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -...
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.291.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c)^2, x)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Bc\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Ad\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+2Bd\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}f\right.}{\sqrt{-cd-d^2}}\right.\right.}{(cd+d^2)\sqrt{-cd-d^2}}\right.}{2f}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A
*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2)
)/((c*d + d^2)*sqrt(-c*d - d^2)) - 2*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
- c - d)*(c*d + d^2)))/f
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^2, x)
```

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

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3.292.1 Optimal result

Integrand size = 37, antiderivative size = 192

$$\begin{aligned}
&\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{\sqrt{a}(3Ad+B(c+4d))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} \\
&\quad + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-1/4*(3*A*d+B*(c+4*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a
*sin(f*x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x
+e)/d/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a*(3*A*d+B*(c+
4*d))*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.292.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.53 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.04

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \left(\tfrac{1}{16}+\tfrac{i}{16}\right)\sqrt{a(1+\sin(e+fx))}\left(\frac{(3Ad+B(c+4d))\left(\cos\left(\tfrac{e}{2}\right)+i\sin\left(\tfrac{e}{2}\right)\right)\left((-1+i)x\cos(e)+\text{RootSum}\left[-d+2icde^{ie}\#1^2+de^{2ie}\#1^4\&,\right.\right.\right.$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x])^3,x]
```

output

```
((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2
] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 +
d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[
E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqr
t[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^
((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] -
Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*L
og[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 +
Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2
)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4
*d))*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootS
um[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((
-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[
d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*
#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x
) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c
 + d)^(5/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*
I)*Sqrt[d]*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + ...
```

3.292.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 3459, 3042, 3251, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3459$$

$$\frac{(3Ad+B(c+4d))\int \frac{\sqrt{\sin(e+fx)a+a}}{(c+d\sin(e+fx))^2}\,dx}{4d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{(3Ad+B(c+4d))\int \frac{\sqrt{\sin(e+fx)a+a}}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3251$$

$$\frac{(3Ad+B(c+4d))\left(\frac{\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{(3Ad+B(c+4d))\left(\frac{\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3252$$

$$\frac{(3Ad+B(c+4d))\left(-\frac{a\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 221$$

$$\frac{(3Ad+B(c+4d))\left(-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c+d)^{3/2}}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3
,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c +
d*Sin[e + f*x])^2) + ((3*A*d + B*(c + 4*d))*(-((Sqrt[a]*ArcTanh[(Sqrt[a]*S
qrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[d]*(c
+ d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))))/(4*d*(c + d))
```

3.292.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.292.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(168) = 336.

Time = 1.09 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.27

method	result
default	$\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2d^2(3dA+Bc+4dB)(\cos^2(fx+e))-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2cd(3dA+Bc+4dB)+$

input
```
int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/4/a*(arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(3*A*
d+B*c+4*B*d)*cos(f*x+e)^2-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^2*c*d*(3*A*d+B*c+4*B*d)+3*A*(a-a*sin(f*x+e))^(3/2)*(a
*(c+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*d^3+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^
(3/2)*(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*
d^2)^(1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^2*c^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*
d^2-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*
(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*
(a*(c+d)*d)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3
*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c
+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2
)/f
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(168) = 336.

Time = 1.29 (sec) , antiderivative size = 1750, normalized size of antiderivative = 9.11

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output

```
[-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)
*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*
c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*
A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e
)^2 + 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(
c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
 + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*co
s(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 +
 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*c
os(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d
 + (A + 4*B)*d^2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*
A + 2*B)*c*d - 2*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*
d^2 + (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a))/((c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c
^2*d^3 + 4*c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d...
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

output

```
Timed out
```

3.292.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c)^3, x)
```

3.292.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(168) = 336$.

Time $= 0.34$ (sec) , antiderivative size $= 422$, normalized size of antiderivative $= 2.20$

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Bc\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+3\,Ad\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+4\,Bd\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.}{\sqrt{-cd-d}}\right.\right.$$
$$\left.\left.\frac{}{(c^2d+2cd^2+d^3)\sqrt{-cd-d^2}}\right.\right.$$

input
```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*d*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^
2))/((c^2*d + 2*c*d^2 + d^3)*sqrt(-c*d - d^2)) + 2*(2*B*c*d*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*d^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + B*c^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B
*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
5*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 - 4*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e))/((c^2*d + 2*c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c -
d)^2))/f
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^3, x)
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

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3.293.1 Optimal result

Integrand size = 37, antiderivative size = 374

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{4a^2(c+d)\,(15c^2+10cd+7d^2)\,(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{8a(5c-d)(c+d)\,(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$
$$+\frac{4(c+d)\,(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}$$
$$+\frac{2a^2(11A(c-17d)d-3B(c^2-9cd+56d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}$$

output
```
4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(
f*x+e))^(3/2)/f+4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*
B*(c^2-9*c*d+56*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^2*(1
1*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f
/(a+a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-11*A*d)*cos(f*x+e)*(c+d*sin(
f*x+e))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A*(c-17*
d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a*
B*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

3.293.2 Mathematica [A] (verified)

Time = 4.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(92400Ac^3+72072Bc^3+216216Ac^2d+1956\text{2}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^3,x]
```

output
```
-1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A
*c*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 +
 351*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*C
os[2*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 184
80*A*c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f
*x] + 100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B
*c*d^2*Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x]
- 5940*B*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*
c*d^2*Sin[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e
 + f*x)] + 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))
```

3.293.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 27$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3460$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3 dx}{9d}}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3042$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d} - \frac{11d}{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3249$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2}{9d}\right)}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3042$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2}{9d}\right)}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3240$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)\right)}{5a}\right)}{9d}\right)}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 27$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\right)}{5a}\right)}{9d}\right)}{11d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3042$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\right.\right.\right.}{5a}}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3230$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-}{5a}\right.\right.}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3042$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-}{5a}\right.\right.}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\downarrow 3125$$

$$\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}} - \frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c}{5a}\right.\right.}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*
d*f) + ((2*a^2*(3*B*(c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^
4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9
*c*d + 56*d^2))*((-2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a +
a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt
[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f
*x]])/(3*f))/(5*a)))/7))/(9*d))/(11*d)
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.293.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.83

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(315B(\cos^4(fx+e))\sin(fx+e)d^3+(385Ad^3+1155d^2cB+735d^3B)(\cos^4(fx+e))+(-1485d^2cA-9}{}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d^2(dA+3Bc)(35(\sin^4(fx+e))+85(\sin^3(fx+e))+102(\sin^2(fx+e))+136\sin(fx+e)+272)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}+\frac{2(1-}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^
3+(385*A*d^3+1155*B*c*d^2+735*B*d^3)*cos(f*x+e)^4+(-1485*A*c*d^2-935*A*d^3
-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-2079*A*c^
2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B*c^2*d-5676*B*c*d^2-2478*B*d^3
)*cos(f*x+e)^2+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3
+6633*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+5775*A*c^3+14553*c^2*d*A
+14157*d^2*c*A+4499*A*d^3+4851*B*c^3+14157*c^2*d*B+13497*d^2*c*B+4431*d^3*
B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.70

$$\int (a + a\sin(e+fx))^{3/2}(A + B\sin(e+fx))(c + d\sin(e+fx))^3\,dx =$$

$$-\frac{2\left(315\,Bad^3\cos(fx+e)^6 + 35\left(33\,Bacd^2 + (11\,A+21\,B)ad^3\right)\cos(fx+e)^5 + 924\left(5\,A+3\,B\right)ac^3 + 396\,(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d
^3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 1
32*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33
*(9*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*
c^3 + 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1
239*B)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a
*c^2*d + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e
)^2 + (231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A +
409*B)*a*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(
f*x + e)^5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A
 + 47*B)*a*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12
*B)*a*d^3)*cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (
187*A + 294*B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c
^2*d + 33*(24*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 +
(231*(5*A + 9*B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c
*d^2 + 17*(143*A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.293.6 Sympy [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \int (a(\sin(e + fx) + 1))^{\frac{3}{2}}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x))**3, x)
```

3.293.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^3, x)
```

3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(350) = 700$.

Time $= 0.45$ (sec) , antiderivative size $= 755$, normalized size of antiderivative $= 2.02$

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/4
*pi + 11/2*f*x + 11/2*e) + 6930*(24*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 16*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*A*a*c^2*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 42*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 42*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 11*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 2310*(8*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 12*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*A*a*c^2*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 30*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 9*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*
x + 3/2*e) + 693*(8*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c^2*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 36*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*
B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e) + 495*(12*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 12*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a*c...
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
, x)
```

3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

3.294.1 Optimal result . . . 2427
3.294.2 Mathematica [A] (verified) . . . 2428
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3.294.1 Optimal result

Integrand size = 37, antiderivative size = 294

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{2a^2(15c^2+10cd+7d^2)\,(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)}{315d^2f\sqrt{a+a\sin(e+fx)}} + \frac{4a(5c-d)\,(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315df} + \frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df}$$

output

```
2/105*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3
/2)/f+2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2)
)*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/63*a^2*(-9*A*d+3*B*c-10*B*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*
(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/
f-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d/f
```

3.294.2 Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(4200Ac^2+3276Bc^2+6552Acd+5928Bcd+$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^2,x]
```

output
```
-1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B
*d^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e +
 f*x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*S
in[e + f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d
^2*Sin[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 9
0*A*d^2*Sin[3*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]))
```

3.294.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$
$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}} -$$

$$\frac{9d}{}$$

$$\downarrow 27$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}} -$$

$$\frac{9d}{}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}} -$$

$$\frac{9d}{}$$

$$\downarrow 3460$$

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}} -$$

$$\frac{9d}{}$$

$$\downarrow 3042$$

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}} -$$

$$\frac{9d}{}$$

$$\downarrow 3240$$

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+\right.\right.}{5a}}{7d}}{\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}}$$

$$\frac{9d}{}$$

$$\downarrow 27$$

$$\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right.}{5a}\right.}{7d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$$\Big\downarrow 3042$$

$$\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right.}{5a}\right.}{7d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$$\Big\downarrow 3230$$

$$\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)}{5a}}{7d}\right.}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$$\Big\downarrow 3042$$

$$\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)}{5a}}{7d}\right.}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$$\Big\downarrow 3125$$

$$\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}} - \frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(e+}{5a}\right.}{7d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d
*f) + ((2*a^2*(3*B*c - 9*A*d - 10*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3
)/(7*d*f*Sqrt[a + a*Sin[e + f*x]]) - (3*a*(3*A*(c - 13*d)*d - B*(c^2 - 7*c
*d + 34*d^2))*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) +
((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))/(5*
a)))/(7*d))/(9*d)
```

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3455

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.294.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(35B(\cos^4(fx+e))d^2+(-45Ad^2-90cdB-85d^2B)(\cos^2(fx+e))\sin(fx+e)+(-126Acd-117Ad^2$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d(dA+2Bc)(15(\sin^3(fx+e))+39(\sin^2(fx+e))+52\sin(fx+e)+104)}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}+\frac{2(1+\sin(fx+e))a^2(\sin(f}{5c}$

input

```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output

```
2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^2+(-45*A*d^2-
90*B*c*d-85*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-126*A*c*d-117*A*d^2-63*B*c^2-
234*B*c*d-172*B*d^2)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2
+402*B*c*d+221*B*d^2)*sin(f*x+e)+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+8
58*c*d*B+409*d^2*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{2\left(35\,Bad^2\cos(fx+e)^5-5\left(18\,Bacd+(9\,A+10\,B)ad^2\right)\cos(fx+e)^4+84\,(5\,A+3\,B)ac^2+24\,(21\,A+\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*co
s(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 4
7*B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*
cos(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A +
134*B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)
*a*c*d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4
+ 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 +
5*(18*B*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*
A + 8*B)*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c
^2 + 6*(63*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.294.6 Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c$$
$$+d\sin(e+fx))^2\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}\,(A+B\sin(e+fx))\,(c+d\sin(e+fx))^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x))**2, x)
```

3.294.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^2, x)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.69

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
 + 9/2*f*x + 9/2*e) + 630*(12*A*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 8*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*A*a*c*d*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + 14*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*d^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*A*a*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 12*A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a*c*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 5*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x
+ 3/2*e) + 126*(2*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*a*c*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(4*B*
a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*
pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
, x)
```

3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

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3.295.1 Optimal result

Integrand size = 35, antiderivative size = 165

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}$$
$$-\frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}$$
$$-\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}$$

output

```
-2/35*(7*A*d+7*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*d*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*
cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*
cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.295.2 Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(700Ac+546Bc+546Ad+494Bd-6(7Bc+$$
$$210f\left(\cos\left(\frac{1}{2}(e+fx\right)\right.$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x]),x]
```

output
```
-1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2
*(e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*
d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.295.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\downarrow 3447$$
$$\int (a\sin(e+fx)+a)^{3/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{3/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$
$$\downarrow 3502$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 27$$

$$\frac{\int(\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3042$$

$$\frac{\int(\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3230$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\int(\sin(e+fx)a+a)^{3/2}dx - \frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\int(\sin(e+fx)a+a)^{3/2}dx - \frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3126$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx - \frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right) - \frac{2a(7Ad+7Bc-2Bd)\cos}{}}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx - \frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right) - \frac{2a(7Ad+7Bc-2Bd)\cos}{}}{7a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\downarrow 3125$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin}{5f}}{7a}$$

$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f) + ((-2*a*(7*B*c +
 7*A*d - 2*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + (a*(35*A*
c + 21*B*c + 21*A*d + 19*B*d)*((-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)))/5)/(7*a)
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.295.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(15B(\sin^3(fx+e))d+21A(\sin^2(fx+e))d+21B(\sin^2(fx+e))c+39B(\sin^2(fx+e))d+35A\sin(fx+e}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(dA+Bc)(\sin^2(fx+e)+3\sin(fx+e)+6)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}+\frac{2Ac(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*B*sin(f*x+e)^3*d+21*A*sin(f*x+
e)^2*d+21*B*sin(f*x+e)^2*c+39*B*sin(f*x+e)^2*d+35*A*sin(f*x+e)*c+63*A*sin(
f*x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*d*A+126*B*c+104*d
*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{2\left(15\,Bad\cos\left(fx+e\right)^4+3\left(7\,Bac+\left(7\,A+13\,B\right)ad\right)\cos\left(fx+e\right)^3-28\left(5\,A+3\,B\right)ac\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x +
e)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42
*A + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*
d)*cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A
+ 19*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B
)*a*c + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.295.6 Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x)), x)
```

3.295.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c), x)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.73

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{\sqrt{2}\left(15\,Bad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+105\left(12\,Aac\mathrm{sgn}\left(\cos\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi + 7/2*f*x + 7/2*e) + 105*(12*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),
x)
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

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3.296.2 Mathematica [A] (verified) . . . 2444
3.296.3 Rubi [A] (verified) . . . 2445
3.296.4 Maple [A] (verified) . . . 2447
3.296.5 Fricas [A] (verification not implemented) . . . 2447
3.296.6 Sympy [F] . . . 2448
3.296.7 Maxima [F] . . . 2448
3.296.8 Giac [A] (verification not implemented) . . . 2448
3.296.9 Mupad [F(-1)] . . . 2449

3.296.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

output
```
-2/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*A+3*B)*cos(f*x+e)/f
/(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)
/f
```

3.296.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(50A+39B-3B\cos(2(e+fx))+2(5A+9$$
$$15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output
```
-1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(50*A + 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.296.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{1}{5}(5A+3B)\int (\sin(e+fx)a+a)^{3/2}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A+3B)\int (\sin(e+fx)a+a)^{3/2}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\downarrow 3126$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx - \frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right) - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx - \frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right) - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\downarrow 3125$$

$$\frac{1}{5}(5A+3B)\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + ((5*A + 3*B)*((-8*a
^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a
 + a*Sin[e + f*x]])/(3*f)))/5
```

3.296.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.296.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(-3B(\cos^2(fx+e))+\sin(fx+e)(5A+9B)+25A+21B)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	77
parts	$\frac{2A(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin^2(fx+e)+3\sin(fx+e)+6)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	118

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2+sin(f*x+e)*(5*A+
9*B)+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36

$$\int (a+a\sin(e+fx))^{3/2}(A$$
$$+B\sin(e+fx))\,dx = \frac{2\left(3\,Ba\cos(fx+e)^3-(5\,A+6\,B)a\cos(fx+e)^2-(25\,A+21\,B)a\cos(fx+e)-4(\right.}{15\,(f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*
a*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*c
os(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

3.296.6 Sympy [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))\,dx = \int (a(\sin(e + fx) + 1))^{\frac{3}{2}}\,(A + B\sin(e + fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

3.296.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.38

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(3\,Ba\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi + \frac{5}{2}\,fx + \frac{5}{2}\,e\right) + 30\left(3\,Aa\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.\right.\right.$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*
f*x + 5/2*e) + 30*(3*A*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*A*a*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(a)/f
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

3.297.1 Optimal result . . . 2450
3.297.2 Mathematica [C] (verified) . . . 2450
3.297.3 Rubi [A] (verified) . . . 2451
3.297.4 Maple [B] (verified) . . . 2454
3.297.5 Fricas [B] (verification not implemented) . . . 2455
3.297.6 Sympy [F(-1)] . . . 2456
3.297.7 Maxima [F] . . . 2456
3.297.8 Giac [B] (verification not implemented) . . . 2456
3.297.9 Mupad [F(-1)] . . . 2457

3.297.1 Optimal result

Integrand size = 37, antiderivative size = 153

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2a^{3/2}(c-d)(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}f} + \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}$$

output
```
-2*a^(3/2)*(c-d)*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)
/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^2*(-3*A*d+3*B*c-4*B*d
)*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*B*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/d/f
```

3.297.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.35 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.87

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = (a(1+\sin(e+fx)))^{3/2}\left(-6\sqrt{d}(-2Bc+2Ad+3Bd)\cos\right.$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x]),x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e
+ f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(B*c - A*d)*((c
+ d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#
1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/
4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(
e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[
-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
+ c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
 ]))/(c + d)^(3/2) + (3*(c - d)*(B*c - A*d)*(-((c + d)*(e + f*x - 2*Log[Se
c[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3
 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1
+ Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt
[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]
*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + T
an[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqr
t[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(3/2) + 6*
Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*...
```

3.297.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 27$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 3460$$

$$\frac{\frac{3a(c-d)(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} + \frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 3042$$

$$\frac{\frac{3a(c-d)(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} + \frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 3252$$

$$\frac{\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{6a^2(c-d)(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\downarrow 221$$

$$\frac{\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{6a^{3/2}(c-d)(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}}{3d} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f) + ((-6*a^(3/2)*(c -
 d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a
 + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*c - 3*A*d - 4*
B*d)*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]]))/(3*d)
```

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.297.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(131) = 262$.

Time = 1.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.91

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^2cd+3A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^2d^2-B(-a$

input

```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output

```
-2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*A*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*A*arctanh((-a*(sin(f*x+e)-1))^(
1/2)*d/(a*(c+d)*d)^(1/2))*a^2*d^2-B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^
(1/2)*d+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2
-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*A*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d-3*B*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c+6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*
d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(131) = 262$.

Time = 0.85 (sec) , antiderivative size = 880, normalized size of antiderivative = 5.75

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \left[-\frac{3\left(Bac^2-(A+B)acd+Aad^2+(Bac^2-(A+B)ac\right.}{}\right.$$

$$-\frac{3\left(Bac^2-(A+B)acd+Aad^2+\left(Bac^2-(A+B)acd+Aad^2\right)\cos\left(fx+e\right)+\left(Bac^2-(A+B)acd+Aa\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A
*a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (
6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 +
d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*
c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
(a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)
^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*a*d*cos(f*x
+ e)^2 - 3*B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x +
 e) + (B*a*d*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1
/3*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*
d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt
(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3
*B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a
*d*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f...
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.297.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c), x)
```

3.297.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(131) = 262$.

Time $= 0.31$ (sec) , antiderivative size $= 274$, normalized size of antiderivative $= 1.79$

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx =$$

$$\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}(Bac^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-Aacd\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-Bacd\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Aad^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}\right.$$

$$-\frac{}{\sqrt{-cd-d^2}d^2}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/3*sqrt(2)*sqrt(a)*(3*sqrt(2)*(B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan
(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d
^2)*d^2) + 2*(2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^3 + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*si
n(-1/4*pi + 1/2*f*x + 1/2*e) - 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^3)/f
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
, x)
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

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3.298.2 Mathematica [C] (verified) . . . 2458
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3.298.1 Optimal result

Integrand size = 37, antiderivative size = 191

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx =$$
$$-\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f}$$
$$-\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}}+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))}$$

output

```
-a^(3/2)*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^
(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^2*(-A*d+
3*B*c+2*B*d)*cos(f*x+e)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)+a*(-A*d+B*c)*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.59 (sec) , antiderivative size = 922, normalized size of antiderivative = 4.83

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = (a(1+\sin(e+fx)))^{3/2}\left(-8B\sqrt{d}\cos\left(\tfrac{1}{2}(e+fx)\right)+\frac{(Ad$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((A*d*(c +
3*d) + B*(-3*c^2 - 3*c*d + 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)
/4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f
*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt
[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*
#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + ((A*d*(c + 3*d
) + B*(-3*c^2 - 3*c*d + 2*d^2))*(-((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/
4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f
*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt
[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*
#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + 8*B*Sqrt[d]*Si
n[(e + f*x)/2] - (4*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] -...
```

3.298.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3454, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\downarrow 3460$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{a(Ad(c+3d)-B(3c^2+3cd-2d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{a(Ad(c+3d)-B(3c^2+3cd-2d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}}{2d(c+d)}$$

$$\downarrow 3252$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2(Ad(c+3d)-B(3c^2+3cd-2d^2))\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df} + \frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{2d(c+d)}$$

$$\downarrow 221$$

$$\frac{\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}} + \frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*
Sin[e + f*x])) - ((2*a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*A
rcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]
])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]]))/(2*d*(c + d))
```

3.298.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.298.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(173) = 346$.

Time $= 0.97$ (sec) , antiderivative size $= 592$, normalized size of antiderivative $= 3.10$

method	result
default	$\frac{a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\sin(fx+e)d\left(A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)acd+3A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a\,d^2-3B\right.\right.$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e)
)^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a*d^2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a-a*sin(f*x+e))
^(1/2)*(a*(c+d)*d)^(1/2)*d)-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a*c*d^2+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3+
3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-2*B*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+A*(a-a*sin(f*x+e
))^(1/2)*(a*(c+d)*d)^(1/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*
d^2-3*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2-B*(a-a*sin(f*x+e))^(1
/2)*(a*(c+d)*d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(173) = 346$.

Time = 1.00 (sec) , antiderivative size = 1428, normalized size of antiderivative = 7.48

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output

```
[1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d
^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2
+ (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*
B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B
*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x +
e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^
2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d
^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x +
e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
 + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)
^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2
 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2
+ (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B
)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x +
e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 +
 c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*co
s(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c...
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output

```
Timed out
```

3.298.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c)^2, x)
```

3.298.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(173) = 346.

Time = 0.33 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{\sqrt{2}\left(\frac{4\,Ba\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{d^2} + \frac{\sqrt{2}(3\,Bac^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - Aac\,d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + 3\,Bac\,d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 3\,Aa\,d^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - 2\,Ba\,d^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right))\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^{2}}}\right)}{(cd^{2}+d^{3})\sqrt{-cd-d^{2}}} - \frac{2(Bac^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right) - Aac\,d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right) - Bac\,d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right) + Aa\,d^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right))}{(cd^{2}+d^{3})\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c-d\right)}\right)\sqrt{a}}{f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/2*sqrt(2)*(4*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)/d^2 + sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*c*d*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^2 + d^3)*sqrt(-c*d - d^2)) - 2*(B*
a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
 A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
) - B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e))/((c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))*s
qrt(a)/f
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^2, x)
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

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3.299.1 Optimal result

Integrand size = 37, antiderivative size = 221

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2}$$
$$+\frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output

```
-1/4*a^(3/2)*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*arctanh(cos(f*x+e)*a^(1/2
)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+1/4*a^
2*(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*
x+e))/(a+a*sin(f*x+e))^(1/2)+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.84 (sec) , antiderivative size = 957, normalized size of antiderivative = 4.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^3,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2)
)*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c +
 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e +
 f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 +
 Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2
)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan
[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#
1^3) & ]))/(c + d)^(7/2) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(-
((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c +
4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e +
f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)
*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[
(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*
Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1
^3) & ]))/(c + d)^(7/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]...
```

3.299.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))^2}dx}{2d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\downarrow 3459$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{-\frac{a(Ad(c+7d)+3B(c^2+3cd+4d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)} - \frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{-\frac{a(Ad(c+7d)+3B(c^2+3cd+4d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)} - \frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\downarrow 3252$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\frac{a^2(Ad(c+7d)+3B(c^2+3cd+4d^2))\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}} d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df(c+d)} - \frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\downarrow 221$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}} - \frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c +
d*Sin[e + f*x])^2) - ((a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*
x]])])/(d^(3/2)*(c + d)^(3/2)*f) - (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d
- 4*d^2))*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e
 + f*x])))/(4*d*(c + d))
```

3.299.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(197) = 394$.

Time = 1.34 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.05

method	result
default	$-\frac{\left(-\arctan\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2d^2\left(Acd+7A\,d^2+3B\,c^2+9cdB+12d^2B\right)\left(\cos^2(fx+e)\right)+2\sin(fx+e)\arctan\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right.$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*(-arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*
d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*cos(f*x+e)^2+2*sin(f*x+e)*arctanh((a-a
*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9
*B*c*d+12*B*d^2)+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a
^2*c^3*d+7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2
*d^2+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+7*A
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4-A*(a-a*sin(
f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-7*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*
d)^(1/2)*d^3+3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B
*c^4+9*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+1
5*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+9*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+12*B*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+5*B*(a-a*sin(f*x+
e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d+7*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(
1/2)*c*d^2-4*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3-A*(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d
)^(1/2)*a*c*d^2+9*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3-3*B*(a-
a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-12*B*(a-a*sin(f*x+e))^(1/2)*(a
*(c+d)*d)^(1/2)*a*c^2*d-5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d
^2+4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-...
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(197) = 394$.

Time $= 1.49$ (sec) , antiderivative size $= 2208$, normalized size of antiderivative $= 9.99$

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output

```
[-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*
A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^
3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2
*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*
a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (
7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
 + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*
c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*
a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d -
a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 -
(c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c
^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*
x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*
a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x
+ e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a
*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*
B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*...
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output

```
Timed out
```

3.299.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c)^3, x)
```

3.299.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(197) = 394$.

Time $= 0.36$ (sec) , antiderivative size $= 624$, normalized size of antiderivative $= 2.82$

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a*c*d*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + 7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*p
i + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^2*d^2 + 2*c*d^3 + d^4)*sqrt(-c*
d - d^2)) - 2*(10*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 - 2*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 14*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 14*A*a*d^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 8*B*a*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a*c^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c^2*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*
a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 + 8*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) - 5*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e) + 9*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 4*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e))/((c^2*d^2 + 2*c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^3, x)
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

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3.300.9 Mupad [F(-1)] . . . 2487

3.300.1 Optimal result

Integrand size = 37, antiderivative size = 534

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e}{45045d^3f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)\sqrt{a+}}{45045d^2f}$$

$$-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a+a\sin(e+}{15015df}$$

$$-\frac{2a^3(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{9009d^3f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2a^3(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f}$$

$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df}$$

output
```
-4/15015*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*
d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/13*a*B*cos(f*x+e)*(
a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^4/d/f-4/45045*a^3*(c+d)*(15*c^2+10*
c*d+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-41
84*d^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/9009*a^3*(13*A*d*(3*c^2
-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(c+d*
sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/1287*a^3*(-39*A*c*d+299*A*d^2
+15*B*c^2-75*B*c*d+280*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^3/f/(a+a*sin
(f*x+e))^(1/2)-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*
(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d
^2/f+2/143*a^2*(-13*A*d+5*B*c-16*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*s
in(f*x+e))^(1/2)/d^2/f
```

3.300.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 1565, normalized size of antiderivative = 2.93

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^3,x]
```

output

```
(B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78
*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*
Cos[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^
(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3
 + 90*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3
)*((-1/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((8
0*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2
 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos
[(3*(e + f*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B
*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e +
f*x]))^(5/2)*((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[
(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^
3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*
A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e
+ f*x))/2] - (1/320 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d
+ 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]...
```

3.300.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.79, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3 dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 27$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3455$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3((13Ad(c+19d)-B(5c^2-9dc-216d^2))a^2+(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2)\sin(e+fx)a^2)dx}{11d}}{13d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 27$$

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3((13Ad(c+19d)-B(5c^2-9dc-216d^2))a^2+(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2)\sin(e+fx)a^2)dx}{11d}}{13d} + \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3((13Ad(c+19d)-B(5c^2-9dc-216d^2))a^2+(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2)\sin(e+fx)a^2)dx}{11d}}{13d} + \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3460$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(}{9df\sqrt{a\sin(e+fx)+a}}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(}{9df\sqrt{a\sin(e+fx)+a}}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3249$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2a^3\left(-39A}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2a^3\left(-39A}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3240$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin}{5af}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 27$$

$$\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3230$$

$$\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}\right)\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3042$$

$$\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}\right)\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\downarrow 3125$$

$$\frac{2a^2(-13Ad+5Bc-16Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}+\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\right.}{}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

3.300. $\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^4)/(1
3*d*f) + ((2*a^2*(5*B*c - 13*A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f) + ((-2*a^3*(15*B*c^2 - 39*A*c*d -
75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*
d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(13*A*d*(3*c^2 - 38*c*d + 355*d^2) -
B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*((-2*a*Cos[e + f*x]*(c + d*
Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[
e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d +
 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Co
s[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))/(5*a)))/7))/(9*d))/(11*d))/(13
*d)
```

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.300.4 Maple [A] (verified)

Time = 168.51 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(-3465B(\cos^6(fx+e))d^3+(4095Ad^3+12285d^2cB+11970d^3B)(\cos^4(fx+e))\sin(fx+e)+(15015c$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d^2(dA+3Bc)(63(\sin^5(fx+e))+224(\sin^4(fx+e))+355(\sin^3(fx+e))+426(\sin^2(fx+e))+568\sin(f}{693\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-3465*B*cos(f*x+e)^6*d^3+(4095*
A*d^3+12285*B*c*d^2+11970*B*d^3)*cos(f*x+e)^4*sin(f*x+e)+(15015*A*c*d^2+14
560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+(-19305*A*
c^2*d-55770*A*c*d^2-31265*A*d^3-6435*B*c^3-55770*B*c^2*d-93795*B*c*d^2-448
60*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^
2-56810*A*d^3-25740*B*c^3-123981*B*c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f
*x+e)^2+(42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3
+181038*B*c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)+138138*A*c^3+373230
*c^2*d*A+359502*d^2*c*A+116090*A*d^3+124410*B*c^3+359502*c^2*d*B+348270*d^
2*c*B+113818*d^3*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output

```
2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*
B)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*
B)*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d
^3 - 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*
a^2*d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d +
 39*(209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^
4 + (1287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*
A + 4370*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429
*(77*A + 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965
*B)*a^2*c*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*
A + 145*B)*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*
a^2*c*d^2 + (58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*co
s(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d -
 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B
*a^2*c*d^2 + (13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d +
 39*(11*A + 23*B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*
(1287*B*a^2*c^3 + 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2
 + (6253*A + 8972*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3
 + 429*(45*A + 53*B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A +
9083*B)*a^2*d^3)*cos(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(1...
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Timed out}$$

input

```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output

```
Timed out
```

3.300.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{5}{2}}(d\sin(fx + e) + c)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)^3, x)
```

3.300.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(506) = 1012$.

Time $= 0.55$ (sec) , antiderivative size $= 1014$, normalized size of antiderivative $= 1.90$

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/1441440*sqrt(2)*(3465*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-13/4*pi + 13/2*f*x + 13/2*e) + 180180*(40*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 30*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*A*a
^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 78*B*a^2*c^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 78*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 69*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*A*a^2*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 21*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(80*A*a^2*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 88*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 264*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*B*a^2*c^2*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 228*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7
6*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 71*B*a^2*d^3*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 9009*(16*A*a^2*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 40*B*a^2*c^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 120*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
44*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*A*a^2*c*d^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 50*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 51*B*a^2*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) ...
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
, x)
```

3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

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3.301.1 Optimal result

Integrand size = 37, antiderivative size = 429

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{2a^3(15c^2+10cd+7d^2)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)}{3465d^3f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{4a^2(5c-d)\left(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx}}{3465d^2f}$$

$$-\frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155df}$$

$$+\frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f}$$

$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df}$$

output
```
-2/1155*a*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3)
)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/11*a*B*cos(f*x+e)*(a+a*sin(f*x+e
))^(3/2)*(c+d*sin(f*x+e))^3/d/f-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(
c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/
(a+a*sin(f*x+e))^(1/2)+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d
^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a^2
*(5*c-d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/99*a^2*(-11*A*d+5*B*c-14*B*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

3.301.2 Mathematica [A] (verified)

Time = 7.73 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.76

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(164472Ac^2+137280Bc^2+274560Acd+248$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^2,x]
```

output
```
-1/27720*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*
x])]*(164472*A*c^2 + 137280*B*c^2 + 274560*A*c*d + 248732*B*c*d + 124366*A
*d^2 + 114640*B*d^2 - 8*(11*A*(63*c^2 + 360*c*d + 254*d^2) + 2*B*(990*c^2
+ 2794*c*d + 1625*d^2))*Cos[2*(e + f*x)] + 70*d*(22*B*c + 11*A*d + 32*B*d)
*Cos[4*(e + f*x)] + 51744*A*c^2*Sin[e + f*x] + 66660*B*c^2*Sin[e + f*x] +
133320*A*c*d*Sin[e + f*x] + 137104*B*c*d*Sin[e + f*x] + 68552*A*d^2*Sin[e
+ f*x] + 69890*B*d^2*Sin[e + f*x] - 1980*B*c^2*Sin[3*(e + f*x)] - 3960*A*c
*d*Sin[3*(e + f*x)] - 11440*B*c*d*Sin[3*(e + f*x)] - 5720*A*d^2*Sin[3*(e +
 f*x)] - 8675*B*d^2*Sin[3*(e + f*x)] + 315*B*d^2*Sin[5*(e + f*x)]))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.301.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.86, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 27$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3455$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a^2(11Ad(c+15d)-B(5c^2-11dc-138d^2))-a^2(11A(3c-19d)d-B(15c^2-65dc+194d^2))\sin(e+fx))dx}{9d} + \frac{11d}{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3} - \frac{11df}{}$$

$$\downarrow 27$$

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2\left(a^2\left(11Ad(c+15d)-B\left(5c^2-11dc-138d^2\right)\right)-a^2\left(11A(3c-19d)d-B\left(15c^2-65dc+194d^2\right)\right)\sin(e+fx)\right)dx}{9d}+\frac{2a}{}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2\left(a^2\left(11Ad(c+15d)-B\left(5c^2-11dc-138d^2\right)\right)-a^2\left(11A(3c-19d)d-B\left(15c^2-65dc+194d^2\right)\right)\sin(e+fx)\right)dx}{9d}+\frac{2a}{}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3460$$

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}+\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c-}{7df\sqrt{a\sin(e+fx)+a}}}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3042$$

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}+\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c-}{7df\sqrt{a\sin(e+fx)+a}}}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3240$$

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/}}{5af}\right)}{7d}}{9d}}{1}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 27$$

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\int \sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}}{9d}}{11}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3042$$

$$\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3230$$

$$\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+f}{7d}\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3042$$

$$\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+f}{7d}\right)}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$$\downarrow 3125$$

$$\frac{2a^2(-11Ad+5Bc-14Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}+\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}+$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

3.301. $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3)/(1
1*d*f) + ((2*a^2*(5*B*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*f) + ((2*a^3*(11*A*(3*c - 19*d)*d - B*
(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*d*f*S
qrt[a + a*Sin[e + f*x]]) + (3*a^2*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c
^3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(
3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(3*f))/(5*a)))/(7*d))/(9*d))/(11*d)
```

3.301.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.301.4 Maple [A] (verified)

Time = 36.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(315B(\cos^4(fx+e))\sin(fx+e)d^2+(385Ad^2+770cdB+1120d^2B)(\cos^4(fx+e))+(-990Acd-143($
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d(dA+2Bc)(35(\sin^4(fx+e))+130(\sin^3(fx+e))+219(\sin^2(fx+e))+292\sin(fx+e)+584)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}+\frac{2(1-}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^
2+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-990*A*c*d-1430*A*d^2-495
*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-693*A*c^2-3960*A*c
*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+(3234*A*c^2+8
580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*sin(f*x+e)+10626*A*
c^2+19140*A*c*d+9218*A*d^2+9570*B*c^2+18436*c*d*B+8930*d^2*B)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{2\left(315\,Ba^2d^2\cos(fx+e)^6+35\left(22\,Ba^2cd+(11\,A+32\,B)a^2d^2\right)\cos(fx+e)^5+1056\,(7\,A+5\,B)a^2c^2+7\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a
^2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*
c*d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c
*d + (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 +
22*(180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (
33*(77*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*
a^2*d^2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*
B)*a^2*c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(
f*x + e)^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(14
3*A + 125*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x +
 e)^4 - 5*(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d
^2)*cos(f*x + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d
 + (583*A + 655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 +
 22*(195*A + 211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.301.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{5}{2}}(d\sin(fx + e) + c)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)^2, x)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.59

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11
/4*pi + 11/2*f*x + 11/2*e) + 6930*(40*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 60*A*a^2*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 52*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 26*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*B*a
^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 + 2310*(20*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 22*B*a^2*c^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 44*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 40*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*A*a^2*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*B*a^2*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 693*(8*A*a^2*c^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 40*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*B*a^2*c*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 25*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*p
i + 5/2*f*x + 5/2*e) + 495*(4*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 8*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 13*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/
2*f*x + 7/2*e) + 385*(4*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +...
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^{5/2}\,(c + d\,\sin(e + f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
, x)
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

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3.302.1 Optimal result

Integrand size = 35, antiderivative size = 212

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f}$$
$$-\frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f}$$
$$-\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f}$$
$$-\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}$$

output

```
-2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f
-2/63*(9*A*d+9*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*B*d*cos(
f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)
*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B
*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.302.2 Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(7476Ac+6240Bc+6240Ad+5653Bd-4($$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x]),x]
```

output
```
-1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180
*A*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[
e + f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e
+ f*x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3
*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.302.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\downarrow 3447$$
$$\int (a\sin(e+fx)+a)^{5/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^{5/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 27$$

$$\frac{\int (\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 3230$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^{5/2}dx - \frac{2a(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}}{9a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^{5/2}dx - \frac{2a(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}}{9a} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 3126$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx - \frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right) - \frac{2a(9Ad+9Bc-2B}{9a}}{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}} - \frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\downarrow 3042$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\int(\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2a(9Ad+9Bc-2B}{}}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$
$$\downarrow 3126$$
$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin}{5f}\right)}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$
$$\downarrow 3042$$
$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin}{5f}\right)}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$
$$\downarrow 3125$$
$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)}{5f}\right)}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f) + ((-2*a*(9*B*c +
 9*A*d - 2*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*f) + (3*a*(21*
A*c + 15*B*c + 15*A*d + 13*B*d)*((-2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(5*f) + (8*a*((-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)))/5))/7)/(9*a)
```

3.302.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.302.4 Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(35B\left(\cos^4(fx+e)\right)d+(-45dA-45Bc-130dB)\left(\cos^2(fx+e)\right)\sin(fx+e)+(-63Ac-180dA-180Bc}{315\cos(fx+e)\sqrt{a+a\sin(fx+e}}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(dA+Bc)\left(3\left(\sin^3(fx+e)\right)+12\left(\sin^2(fx+e)\right)+23\sin(fx+e)+46\right)}{21\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))a^3(\sin(fx-}{15\cos(fx}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d+(-45*A*d-45*B
*c-130*B*d)*cos(f*x+e)^2*sin(f*x+e)+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(
f*x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+966*A*c+870*d*A+870*
B*c+838*d*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{2\left(35\,Ba^2d\cos(fx+e)^5-5\left(9\,Ba^2c+(9\,A+19\,B)a^2d\right)\cos(fx+e)^4+96\,(7\,A+5\,B)a^2c+32\,(15\,A+1\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos
(f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20
*B)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c
 + (255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (4
35*A + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A +
 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*
cos(f*x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x +
e)^2 - 2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.302.6 Sympy [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{5}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x)), x)
```

3.302.7 Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c), x)
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(192) = 384$.

Time $= 0.36$ (sec) , antiderivative size $= 404$, normalized size of antiderivative $= 1.91$

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{\sqrt{2}(35\,Ba^2d\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\sin(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e)+630\,(20\,Aa^2c

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
 + 9/2*f*x + 9/2*e) + 630*(20*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 15*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 13*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(10*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 11*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*A*a^2*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(2*A*a^2*c*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 5*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A
*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*d*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*B*a^2*c*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 5*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x +
7/2*e))*sqrt(a)/f
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^{5/2}\,(c + d\,\sin(e + f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),
x)
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

3.303.1 Optimal result . . . 2506
3.303.2 Mathematica [A] (verified) . . . 2506
3.303.3 Rubi [A] (verified) . . . 2507
3.303.4 Maple [A] (verified) . . . 2509
3.303.5 Fricas [A] (verification not implemented) . . . 2510
3.303.6 Sympy [F] . . . 2510
3.303.7 Maxima [F] . . . 2510
3.303.8 Giac [A] (verification not implemented) . . . 2511
3.303.9 Mupad [F(-1)] . . . 2511

3.303.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}$$

output
```
-2/35*a*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*cos(f*x+e)*(a+
a*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*A+5*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(
1/2)-16/105*a^2*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.303.2 Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(1246A+1040B-6(7A+20B)\cos(2(e+f$$
$$210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output
```
-1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(1246*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Si
n[e + f*x] - 15*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]))
```

3.303.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{1}{7}(7A+5B)\int (\sin(e+fx)a+a)^{5/2}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+5B)\int (\sin(e+fx)a+a)^{5/2}dx - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$$\downarrow 3126$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx - \frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right) - \frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int(\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$$\downarrow 3126$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a}{5f}\right.$$

$$\left.\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a}{5f}\right.$$

$$\left.\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}\right.$$

$$\downarrow 3125$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/}}{5f}\right.$$

$$\left.\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*f) + ((7*A + 5*B)*((-2*a
*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + (8*a*((-8*a^2*Cos[e + f*
x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f)))/5))/7
```

3.303.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.303.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(-15B(\cos^2(fx+e))\sin(fx+e)+(-21A-60B)(\cos^2(fx+e))+(98A+130B)\sin(fx+e)+322A+29}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2A(1+\sin(fx+e))a^3(\sin(fx+e)-1)(3(\sin^2(fx+e))+14\sin(fx+e)+43)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^3(\sin(fx+e)-1)(3(\sin^3(fx+e))+12}{21\cos(fx+e)\sqrt{a+a\sin(fx}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(-2
1*A-60*B)*cos(f*x+e)^2+(98*A+130*B)*sin(f*x+e)+322*A+290*B)/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e+fx))\,dx = \frac{2\left(15\,Ba^2\cos(fx+e)^4 + 3\,(7\,A+20\,B)a^2\cos(fx+e)^3 - (77\,A+85\,B)a^2\cos(fx+e\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
 + 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A
 + 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2
 - 2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

3.303.6 Sympy [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{5}{2}}\,(A + B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)
```

3.303.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(15\,Ba^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}fx+\frac{7}{2}e\right)+525\left(4\,Aa^2\mathrm{sgn}\left(\cos\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi +
7/2*f*x + 7/2*e) + 525*(4*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*
a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) +
35*(10*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

3.304.1 Optimal result . . . 2512
3.304.2 Mathematica [C] (verified) . . . 2513
3.304.3 Rubi [A] (verified) . . . 2513
3.304.4 Maple [B] (verified) . . . 2517
3.304.5 Fricas [B] (verification not implemented) . . . 2517
3.304.6 Sympy [F(-1)] . . . 2518
3.304.7 Maxima [F] . . . 2519
3.304.8 Giac [B] (verification not implemented) . . . 2519
3.304.9 Mupad [F(-1)] . . . 2520

3.304.1 Optimal result

Integrand size = 37, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2a^{5/2}(c-d)^2(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}f}$$
$$+\frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df}$$

output

```
-2/5*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f+2*a^(5/2)*(c-d)^2*(-A*d+B*c
)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d
^(7/2)/f/(c+d)^(1/2)+2/15*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*c
os(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/15*a^2*(-5*A*d+5*B*c-8*B*d)*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

3.304.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.38 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.55

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x]),x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 -
5*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[
(3*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] - (15*(c - d)^2*(B*c -
 A*d)*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum
[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[
(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^
(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*
x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 +
 Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 -
 c*#1^3) & ]))/(c + d)^(3/2) + (15*(c - d)^2*(B*c - A*d)*((c + d)*(e + f*x
 - 2*Log[Sec[(e + f*x)/4]^2]) - Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2
- 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2
)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]
- 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e
+ f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*L
og[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#
1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + ...
```

3.304.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\downarrow 3455$$

$$\frac{2\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 27$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3455$$

$$\frac{2\int -\frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+fx)a^2\right)}{2(c+d\sin(e+fx))}dx}{3d} + \frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(}}{3df}}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 27$$

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \int \frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}}{3d}}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3042$$

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \int \frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}}{3d}}{5d} - \frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3460$$

$$\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \frac{\frac{15a^2(c-d)^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} - \frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3042$$

$$\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \frac{\frac{15a^2(c-d)^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} - \frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 3252$$

$$\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \frac{-\frac{30a^3(c-d)^2(Bc-Ad)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df} - \frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-\right.\right.}{df\sqrt{a\sin(e}}}{3d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\downarrow 221$$

$$\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} - \frac{-\frac{30a^{5/2}(c-d)^2(Bc-Ad)\arctanh\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}} - \frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd-\right.\right.}{df\sqrt{a\sin(e+fx)-}}}{3d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*d*f) + ((2*a^2*(5*B*c
- 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f) - ((-30*a^
(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^3*(5*A*(3
*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*
Sin[e + f*x]]))/(3*d))/(5*d)
```

3.304. $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

3.304.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.304.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(192) = 384.

Time = 2.94 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.49

method	result
default	$\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3B(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{a(c+d)d}\,d^2+5A(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{a(c+d)d}\,a\,d^2-15A\,\operatorname{arctanh}\left($

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)*(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^3-30*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c^2+45*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-60*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2)/d^3/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(192) = 384.

Time = 1.30 (sec) , antiderivative size = 1314, normalized size of antiderivative = 6.03

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

output

```
[1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d
^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*c
os(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a
^2*d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^
2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*
d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos
(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9
*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2
*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
 (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e
))) + 4*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d
 - (35*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x +
e)^2 - (15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*co
s(f*x + e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^
2*c*d - (35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3
*f*sin(f*x + e) + d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A
 + B)*a^2*c*d^2 - A*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A +...
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output

```
Timed out
```

3.304.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e)
+ c), x)
```

3.304.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(192) = 384$.

Time $= 0.33$ (sec) , antiderivative size $= 523$, normalized size of antiderivative $= 2.40$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \sqrt{2}\sqrt{a}\left(\frac{15\sqrt{2}(Ba^2c^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-Aa^2c^2d\mathrm{sgn}(\cos(-\frac{1}{4}\pi-\right.$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/15*sqrt(2)*sqrt(a)*(15*sqrt(2)*(B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^2*d*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^2*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d^3) + 2*(12*B*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 10*B*a^
2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^3 - 10*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 - 40*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 15*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 15*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45*B*a^2*c*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 45*A*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*B*a^2*
d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^
5)/f
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
, x)
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

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3.305.1 Optimal result

Integrand size = 37, antiderivative size = 265

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\operatorname{arctanh}}{d^{7/2}(c+d)^{3/2}f}$$
$$-\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))}$$

output

```
a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(
1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f+a*(
-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))-1/3
*a^3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*s
in(f*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/d^2/(c+d)/f
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.43 (sec) , antiderivative size = 1002, normalized size of antiderivative = 3.78

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
 + f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(-(A*d*(3*c + 5
*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/
4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & ,
 (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*
x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 +
 Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/
4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#
1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + (3*(c - d)*(-(A
*d*(3*c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(-((c + d)*(e + f*x - 2*Log[S
ec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^
3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1
 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqr
t[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(
e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) ...
```

3.305.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3454, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\downarrow 3455$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2\int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\sin(e+fx)\right)}{2(c+d\sin(e+fx))}dx}{3d} + \frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin}}{3df}}{2d(c+d)}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{\int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{3d} + \frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin}}{3df}}{2d(c+d)}$$

$$\downarrow 3042$$

3.305. $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{3d} + \frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin}}{3df}}{2d(c+d)}$$

$$\downarrow 3460$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{3a^2(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} + \frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d} + \frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)}{3df}}{2d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{3a^2(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d} + \frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d} + \frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)}{3df}}{2d(c+d)}$$

$$\downarrow 3252$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{6a^3(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}}{3d} + \frac{2a^2(-3Ad+5}{2d(c+d)}$$

$$\downarrow 221$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df} + \frac{\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{6a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)a}{d^{3/2}f\sqrt{c+}}}{3d}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

3.305. $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c +
d*Sin[e + f*x])) - ((2*a^2*(5*B*c - 3*A*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a
*Sin[e + f*x]])/(3*d*f) + ((-6*a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2
 + 5*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqr
t[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^3*(3*A*d*(3*c + d)
 - B*(15*c^2 - 5*c*d - 14*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]
]))/(3*d))/(2*d*(c + d))
```

3.305.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.305.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(241) = 482$.

Time = 10.18 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	932

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(-9*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-6*A*arctanh((a-a
*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+15*A*arctanh((a-a*sin(
f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-2*B*(a-a*sin(f*x+e))^(3/2)*(a
*(c+d)*d)^(1/2)*c*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2+15*a^
2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3-21*B*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+6*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+6*A*(a-a*sin(f*x+e))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c*d+6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d
^2-12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2+6*B*(a-a*sin(f*x+e)
)^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d+18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1
/2)*a*d^2)-9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^3*d-6*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2
+15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-2*B*
(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-2*B*(a-a*sin(f*x+e))^(3/2)*
(a*(c+d)*d)^(1/2)*c*d^2+15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*B*c^4-21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^2*c^2*d^2+6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*c*d^3+9*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c...
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(241) = 482$.

Time $= 1.49$ (sec) , antiderivative size $= 2046$, normalized size of antiderivative $= 7.72$

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output

```
[-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
 (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^
2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B
*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3
)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^
2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A
*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*
sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*
c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*
d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e
) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*
d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*co
s(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*
(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A +
14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^
2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*
a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3...
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input

```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output

```
Timed out
```

3.305.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e)
+ c)^2, x)
```

3.305.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(241) = 482$.

Time $= 0.36$ (sec) , antiderivative size $= 597$, normalized size of antiderivative $= 2.25$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*(5*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*A*a^2*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^2
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^3 + d^4)*sqrt(-c*d - d^2)) - 6*(B*
a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) - 2*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2 - c - d)) + 4*(2*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*B*a^2*c*d^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*d^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*B*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^6)/f
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^2, x)
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

3.306.1 Optimal result 2531
3.306.2 Mathematica [C] (warning: unable to verify) 2532
3.306.3 Rubi [A] (verified) 2532
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3.306.9 Mupad [F(-1)] 2539

3.306.1 Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{7/2}(c+d)^{5/2}f}$$
$$+\frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2}$$
$$-\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))}$$

output

```
-1/4*a^(5/2)*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3)
)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d
^(7/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/
(c+d)/f/(c+d*sin(f*x+e))^2+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))
*cos(f*x+e)/d^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(A*d*(c+7*d)-B*(5
*c^2+7*c*d-4*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/(c+d*si
n(f*x+e))
```

3.306.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 12.86 (sec) , antiderivative size = 1046, normalized size of antiderivative = 3.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^3,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(((A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3
 + 30*c^2*d + 7*c*d^2 - 20*d^3))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & ,
(-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x
)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c
 + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(7/2) + ((A*d*(3*c^2 + 1
0*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(-((c + d)*(e
+ f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c
*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) -
d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x
)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + T
an[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqr
t[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*
Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c
 + d)^(7/2) - (4*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^...
```

3.306.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))^2}dx}{2d(c+d)} + \frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\downarrow 3454$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\int -\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+25dc+4d^2\right)\right)\sin(e+fx)\right)}{2(c+d\sin(e+fx))}dx}{d(c+d)} + \frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(}{df(c+d)(c+d\sin(e+}$$

$$\downarrow 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} - \frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+25\right.\right.\right.}{c+d\sin(e+fx)}}{2d(c+d)}}{4d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+25\right.\right.}{c+d\sin(e+fx)}}{2d(c+d)}}{4d(c+d)}$$

$$\downarrow 3460$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{a^2\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^3(}{2d(c+d)}}{4d(c+d)}$$

$$\downarrow 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{a^2\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^3(}{2d(c+d)}}{4d(c+d)}$$

$$\downarrow 3252$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{2a^3\left(Ad\left(3c^2+10cd+19d^2\right)-B(15c\right.}{2d(c+d)}}{4d(c+d)}$$

$$\downarrow 221$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2} -$$

$$\frac{\frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))} - \frac{\frac{2a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}} - \frac{2a^{5/2}\left(Ad\left(3c^2+10cd+19d^2\right)-B(\right.}{2d(c+d)}}{4d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

3.306. $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - ((a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d^2))*C
os[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((-2*a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*
d^2 - 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a
+ a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^3*(3*A*d*(c + 3*d) - B
*(15*c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]]))/
(2*d*(c + d)))/(4*d*(c + d))
```

3.306.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.306.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(280) = 560$.

Time = 63.26 (sec) , antiderivative size = 1587, normalized size of antiderivative = 5.15

method	result	size
default	Expression too large to display	1587

input

```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output

```
-1/4*a*(-7*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*
x+e)^2*a^2*c*d^4+6*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*sin(f*x+e)*a^2*c^3*d^2+13*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a*d^4+15*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^4+10*A*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+19*A*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3+4*B*(-a*(sin(f*x+
e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+5*A*(-a*(sin(f*x+e)-1))
^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3+38*A*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c*d^4+8*B*(-a*(sin(f*x+e)-1))^(
1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^4-30*B*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^4*d+6*A*(-a*(sin(f*x+e)-1))^(
3/2)*(a*(c+d)*d)^(1/2)*c*d^3-15*a^2*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*B*c^5-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4
-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-9*B*(-a*(sin(f*x+e)-
1))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d-2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d
)^(1/2)*c^2*d^2+15*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3-30*
B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d-7*B*...
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. 2(280) = 560.

Time = 1.73 (sec) , antiderivative size = 3046, normalized size of antiderivative = 9.89

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
[1/16*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2
 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a
^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c
*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A -
25*B)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4
- (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2
*c^4*d - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A -
 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*
(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*
d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^
2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*
d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A
 - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))
*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 -
 (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2
+ d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d +
4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e)
+ (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*
cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x...
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.306.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e)
+ c)^3, x)
```

3.306.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(280) = 560$.

Time $= 0.41$ (sec) , antiderivative size $= 895$, normalized size of antiderivative $= 2.91$

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output

```
1/8*sqrt(2)*(16*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)/d^3 + sqrt(2)*(15*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a^2*c^2*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*A*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*B*a^2*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt
(-c*d - d^2))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(-c*d - d^2)) - 2*(18*B*a^2*c
^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 10*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 + 4*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 12*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 30*B*a^2*c*d^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 22*A*a^2*d^4*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*a^2*d^4
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 7*
B*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 3*A*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 13*B*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) + 13*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/...
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^3,x)
```

output

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^3, x)
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

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3.307.1 Optimal result

Integrand size = 37, antiderivative size = 284

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= -\frac{\sqrt{2}(A-B)(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} \\
&\quad - \frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af} \\
&\quad - \frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

output

```
-(A-B)*(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))*2^(1/2)/f/a^(1/2)-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*
d+144*c*d^2-37*d^3))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/35*(7*A*d+6*B*c
-B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/7*B*cos(f*x
+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-2/105*d*(7*A*(9*c-d)*d+B*(
24*c^2-15*c*d+31*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

3.307.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.32

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((840+840i)(-1)^{3/4}(A-B)(c-d)^3\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1-\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(
c - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*
A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos
[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3
*(e + f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*
d^3*Cos[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 -
 12*c^2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d +
B*(12*c^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*
c + d))*Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt
[a*(1 + Sin[e + f*x])])
```

3.307.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a\sin(e+fx)+a}}dx$$
$$\downarrow 3462$$
$$\frac{2\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{7a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{7a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{7a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3462$$

$$\frac{2\int \frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+}{5f\sqrt{a\sin(e+}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\Bigg|_0^{7a}$$

$$\downarrow 27$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\Bigg|_0^{7a}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\Bigg|_0^{7a}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\Bigg|_0^{7a}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{\frac{7a}{3}}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx)}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e}{5f\sqrt{a\sin(}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2a(7Ad+6Bc-Bd)\cos(e+fx)}{5f\sqrt{a\sin(e+fx}}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^{7a}}{7f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

output
```
$Aborted
```

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3462

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

3.307.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(259) = 518$.

Time = 3.56 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.97

method	result
parts	$-\frac{A c^3(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f} + \frac{c^2(3dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1}}{a\cos(fx}$
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(105A\,a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^3-315A\,a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^2\right.$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
-A*c^3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(
1/2)/f+c^2*(3*A*d+B*c)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(f*x+
e))^(1/2))/a/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/15*d^2*(A*d+3*B*c)*(1+s
in(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-6*(a-a*sin(f*x+e))^(5/2)+10*a*(a-a*sin(
f*x+e))^(3/2)-30*a^2*(a-a*sin(f*x+e))^(1/2))/a^3/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f-1/105*d^3*B*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*a^(7/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-30*(a-a*sin
(f*x+e))^(7/2)+84*(a-a*sin(f*x+e))^(5/2)*a-140*a^2*(a-a*sin(f*x+e))^(3/2))
/a^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+c*d*(A*d+B*c)*(1+sin(f*x+e))*(-a*
(sin(f*x+e)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))+2*(a-a*sin(f*x+e))^(3/2))/a^2/cos(f*x+e)/(a+a*sin(f*x+
e))^(1/2)/f
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(259) = 518$.

Time = 0.29 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.21

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{105\sqrt{2}((A-B)ac^3-3(A-B)ac^2d+3(A-B)acd^2-(A-B)ad^3+((A-B)ac^3-3(A-B)ac^2d+3(A-B)acd^2-(A-B)ad^3)\cos(fx+e)+((A-B)ac^{3}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2
- (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A -
 B)*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
 - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(
f*x + e)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 -
(119*A - 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*
B*c^2*d + 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105
*B*c^3 + 105*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3
)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^
2*d - 21*(10*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A -
 6*B)*d^3)*cos(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*
B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x
 + e) + a*f*sin(f*x + e) + a*f)
```

3.307.6 Sympy [F]

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

3.307.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e)
+ a), x)
```

3.307.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(259) = 518$.

Time $= 0.41$ (sec) , antiderivative size $= 555$, normalized size of antiderivative $= 1.95$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\frac{105\sqrt{2}(A\sqrt{a}c^3-B\sqrt{a}c^3-3A\sqrt{a}c^2d+3B\sqrt{a}c^2d+3A\sqrt{a}cd^2-3B\sqrt{a}cd^2-A\sqrt{a}d^3+B\sqrt{a}d^3)\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{105\sqrt{2}(A\sqrt{a}c^3-}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/210*(105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sqrt(a)*c^2*d + 3*
B*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 - A*sqrt(a)*d^3 +
B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))) - 105*sqrt(2)*(A*sqrt(a)*c^3 - B*sqrt(a)*c^3 - 3*A*sq
rt(a)*c^2*d + 3*B*sqrt(a)*c^2*d + 3*A*sqrt(a)*c*d^2 - 3*B*sqrt(a)*c*d^2 -
A*sqrt(a)*d^3 + B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sqrt(2)*(120*B*a^(13/2)*d^3*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^7 - 252*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^5 - 84*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 168*B*a^
(13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 210*B*a^(13/2)*c^2*d*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 + 210*A*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^3 + 210*B*a^(13/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 70*A*a^(
13/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 140*B*a^(13/2)*d^3*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 - 105*B*a^(13/2)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 - 315*A*a^(13/2)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 315*B*a^(13/2)*c*
d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 105*A*a^(13/2)*d^3*sin(-1/4*pi + 1/2*
f*x + 1/2*e))/(a^7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^3}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/
2), x)
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

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3.308.1 Optimal result

Integrand size = 37, antiderivative size = 200

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= -\frac{\sqrt{2}(A-B)(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} \\
&\quad - \frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} \\
&\quad - \frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} \\
&\quad - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

output

```
-(A-B)*(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))*2^(1/2)/f/a^(1/2)-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*cos(f*x+e
)/f/(a+a*sin(f*x+e))^(1/2)-2/5*B*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(
f*x+e))^(1/2)-2/15*d*(5*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a
/f
```

3.308.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((60+60i)(-1)^{3/4}(A-B)(c-d)^2\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+t\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c
- d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*
c - d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c
 + d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c -
d)*d + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d
))*Sin[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

3.308.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\downarrow 3462$$

$$\frac{2\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3447$$

$$\frac{\int \frac{ad(4Bc+5Ad-Bd)\sin^2(e+fx)+(ac(4Bc+5Ad-Bd)+ad(5Ac-Bc+4Bd))\sin(e+fx)+ac(5Ac-Bc+4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{ad(4Bc+5Ad-Bd)\sin(e+fx)^2+(ac(4Bc+5Ad-Bd)+ad(5Ac-Bc+4Bd))\sin(e+fx)+ac(5Ac-Bc+4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3502$$

$$\frac{2\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}}dx}{3a} - \frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a} - \frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3230$$

$$\frac{\frac{15a^2(A-B)(c-d)^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$\frac{\frac{15a^2(A-B)(c-d)^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3128$$

$$\frac{-\frac{30a^2(A-B)(c-d)^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 219$$

$$\frac{-\frac{15\sqrt{2}a^{3/2}(A-B)(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

3.308. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}dx$

output
```
(-2*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
+ ((-2*d*(4*B*c + 5*A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f
) + ((-15*Sqrt[2]*a^(3/2)*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])
/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f - (4*a^2*(5*A*(3*c - d)*d + B*(6*c
^2 - 7*c*d + 7*d^2))*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(3*a))/(5
*a)
```

3.308.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.308.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(179) = 358.

Time = 2.62 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.98

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(15Aa^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^2-30Aa^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)cd+\right.$
parts	$-\frac{Ac^2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{c(2dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}}{a\cos(fx}$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a
*sin(f*x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x
+e))^(3/2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*(a-a*sin(f*x+e))^(1/
2)*A*a^2*c*d+30*(a-a*sin(f*x+e))^(1/2)*B*a^2*c^2+30*(a-a*sin(f*x+e))^(1/2)
*B*a^2*d^2)/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.24

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{15\sqrt{2}\left((A-B)ac^2-2(A-B)acd+(A-B)ad^2+\left((A-B)ac^2-2(A-B)acd+(A-B)ad^2\right)\cos(fx+e)+\left((A-B)ac^2-2(A-B)acd+(A-B)ad^2\right)\sin\right.}{\sqrt{a}}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
 B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2
 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f
*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^
2*cos(f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10
*B*c*d + (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d -
(5*A - 16*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(
3*A - 2*B)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*
x + e) + a*f)
```

3.308.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

3.308.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e)
+ a), x)
```

3.308.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(179) = 358$.

Time $= 0.35$ (sec) , antiderivative size $= 363$, normalized size of antiderivative $= 1.82$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{15\sqrt{2}(A\sqrt{a}c^2-B\sqrt{a}c^2-2A\sqrt{a}cd+2B\sqrt{a}cd+A\sqrt{a}d^2-B\sqrt{a}d^2)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{15\sqrt{2}(A\sqrt{a}c^2-B\sqrt{a}c^2-2A\sqrt{a}cd+2B\sqrt{a}cd-}{a\operatorname{sgn}\left(\cos(-\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
1/30*(15*sqrt(2)*(A*sqrt(a)*c^2 - B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sq
rt(a)*c*d + A*sqrt(a)*d^2 - B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c
^2 - B*sqrt(a)*c^2 - 2*A*sqrt(a)*c*d + 2*B*sqrt(a)*c*d + A*sqrt(a)*d^2 - B
*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))) + 4*sqrt(2)*(12*B*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^5 - 20*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*A*a^(9/
2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 10*B*a^(9/2)*d^2*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^3 + 15*B*a^(9/2)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 30*A
*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15*B*a^(9/2)*d^2*sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(a^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^2}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/
2), x)
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

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3.309.1 Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{\sqrt{2}(A-B)(c-d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}$$
$$-\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

output
```
-(A-B)*(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)*2^(1/2)/f/a^(1/2)-2/3*(3*A*d+3*B*c-2*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^
(1/2)-2/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

3.309.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-6-6i)(-1)^{3/4}(A-B)(c-d)\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}(-1+\right.\right.}{3f\sqrt{a(1+\sin}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)
*(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x]))
)/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

3.309.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3502$$

$$\frac{2\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}\,dx}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}\,dx}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 3230$$

$$\frac{3a(A-B)(c-d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 3042$$

$$\frac{3a(A-B)(c-d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 3128$$

$$\frac{-\frac{6a(A-B)(c-d)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f} - \frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\downarrow 219$$

$$\frac{-\frac{3\sqrt{2}\sqrt{a}(A-B)(c-d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f} - \frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} - \frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x
]
```

output
```
(-2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f) + ((-3*Sqrt[2]*Sqrt
[a]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/f - (2*a*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(f*Sqrt[a + a
*Sin[e + f*x]]))/(3*a)
```

3.309.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.309.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(113) = 226$.

Time = 2.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(3Aa^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)c-3Aa^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)d-3Ba\right.}{3a^2}$
parts	$-\frac{Ac(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}-\frac{(dA+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\right.}{a\cos(fx+}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*d+6*(a-a*sin(f*x+e))^(1/2)*A*a*d+6*(a-a*sin(f*x+e))^(1/2)*B*a*c)/a^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(113) = 226$.

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.33

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{3\sqrt{2}((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos(fx+e)+((A-B)ac-(A-B)ad)\sin(fx+e))\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-2}{\cos(fx+e)^2-(c}\right.}{\sqrt{a}}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*co
s(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(
cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B
*d*cos(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x
+ e) + (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.309.6 Sympy [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{\sqrt{a + a\sin(e + fx)}}\,dx = \int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) +
1)), x)
```

3.309.7 Maxima [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{\sqrt{a + a\sin(e + fx)}}\,dx = \int \frac{(B\sin(fx + e) + A)(d\sin(fx + e) + c)}{\sqrt{a\sin(fx + e) + a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) +
a), x)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{3\sqrt{2}(A\sqrt{ac}-B\sqrt{ac}-A\sqrt{ad}+B\sqrt{ad})\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{3\sqrt{2}(A\sqrt{ac}-B\sqrt{ac}-A\sqrt{ad}+B\sqrt{ad})\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} -$$

$$\frac{}{6f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
1/6*(3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log
(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)) - 3*sqrt(2)*(A*sqrt(a)*c - B*sqrt(a)*c - A*sqrt(a)*d + B*sqrt(a)*d)*log
(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))) - 4*sqrt(2)*(2*B*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a^(5
/2)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^(5/2)*d*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2)
, x)
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

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3.310.2 Mathematica [C] (verified) . . . 2568
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3.310.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
-(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1
/2)/f/a^(1/2)-2*B*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

3.310.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}(A-B)\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+\right.\right.\right.}{f\sqrt{a(1+\sin(e+fx))}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTa
nh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

3.310.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3230$$

$$(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3128$$

$$-\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f} - \frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 219$$

$$-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f} - \frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]
)
```

3.310.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.310.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)A-\sqrt{a}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)B+2B\sqrt{a-}\right.}{a\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}$
parts	$-\frac{A(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\cos(fx+e)\sqrt{a+a\sin(fx+e)}f}+\frac{B(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\sqrt{2}\right.\text{ arc}}{a\cos(fx+e)\sqrt{a}}$
risch	$-\frac{\left(-2iA+iB+B\,\mathrm{e}^{i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+i\right)\sqrt{2}\,\mathrm{e}^{-i(fx+e)}}{f\sqrt{-a\left(-2\,\mathrm{e}^{i(fx+e)}+i\mathrm{e}^{2i(fx+e)}-i\right)\mathrm{e}^{-i(fx+e)}}}-\frac{2i(A-B)\left(\mathrm{e}^{i(fx+e)}+i\right)\left(\arctan\left(\frac{\sqrt{-i\mathrm{e}^{i(fx+e)}a}}{\sqrt{a}}\right)a\sqrt{-i\mathrm{e}^{i(fx+e)}a}+a^{\frac{3}{2}}\right)\sqrt{}}{fa^{\frac{3}{2}}\sqrt{-a\left(-2\,\mathrm{e}^{i(fx+e)}+i\mathrm{e}^{2i(fx+e)}-i\right)\mathrm{e}^{-i(fx+e)}}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*B*(a-a*sin(f*x+e))^(1/2))/a/cos(f*x+e
)/(a+a*sin(f*x+e))^(1/2)/f
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(68) = 136$.

Time $= 0.26$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 2.66$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}}\,dx =$$

$$-\frac{\sqrt{2}((A-B)a\cos(fx+e)+(A-B)a\sin(fx+e)+(A-B)a)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+}{\sqrt{a}}}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{a}}$$
$$\overline{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a
)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(
a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

3.310.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}}\,dx = \int \frac{A + B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.310.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)
```

3.310.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{4\sqrt{2}B\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{2f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/2*(4*sqrt(2)*B*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(sqrt(a)*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))) + sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*s
qrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.310.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{A\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{fx}{2}\middle|1\right)\sqrt{\frac{2\,(a+a\sin(e+fx))}{a}}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\left(4\,\mathrm{E}\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-\sin(e+fx)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-\sin(e+fx)}}{2}\right)\middle|1\right)\right)\sqrt{\cos(e+fx)^2}\sqrt{\frac{a+a\sin(e+fx)}{2a}}}{f\cos(e+fx)\sqrt{a+a\sin(e+fx)}}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
- (A*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2)
)/(f*(a + a*sin(e + f*x))^(1/2)) - (B*(4*ellipticE(asin((2^(1/2)*(1 - sin(
e + f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/
2))/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*
cos(e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

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3.311.1 Optimal result

Integrand size = 37, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

output
```
-(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1
/2)/(c-d)/f/a^(1/2)-2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

3.311.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.09 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.55

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{(-1)^{3/4}\left((4+4i)(A-B)\sqrt{d}\sqrt{c+d}\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt[4]{-1}(Bc-Ad)\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])),x]
```

output
```
((-1)^(3/4)*((4 + 4*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*
#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) +
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x
)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*
#1^3) & ] - (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^
3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]
*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 +
 Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]))/(2*(c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[a*(1 + Si
n[e + f*x])])
```

3.311.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}dx$$

$$\downarrow 3464$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d} + \frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}$$

$$\downarrow 3042$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}$$

$$\downarrow 3128$$

$$\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}$$

$$\downarrow 219$$

$$\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\downarrow 3252$$

$$-\frac{2(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\downarrow 221$$

$$-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x
]
```

output
```
-((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d
]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*
Sqrt[d]*Sqrt[c + d]*f)
```

3.311.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.311.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a(c+d)d}\,A-2A\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)\sqrt{a}\,d\right.}{(c-d)\sqrt{a(c+d)d}\sqrt{a}\cos(fx+e)\sqrt{a+a\sin($}$

input
```
int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*A-2*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d-2^(1/2)*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*B+2*B*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c)/(c-d)/(a*(c+d)*d)^(1/2
)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(113) = 226$.

Time $= 0.84$ (sec) , antiderivative size $= 744$, normalized size of antiderivative $= 5.47$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$

$$= \left[\frac{\sqrt{acd+ad^2}(Bc-Ad)\log\left(\frac{ad^2\cos(fx+e)^3 - ac^2 - 2\,acd - ad^2 - (6\,acd+7\,ad^2)\cos(fx+e)^2 - 4\sqrt{acd+ad^2}\left(d\cos(fx+e)^2 - (c+2\,d)\cos\right.\right.}{d^2\cos(fx+e)^3 + (2\,cd+d^2)\cos} \right.$$

$$\left. -\frac{2\sqrt{-acd-ad^2}(Bc-Ad)\arctan\left(\frac{\sqrt{-acd-ad^2}\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)-c-2\,d)}{2\,(acd+ad^2)\cos(fx+e)}\right) - \frac{\sqrt{2}((A-B)acd+(A-B)ad^2)\log\left(\right.}{2\,(ac^2d-ad^3)f}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^
2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)
*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a
*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(
3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*
d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (
d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*
x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e
) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^
3)*f), -1/2*(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a
*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)
*cos(f*x + e))) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x +
e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((
a*c^2*d - a*d^3)*f)]
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.311.7 Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))}\,dx = \int \frac{B\sin(fx + e) + A}{\sqrt{a\sin(fx + e) + a}(d\sin(fx + e) + c)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
 c)), x)
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(113) = 226.

Time = 0.33 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.85

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}(c + d\sin(e+fx))}\,dx =$$
$$-\frac{\frac{2\sqrt{2}(B\sqrt{a}c - A\sqrt{a}d)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{\left(\sqrt{2}a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \sqrt{2}a d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)\sqrt{-cd-d^2}} - \frac{(A\sqrt{a} - B\sqrt{a})\log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right.}{\sqrt{2}a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \sqrt{2}a d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi\right.\right.}}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-(2*sqrt(2)*(B*sqrt(a)*c - A*sqrt(a)*d)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2
*f*x + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2
)) - (A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt
(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))) + (A*sqrt(a) - B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(sqrt(2)*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a
*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}(c + d\sin(e+fx))}\,dx$$
$$= \int \frac{A + B\,\sin(e+f\,x)}{\sqrt{a + a\,\sin(e+f\,x)}\,(c + d\,\sin(e+f\,x))}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)))
, x)
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$

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3.312.1 Optimal result

Integrand size = 37, antiderivative size = 207

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx \\
&= -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2 f} \\
&\quad + \frac{(Ad(3c+d)-B(c^2+cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2\sqrt{d}(c+d)^{3/2} f} \\
&\quad - \frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1
/2)/(c-d)^2/f/a^(1/2)+(A*d*(3*c+d)-B*(c^2+c*d+2*d^2))*arctanh(cos(f*x+e)*a
^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)^2/(c+d)^(3/2)/f/a
^(1/2)/d^(1/2)-(-A*d+B*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))/(a+a*sin
(f*x+e))^(1/2)
```

3.312.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.97 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.56

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{(\cos(\frac{1}{2}(e+fx))+\sin(\frac{1}{2}(e+fx)))\left((8+8i)(-1)^{3/4}(A-B)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTa
nh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) +
B*(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c +
4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]
) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e +
 f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*
Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2
- c*#1^3) & ]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*
d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*
c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#
1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + T
an[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
 ]))/(Sqrt[d]*(c + d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*
(1 + Sin[e + f*x])])
```

3.312.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3464$$

$$\frac{\frac{2a(A-B)(c+d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{2a(A-B)(c+d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3128$$

$$\frac{-\frac{4a(A-B)(c+d)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 219$$

$$\frac{-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 3252$$

$$\frac{\frac{2a\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\downarrow 221$$

$$\frac{\frac{2\sqrt{a}(Ad(3c+d)-B(c^2+cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2)
,x]
```

output
```
((-2*Sqrt[2]*Sqrt[a]*(A - B)*(c + d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[
2]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*f) + (2*Sqrt[a]*(A*d*(3*c + d) - B
*(c^2 + c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[d]*Sqrt[c + d]*f))/(2*a*(c^2 - d
^2)) - ((B*c - A*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x]))
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.312.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(182) = 364$.

Time = 1.00 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.34

method	result	size
default	Expression too large to display	899

input
```
int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(-sin(f*x+e)*d*(-3*A*a^(5
/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d-A*a^(5/2)*ar
ctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+B*a^(5/2)*arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2+B*a^(5/2)*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d+2*B*a^(5/2)*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+A*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d-B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c-B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d)+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a
^(5/2)*c^2*d+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/
2)*c*d^2-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c
^3-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d-2
*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+A*(
a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-A*(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-B*(a-a*sin(f*
x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(...
```

3.312.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(182) = 364$.

Time = 2.12 (sec) , antiderivative size = 2159, normalized size of antiderivative = 10.43

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
[-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B
*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A -
 B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (
4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*
d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x +
e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sq
rt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x
+ e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2
+ 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*
d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x
 + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*c
os(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2
)*sin(f*x + e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A
 - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (
A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A
 - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(
A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 +
(A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
 - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (c...
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.312.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$$
$$= \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
 c)^2), x)
```

3.312.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(182) = 364$.

Time $= 0.38$ (sec) , antiderivative size $= 492$, normalized size of antiderivative $= 2.38$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx =$$
$$-\frac{\sqrt{2}\left(\sqrt{2}B\sqrt{a}c^2-3\sqrt{2}A\sqrt{a}cd+\sqrt{2}B\sqrt{a}cd-\sqrt{2}A\sqrt{a}d^2+2\sqrt{2}B\sqrt{a}d^2\right)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(ac^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-ac^2d\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))-acd^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+ad^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\sqrt{-cd}}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/2*(sqrt(2)*(sqrt(2)*B*sqrt(a)*c^2 - 3*sqrt(2)*A*sqrt(a)*c*d + sqrt(2)*B
*sqrt(a)*c*d - sqrt(2)*A*sqrt(a)*d^2 + 2*sqrt(2)*B*sqrt(a)*d^2)*arctan(sqr
t(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)))*sqrt(-c*d - d^2)) - 2*(A*sqrt(a) - B*sqrt(a))*log(sin(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e)) - 2*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a*d^2*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*(A*sqrt(a) - B*sqrt(a))*log(-sin(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - 2*sqrt(2)*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(B*sqrt(a)*c*sin(-1/4*pi + 1
/2*f*x + 1/2*e) - A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a*
c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))/f
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
2), x)
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

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3.313.1 Optimal result

Integrand size = 37, antiderivative size = 309

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{\sqrt{2}(A-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3 f} \\
&\quad + \frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{a}(c-d)^3\sqrt{d}(c+d)^{5/2} f} \\
&\quad - \frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right) f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&\quad + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4\left(c^2-d^2\right)^2 f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

output

```
-(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1
/2)/(c-d)^3/f/a^(1/2)+1/4*(A*d*(15*c^2+10*c*d+7*d^2)-B*(3*c^3+6*c^2*d+19*c
*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e
))^(1/2))/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)/d^(1/2)-1/2*(-A*d+B*c)*cos(f*x+e)/
(c^2-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*(A*d*(7*c+d)-B*(
3*c^2+c*d+4*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e
))^(1/2)
```

3.313.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.96 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.76

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((32+32i)(-1)^{3/4}(A-B)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^3),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((32 + 32*I)*(-1)^(3/4)*(A - B)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + ((A*d*(15*c^2 + 10*
c*d + 7*d^2) - B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Se
c[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & ,
(-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e +
 f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/
4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(Sqrt[d]*(c + d)^(5/2)) +
((-(A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 -
4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c +
 d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqr
t[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f
*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Lo
g[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(Sqr
t[d]*(c + d)^(5/2)) - (8*(c - d)^2*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*(c - d)*(-(A*d*(7*c + d)
) + B*(3*c^2 + c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
d)^2*(c + d*Sin[e + f*x])))/(16*(c - d)^3*f*Sqrt[a*(1 + Sin[e + f*x])]...
```

3.313.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}\,dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}\,dx}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}\,dx}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}\,dx}{a\left(c^2-d^2\right)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}\,dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}-$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}-$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}-$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int-\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int-\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

3.313. $\int\frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}}$$

$$\downarrow 25$$

$$\frac{\int-\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}}$$

$$\downarrow 25$$

$$\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}}{\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}}$$

$$\downarrow 25$$

$$\frac{\int-\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{\frac{4a\left(c^2-d^2\right)}{\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)
,x]
```

output
```
$Aborted
```

3.313. $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

3.313.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

3.313.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(276) = 552$.

Time = 1.59 (sec) , antiderivative size = 2275, normalized size of antiderivative = 7.36

method	result	size
default	Expression too large to display	2275

input
```
int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/4*(-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c
^5-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*a^4*c^3*d-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^3*d+4*B*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
/2))*a^4*c^2*d^2-4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4+4*B*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a
^4*d^4-6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*
c^4*d+A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4-5*B*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^4+4*B*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4+A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d
)^(1/2)*a^(5/2)*d^4-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2
)*d^4-4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*s
in(f*x+e)^2*d^5+7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(9/2)*sin(f*x+e)^2*d^5-19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(9/2)*c^3*d^2-4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^(9/2)*c^2*d^3+15*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(9/2)*c^4*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(...
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(276) = 552$.

Time $= 4.28$ (sec) , antiderivative size $= 4180$, normalized size of antiderivative $= 13.53$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}(c + d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
[1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A -
 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(
5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3
 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*
B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*
d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4
 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A
 - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*
B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A
 - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A
 - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c
*d + a*d^2)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
 + 7*a*d^2)*cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c
+ 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*s
qrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
 + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 -
c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d
*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c
^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^...
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a + a\sin(e+fx)}(c + d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

output

```
Timed out
```

3.313.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

3.313.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(276) = 552$.

Time $= 0.48$ (sec) , antiderivative size $= 876$, normalized size of antiderivative $= 2.83$

$$\int \frac{A + B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/8*(sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^3 - 15*sqrt(2)*A*sqrt(a)*c^2*d + 6*sq
rt(2)*B*sqrt(a)*c^2*d - 10*sqrt(2)*A*sqrt(a)*c*d^2 + 19*sqrt(2)*B*sqrt(a)*
c*d^2 - 7*sqrt(2)*A*sqrt(a)*d^3 + 4*sqrt(2)*B*sqrt(a)*d^3)*arctan(sqrt(2)*
d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a*c^5*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) - a*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a*c
^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*a*c^2*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + a*c*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a*d^5*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - 8*(A*sqrt(a) - B*s
qrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 3*sqrt(2)*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 8*(A*sqrt(a) - B*sqrt(a))*log(
-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a*c^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) - 3*sqrt(2)*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*
sqrt(2)*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(6*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 - 14*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 2*B*s
qrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*A*sqrt(a)*d^3*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 8*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 5*B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 9*A*sqrt(a)*c^2*d*si...
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
3), x)
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.314.1 Optimal result . . . 2604
3.314.2 Mathematica [C] (verified) . . . 2605
3.314.3 Rubi [A] (verified) . . . 2605
3.314.4 Maple [B] (verified) . . . 2611
3.314.5 Fricas [B] (verification not implemented) . . . 2612
3.314.6 Sympy [F(-1)] . . . 2612
3.314.7 Maxima [F] . . . 2613
3.314.8 Giac [B] (verification not implemented) . . . 2613
3.314.9 Mupad [F(-1)] . . . 2614

3.314.1 Optimal result

Integrand size = 37, antiderivative size = 283

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \\
&-\frac{(c-d)^2(3B(c-5d)+A(c+11d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} \\
&+\frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} \\
&+\frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} \\
&+\frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} \\
&-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

output

```
-1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d
)^2*(3*B*(c-5*d)+A*(c+11*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/15*d*(15*A*c^2-120*A*c*d+65*A*d^2-99
*B*c^2+168*B*c*d-93*B*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/10*(5*A
-9*B)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(1/2)+1/30*d^2*
(15*A*c-35*A*d-51*B*c+39*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

3.314.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-30Ac^3\cos\left(\frac{1}{2}(e+f\right.\right.$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*
c^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e +
f*x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 11
0*A*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3
*(e + f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e
+ f*x))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2
] + 30*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*
d^3*Cos[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e
+ f*x)/2] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*
B*c^2*d*Sin[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[
(e + f*x)/2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (
30 + 30*I)*(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^2 - 180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*
x))/2] + 210*B*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2]
- 123*B*d^3*Sin[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*
d^3*Sin[(5*(e + f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(
e + f*x))/2]))/(60*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.314.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3456, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}\,dx}{2a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}\,dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}\,dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3462$$

$$\frac{2\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}\,dx}{5a} + \frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e+}{5f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}\,dx}{5a} + \frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e+f}{5f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a} + \frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e+f}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-d^2(15Ac-51Bc-35Ad+39Bd)\sin^2(e+fx)a^2+c((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))a^2+\left(a^2d((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2cd(15A\right.}{\sqrt{\sin(e+fx)a+a}}}{5a}$$

$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-d^2(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)^2a^2+c((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))a^2+\left(a^2d((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2cd(15A\right.}{\sqrt{\sin(e+fx)a+a}}}{5a}$$

$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3502$$

$$\frac{2\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a} + \frac{2ad^2(15Ac-35A}{5a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a} + \frac{2ad^2(15Ac-35Ad}{5a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(15Ac-35Ad}{}}{5a}$$

$$4a^2$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3230$$

$$\frac{15a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(15Ac-35Ad-51Bc+39Bd)}{3f}}{5a}$$

$$4a^2$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{15a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(15Ac-35Ad-51Bc+39Bd)}{3f}}{5a}$$

$$4a^2$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3128$$

$$\frac{\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{30a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{3a}+\frac{2ad^2(15Ac-}{}}{5a}$$

$$4a^2$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 219$$

$$\frac{\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{15\sqrt{2}a^{5/2}(c-d)^2(A(c+11d)+3B(c-5d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{3a}+\frac{2ad^2(15Ac-3}{}}{5a}$$

$$4a^2$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^(3/2)) + ((2*a*(5*A - 9*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sq
rt[a + a*Sin[e + f*x]]) + ((2*a*d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*Co
s[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + ((-15*Sqrt[2]*a^(5/2)*(c - d)
^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/f + (4*a^3*d*(15*A*c^2 - 99*B*c^2 - 120*A*c*d
+ 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x
]]))/(3*a))/(5*a))/(4*a^2)
```

3.314.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.314.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(256) = 512.

Time = 3.18 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.89

method	result	size
parts	Expression too large to display	817
default	Expression too large to display	1030

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*A*c^3/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*c^2*(3*A*d+B*c)/a^(5/2)*(-3*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)-3*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+2*(a-a*sin(f*x+e))^(
1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/
f-1/12*d^2*(A*d+3*B*c)*(-8*(a-a*sin(f*x+e))^(3/2)*sin(f*x+e)*a^(1/2)-24*(a
-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)-8*(a-a*sin(f*x+e))^(3/2)*a^(1/
2)-30*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)/cos(f*x+e
)/(a+a*sin(f*x+e))^(1/2)/f+1/20*d^3*B*(-8*(a-a*sin(f*x+e))^(5/2)*a^(1/2)*s
in(f*x+e)-80*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*sin(f*x+e)-8*(a-a*sin(f*x+e))^
(5/2)*a^(1/2)-90*(a-a*sin(f*x+e))^(1/2)*a^(5/2)+75*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+75*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^
(9/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-3/4*c*d*(A*d+B*c)/a^(5/2)*(-7*2^
(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)+8*(
a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(1/2)-7*2^(1/2)*arctanh(1/2*(a-a*sin...
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(256) = 512$.

Time = 0.29 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}(2(A+3B)c^3 + 6(3A-7B)c^2d - 6(7A-11B)c$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*
c*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*
A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(
3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)
 + (2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A
 - 15*B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2
 + (11*A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x +
 e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x
 + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)
/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) -
4*(12*B*d^3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B
)*c*d^2 + 15*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3
 - 4*(45*B*c^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^
2 - 15*((A - B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d
^3)*cos(f*x + e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*
c^2*d + 45*(A - B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^
3)*cos(f*x + e)^2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a
^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)
^(3/2), x)
```

3.314.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(256) = 512$.

Time $= 0.40$ (sec) , antiderivative size $= 650$, normalized size of antiderivative $= 2.30$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}(A\sqrt{ac}^3+3B\sqrt{ac}^3+9A\sqrt{ac}^2d-21B\sqrt{ac}^2d-21A\sqrt{ac}d^2+33B\sqrt{ac}d^2+11}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
1/120*(15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)*c^3 + 9*A*sqrt(a)*c^2*d - 2
1*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*sqrt(a)*c*d^2 + 11*A*sqrt(a)
*d^3 - 15*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 15*sqrt(2)*(A*sqrt(a)*c^3 + 3*B*sqrt(a)
*c^3 + 9*A*sqrt(a)*c^2*d - 21*B*sqrt(a)*c^2*d - 21*A*sqrt(a)*c*d^2 + 33*B*
sqrt(a)*c*d^2 + 11*A*sqrt(a)*d^3 - 15*B*sqrt(a)*d^3)*log(-sin(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 30*sqrt(2)
*(A*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*c^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) - 3*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3
*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*A*sqrt(a)*c*d^2*sin(-1
/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e
) - A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + B*sqrt(a)*d^3*sin(-1/4*
pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) + 16*sqrt(2)*(12*B*a^(17/2)*d^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^5 - 30*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
3 - 10*A*a^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 45*B*a^(17/2)*c^2
*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 45*A*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*
f*x + 1/2*e) - 45*B*a^(17/2)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 15*A*a
^(17/2)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 30*B*a^(17/2)*d^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(a^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^3}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/
2), x)
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

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3.315.1 Optimal result

Integrand size = 37, antiderivative size = 203

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx &= \\ -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} &\\ +\frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} &\\ +\frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} &\\ -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} &\end{aligned}$$

output

```
-1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d
)*(A*c+7*A*d+3*B*c-11*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/3*d*(3*A*c-9*A*d-15*B*c+13*B*d)*cos(f*
x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/6*(3*A-7*B)*d^2*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/a^2/f
```

3.315.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2\sin\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)
*(-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 6*d*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*
Sin[(3*(e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.315.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-a(3A-7B)d^2\sin^2(e+fx)+(ad(Ac+3Bc+4Ad-4Bd)-a(3A-7B)cd)\sin(e+fx)+ac(Ac+3Bc+4Ad-4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-a(3A-7B)d^2\sin(e+fx)^2+(ad(Ac+3Bc+4Ad-4Bd)-a(3A-7B)cd)\sin(e+fx)+ac(Ac+3Bc+4Ad-4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3502$$

$$\frac{2\int -\frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}}dx}{3a} + \frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{\int \frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a} -$$
$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3042$$

$$\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{\int \frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a} -$$
$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3230$$

$$\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{-3a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} -$$
$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3042$$

$$\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{-3a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a} -$$
$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 3128$$

$$\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{\frac{6a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f} - \frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(}{f\sqrt{a\sin(e+fx)+a}}}{3a}$$
$$\frac{4a^2}{\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}}$$

$$\downarrow 219$$

3.315. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}dx$

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{\frac{3\sqrt{2}a^{3/2}(c-d)(Ac+7Ad+3Bc-11Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f} - \frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e}{f\sqrt{a\sin(e+fx)+a}}}{3a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^(3/2)) + ((2*(3*A - 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
 - ((3*Sqrt[2]*a^(3/2)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f - (4*a^2*d*(3*A*
c - 15*B*c - 9*A*d + 13*B*d)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(
3*a))/(4*a^2)
```

3.315.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.315.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(180) = 360.

Time = 2.59 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{A\,c^2\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)\sqrt{-a(\sin(fx}$
	$4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f$
default	$-\frac{\left(\sin(fx+e)\left(3A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2c^2+18A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2cd-21A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx}\right.\right.\right.$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*A*c^2/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*c*(2*A*d+B*c)/a^(5/2)*(-3*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)-3*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+2*(a-a*sin(f*x+e))^(1/
2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+
1/4*d*(A*d+2*B*c)/a^(5/2)*(7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a*sin(f*x+e)+7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a-8*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(1/2)-10*(a-a*sin(
f*x+e))^(1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f+1/12*d^2*B*(-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^2*sin(f*x+e)+24*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+8
*(a-a*sin(f*x+e))^(3/2)*sin(f*x+e)*a^(1/2)-33*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+30*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+8*(a
-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(180) = 360$.

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.88

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(2\left(A+3B\right)c^2+4\left(3A-7B\right)cd-2\left(7A-11B\right)d^2-\left(\left(A+3B\right)c^2+2\left(3A-7B\right)cd-\left(7A-11B\right)\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
 - ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 +
 ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*
(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 +
2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*si
n(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d
 - 3*(A - B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A -
B)*c^2 - 2*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x
+ e)^2 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A
- B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos
(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f
)*sin(f*x + e))
```

3.315.6 Sympy [F]

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A + B\sin(e+fx))\left(c+d\sin(e+fx)\right)^2}{\left(a\left(\sin(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1
))**(3/2), x)
```

3.315.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)
^(3/2), x)
```

3.315.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(180) = 360$.

Time $= 0.35$ (sec) , antiderivative size $= 461$, normalized size of antiderivative $= 2.27$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}(A\sqrt{a}c^2+3B\sqrt{a}c^2+6A\sqrt{a}cd-14B\sqrt{a}cd-7A\sqrt{a}d^2+11B\sqrt{a}d^2)\log(\sin(-}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
1/24*(3*sqrt(2)*(A*sqrt(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*
sqrt(a)*c*d - 7*A*sqrt(a)*d^2 + 11*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*
x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(A*s
qrt(a)*c^2 + 3*B*sqrt(a)*c^2 + 6*A*sqrt(a)*c*d - 14*B*sqrt(a)*c*d - 7*A*sq
rt(a)*d^2 + 11*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*sqrt(2)*(A*sqrt(a)*c^2*sin(-1/4
*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*
A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 2*B*sqrt(a)*c*d*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + A*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqr
t(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^
2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 16*sqrt(2)*(2*B*a^(9/2)*
d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 6*B*a^(9/2)*c*d*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 3*A*a^(9/2)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 3*B*a^(9/2)
*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))))/f
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^2}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/
2), x)
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

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3.316.9 Mupad [F(-1)] . . . 2631

3.316.1 Optimal result

Integrand size = 35, antiderivative size = 133

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(Ac+3Bc+3Ad-7Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/2*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*c+3*A*d+3*B*c-
7*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3
/2)/f*2^(1/2)-2*B*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin(\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3
/4)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan
[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.316.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3498$$

$$-\frac{\int -\frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}\,dx}{2a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}\,dx}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3230$$

$$\frac{a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3128$$

$$\frac{-\frac{2a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f} - \frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 219$$

$$\frac{-\frac{\sqrt{2}\sqrt{a}(Ac+3Ad+3Bc-7Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f} - \frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2)
,x]
```

output
```
-1/2*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)) + (-((S
qrt[2]*Sqrt[a]*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x]
)/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f) - (8*a*B*d*Cos[e + f*x])/(f*Sqrt
[a + a*Sin[e + f*x]]))/(4*a^2)
```

3.316.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.316.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(116) = 232.

Time = 2.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.92

method	result
default	$-\frac{\left(\sin(fx+e)\left(A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)ac+3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)ad+3B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.$
parts	$-\frac{Ac\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)\sqrt{-a(\sin(fx+}$
	$4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+
8*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2
)*a^(1/2)*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/
2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/
f
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(116) = 232.

Time = 0.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(((A+3\,B)c+(3\,A-7\,B)d)\cos(fx+e)^2-2\,(A+3\,B)c-2\,(3\,A-7\,B)d-((A+3\,B)c+(3\,A-7\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*
c - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A +
 3*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*si
n(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*co
s(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c -
 (A - B)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e)
- (A - B)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*co
s(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*
f)*sin(f*x + e))
```

3.316.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))*
*(3/2), x)
```

3.316.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(
3/2), x)
```

3.316.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{%%{[268435456,0]:[1,0,-2]%%},[2]%%%},0]:[1,0,%%%{-
1,[1]%%%}
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2)
, x)
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

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3.317.2 Mathematica [C] (verified) . . . 2632
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3.317.8 Giac [B] (verification not implemented) . . . 2636
3.317.9 Mupad [F(-1)] . . . 2636

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A+3B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A+3*B)*arctanh(1/2*cos
(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.317.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\downarrow 3229$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3128$$

$$-\frac{(A+3B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 219$$

$$-\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/2*((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e +
 f*x])^(3/2))
```

3.317.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.317.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(72) = 144$.

Time = 1.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

method	result
default	$-\dfrac{\left(\sin(fx+e)\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a(A+3B)+A\sqrt{2}\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a+3B\sqrt{2}\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}}{2\sqrt{a}}\right.\right.}{4a^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$-\dfrac{A\left(\sqrt{2}\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+\sqrt{2}\arctanh\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\right)\sqrt{-a(\sin(fx+}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*2^(1/2)*a*(A+3*B)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a+2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*
(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(72) = 144$.

Time $= 0.27$ (sec) , antiderivative size $= 293$, normalized size of antiderivative $= 3.37$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}((A+3\,B)\cos(fx+e)^2 - (A+3\,B)\cos(fx+e) - ((A+3\,B)\cos(fx+$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*
B)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos
(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - s
in(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
+ 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f
*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*
f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

3.317.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{A + B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.317.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

3.317.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(72) = 144$.

Time $= 0.31$ (sec) , antiderivative size $= 184$, normalized size of antiderivative $= 2.11$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{2}(A\sqrt{a}+3\,B\sqrt{a})\log(\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))} - \frac{\sqrt{2}(A\sqrt{a}+3\,B\sqrt{a})\log(-\sin(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e))}}{8\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(2)*(A*sqrt(a) + 3*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(A*sqrt(a) + 3*B*
sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))) - 2*sqrt(2)*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
B*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

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3.318.2 Mathematica [C] (verified) . . . 2637
3.318.3 Rubi [A] (verified) . . . 2638
3.318.4 Maple [B] (verified) . . . 2641
3.318.5 Fricas [B] (verification not implemented) . . . 2642
3.318.6 Sympy [F(-1)] . . . 2643
3.318.7 Maxima [F] . . . 2644
3.318.8 Giac [B] (verification not implemented) . . . 2644
3.318.9 Mupad [F(-1)] . . . 2645

3.318.1 Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A(c-5d)+B(3c+d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2 f} + \frac{2\sqrt{d}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*(c-5*d)+B*(3*c
+d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2
)/(c-d)^2/f*2^(1/2)+2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

3.318.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.96 (sec) , antiderivative size = 781, normalized size of antiderivative = 4.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(
3/4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A
*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 -
 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sq
rt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*L
og[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*
d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[
d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x
)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[
-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin
[e + f*x]))^(3/2))
```

3.318.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3464$$

$$\frac{a(A(c-5d)+B(3c+d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3042$$

$$\frac{a(A(c-5d)+B(3c+d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3128$$

$$\frac{-\frac{2a(A(c-5d)+B(3c+d))\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 219$$

$$\frac{-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 3252$$

$$\frac{\frac{8ad(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\downarrow 221$$

$$\frac{\frac{8\sqrt{a}\sqrt{d}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}} - \frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]))
,x]
```

output
```
(-((Sqrt[2]*Sqrt[a]*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f
*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*f)) + (8*Sqrt[a]*Sqrt[d
]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a +
 a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f))/(4*a^2*(c - d)) - ((A - B)*Co
s[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.318.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(158) = 316$.

Time $= 0.98$ (sec) , antiderivative size $= 624$, normalized size of antiderivative $= 3.34$

method	result
default	$-\frac{\left(\sin(fx+e)\left(8A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^{\frac{3}{2}}d^2-8B\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^{\frac{3}{2}}cd+A\sqrt{a(c+d)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(}}{2\sqrt{}}\right.\right.\right.$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*(8*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c+B*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*d+2*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1
/2)*c-2*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d-2*B*(a*(c+d)*
d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f
*x+e))^(1/2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^
2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(158) = 316$.

Time $= 1.81$ (sec) , antiderivative size $= 1561$, normalized size of antiderivative $= 8.35$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
+ 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*
c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt
(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x
+ e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*
d)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3
 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos
(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (
c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d
^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*si
n(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x
+ e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A
- B)*c - (A - B)*d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2
 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*...
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.318.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)), x)
```

3.318.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(158) = 316$.

Time $= 0.36$ (sec) , antiderivative size $= 471$, normalized size of antiderivative $= 2.52$

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{8\sqrt{2}(Bcd-Ad^2)\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.}{\sqrt{-cd-d}}\right.}{\left(\sqrt{2}a^{\frac{3}{2}}c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.fx+\frac{1}{2}e\Big)\Big)-2\sqrt{2}a^{\frac{3}{2}}cd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.fx+\frac{1}{2}e\Big)\Big)+}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/4*(8*sqrt(2)*(B*c*d - A*d^2)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/
2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(3/2)*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) - 2*sqrt(2)*a^(3/2)*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(
2)*a^(3/2)*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A
*sqrt(a)*c + 3*B*sqrt(a)*c - 5*A*sqrt(a)*d + B*sqrt(a)*d)*log(sin(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - 2*sqrt(2)*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 5
*A*sqrt(a)*d + B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt
(2)*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*a^2*c*d*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))) - 2*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B*sqrt(a)*sin(-
1/4*pi + 1/2*f*x + 1/2*e))/((sqrt(2)*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - sqrt(2)*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*(sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2 - 1)))/f
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)))
, x)
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

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3.319.1 Optimal result

Integrand size = 37, antiderivative size = 292

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx &= \\ -\frac{(Ac+3Bc-9Ad+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} &\\ -\frac{\sqrt{d}(Ad(5c+3d)-B(3c^2+3cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f} &\\ -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} &\\ +\frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} &\end{aligned}$$

output

```
-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*
(A*c-9*A*d+3*B*c+5*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-(A*d*(5*c+3*d)-B*(3*c^2+3*c*d+2*d^2
))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*
d^(1/2)/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f+1/2*d*(B*(3*c+d)-A*(c+3*d))*cos(f*x+
e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.319.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.57 (sec) , antiderivative size = 904, normalized size of antiderivative = 3.10

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)(c-d)\,\mathrm{s}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-
1)^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-
(A*d*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f
*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log
[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan
[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3
)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2)/(c + d)^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 -
4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c +
 d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqr
t[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f
*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Lo
g[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*
d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f...
```

3.319.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 3463$$

$$\frac{\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))} - \frac{\int -\frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)} + \frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{4a^2(c-d)(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 3464$$

$$\frac{\frac{a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+$$

$$\downarrow 3042$$

$$\frac{\frac{a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+$$

$$\downarrow 3128$$

$$\frac{\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A}{f(c^2-d^2)\sqrt{a\sin(e+f}}$$

$$\downarrow 219$$

$$\frac{\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c}{f(c^2-d^2)\sqrt{a\sin(e+fx}}$$

$$\downarrow 3252$$

$$\frac{-\frac{4a^2d\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\downarrow 221$$

$$\frac{-\frac{4a^{3/2}\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}}{4a^2(c-d)}+\frac{}{f}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*S
in[e + f*x])) + ((-((Sqrt[2]*a^(3/2)*(c + d)*(A*c + 3*B*c - 9*A*d + 5*B*d)
*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c -
 d)*f)) - (4*a^(3/2)*Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*
x]])])/((c - d)*Sqrt[c + d]*f))/(a*(c^2 - d^2)) + (2*a*d*(B*(3*c + d) - A*
(c + 3*d))*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Si
n[e + f*x])))/(4*a^2*(c - d))
```

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.319.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(259) = 518$.

Time = 1.40 (sec) , antiderivative size = 2049, normalized size of antiderivative = 7.02

method	result	size
default	Expression too large to display	2049

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output

```
-1/4/a^(5/2)*(4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(
f*x+e)*c*d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2-6*B*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d+4*B*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d+5*B*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
/2))*a*c*d^2+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3-9*A*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)*a*d^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3+5*B*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a
*d^3-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)^2*a*d^3+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*
d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-8*A*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/...
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. 2(259) = 518.

Time = 4.26 (sec) , antiderivative size = 3403, normalized size of antiderivative = 11.65

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*
d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)
*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B
)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B
)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*
B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3
- ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + (
(A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*co
s(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5
*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c
^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^
3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(
f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A
 - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A
- 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2
- (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (
8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqr...
```

3.319.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.319.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Timed out
```

3.319.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.

Time $= 0.45$ (sec) , antiderivative size $= 866$, normalized size of antiderivative $= 2.97$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/4*(2*sqrt(2)*(3*sqrt(2)*B*sqrt(a)*c^2*d - 5*sqrt(2)*A*sqrt(a)*c*d^2 + 3*
sqrt(2)*B*sqrt(a)*c*d^2 - 3*sqrt(2)*A*sqrt(a)*d^3 + 2*sqrt(2)*B*sqrt(a)*d^
3)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2
*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^3*d*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 2*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d
^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + (A*sqrt(a)*c +
 3*B*sqrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqr
t(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^2*c*d^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))) - (A*sqrt(a)*c + 3*B*sqrt(a)*c - 9*A*sqrt(a)*d + 5*B*sqrt(
a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 3*sqrt(2)*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt
(2)*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(2*A*sqrt(a)*c*d*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 - 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^3 + 6*A*sqrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 2*B*sqrt(a)*d^2
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x +
1/2*e) + B*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 2*A*sqrt(a)*c*d*si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 6*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + ...
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
2), x)
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

3.320.1 Optimal result 2656
3.320.2 Mathematica [C] (warning: unable to verify) 2657
3.320.3 Rubi [A] (verified) 2657
3.320.4 Maple [B] (verified) 2663
3.320.5 Fricas [B] (verification not implemented) 2663
3.320.6 Sympy [F(-1)] 2664
3.320.7 Maxima [F(-1)] 2664
3.320.8 Giac [B] (verification not implemented) 2664
3.320.9 Mupad [F(-1)] 2665

3.320.1 Optimal result

Integrand size = 37, antiderivative size = 402

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{(A(c-13d)+3B(c+3d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4 f}$$
$$-\frac{\sqrt{d}(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{3/2}(c-d)^4(c+d)^{5/2}f}$$
$$-\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}$$
$$+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}$$
$$+\frac{d(3B(3c^2+3cd+2d^2)-A(2c^2+15cd+7d^2))\cos(e+fx)}{4a(c-d)^3(c+d)^2 f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output

```
-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/
4*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1/2)-1/4*(A*d*(35*c^2+42*c*d+19*d^2)-
3*B*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c
+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f+1/
2*d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/
(a+a*sin(f*x+e))^(1/2)+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^
2))*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.320.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 12.80 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.37

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^3),x]
```

output
```
((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec
[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4
)*c^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x])
)^(3/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c
^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c
 + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/
4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(
e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3
*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1
^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(
c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 4
2*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1
^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + T
an[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 +
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e
+ f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2]...
```

3.320.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}\,dx}{2a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}\,dx}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}\,dx}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$\frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2} - \frac{\int -\frac{2\left(a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}\,dx}{2a(c^2-d^2)}}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}\,dx}{a(c^2-d^2)} + \frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{a(c^2-d^2)}+\frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}-$$

$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$\frac{a^2d\left(3B\left(3c^2+3cd+2d^2\right)-A\left(2c^2+15cd+7d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int-\frac{a^3\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)-a^3d\left(3B\left(3c^2+3dc+2d^2\right)-A\left(2c^2\right.\right.}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}$$

$$a(c^2-d^2)$$

$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int\frac{\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)a^3+d\left(2Ac^2-9Bc^2+15Adc-9Bdc+7Ad^2-6Bd^2\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3cd+2d^2\right)-A\left(2\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx}}$$

$$a(c^2-d^2)$$

$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int\frac{\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)a^3+d\left(2Ac^2-9Bc^2+15Adc-9Bdc+7Ad^2-6Bd^2\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3cd+2d^2\right)-A\left(2\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx}}$$

$$a(c^2-d^2)$$

$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3464$$

$$\frac{2a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}+\frac{a^2d\left(3B\left(3c^2+3cd+\right.\right.}{f\left(c^2-d^2\right)\sqrt{}}$$

$$2a\left(c^2-d^2\right)$$

$$a(c^2-d^2)$$

$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

3.320. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

$$\frac{\frac{2a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3cd+\right.\right.}{f\left(c^2-d^2\right)\sqrt{}}$$
$$\frac{}{a(c^2-d^2)}$$
$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3128$$

$$\frac{\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{4a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{2a\left(c^2-d^2\right)}+$$
$$\frac{}{a(c^2-d^2)}$$
$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 219$$

$$\frac{\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}+$$
$$\frac{}{a(c^2-d^2)}$$
$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3252$$

$$\frac{-\frac{2a^3d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\operatorname{arctanh}}{f(c-d)}}{2a\left(c^2-d^2\right)}$$
$$\frac{}{a(c^2-d^2)}$$
$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 221$$

$$\frac{-\frac{2a^{5/2}\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\operatorname{arctanh}\left(\frac{}{f(c-d)}\right.}{2a\left(c^2-d^2\right)}$$
$$\frac{}{a(c^2-d^2)}$$
$$4a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

3.320. $\int\frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}dx$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
3),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*S
in[e + f*x])^2) + ((2*a*d*(B*(2*c + d) - A*(c + 2*d))*Cos[e + f*x])/((c^2
- d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (((-2*Sqrt[2]*
a^(5/2)*(c + d)^2*(A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e +
f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*f) - (2*a^(5/2)*Sqrt[d
]*(A*d*(35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d
^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e
+ f*x]])])/((c - d)*Sqrt[c + d]*f))/(2*a*(c^2 - d^2)) + (a^2*d*(3*B*(3*c^2
 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c*d + 7*d^2))*Cos[e + f*x])/((c^2 - d^2)
*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(a*(c^2 - d^2)))/(4*a^2
*(c - d))
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.320.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4706 vs. 2(363) = 726.

Time = 2.07 (sec) , antiderivative size = 4707, normalized size of antiderivative = 11.71

method	result	size
default	Expression too large to display	4707

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*(-a*(sin(f*x+e)-1))^(1/2)*(-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)
^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)
*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^4-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3+15*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d+9*B*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^3*a^2*d^5-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5+9*B*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^
2*a^2*d^5+4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+
e)*c^4*d+17*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+
e)*c^3*d^2-A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e
)*c^2*d^3-17*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x
+e)*c*d^4-13*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x
+e)*c^4*d-7*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+
e)*c^3*d^2+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*c*d^4+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5-13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5-11*
A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/...
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2789 vs. 2(362) = 724.

Time = 8.46 (sec) , antiderivative size = 5864, normalized size of antiderivative = 14.59

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
Too large to include
```

3.320.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.320.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

3.320.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(362) = 724$.

Time $= 0.60$ (sec) , antiderivative size $= 1263$, normalized size of antiderivative $= 3.14$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/8*(sqrt(2)*(15*sqrt(2)*B*sqrt(a)*c^3*d - 35*sqrt(2)*A*sqrt(a)*c^2*d^2 +
30*sqrt(2)*B*sqrt(a)*c^2*d^2 - 42*sqrt(2)*A*sqrt(a)*c*d^3 + 39*sqrt(2)*B*s
qrt(a)*c*d^3 - 19*sqrt(2)*A*sqrt(a)*d^4 + 12*sqrt(2)*B*sqrt(a)*d^4)*arctan
(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^2*c^6*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*a^2*c^5*d*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) - a^2*c^4*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*a^2*c^3*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*c^2*d^4*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) - 2*a^2*c*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + a^2*d^6*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + 2*(A*sqrt(a)*c + 3*
B*sqrt(a)*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(sin(-1/4*pi + 1/2*f*x +
1/2*e) + 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(
2)*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^2*c^2*d^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c*d^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
- 2*(A*sqrt(a)*c + 3*B*sqrt(a)*c - 13*A*sqrt(a)*d + 9*B*sqrt(a)*d)*log(-si
n(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) - 4*sqrt(2)*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6
*sqrt(2)*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^2*c
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^2*d^4*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))) - 4*(A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - B...
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
3), x)
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

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3.321.1 Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{(c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}$$
$$+\frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f}$$
$$-\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}}$$
$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
-1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*si
n(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
^(5/2)-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*arctan
h(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
+1/24*d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*cos(f*x+e)/a^
2/f/(a+a*sin(f*x+e))^(1/2)+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/a^3/f
```

3.321.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.77 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)(c-d)^3\,\mathrm{s}\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)
/2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c -
d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2
 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 -
 16*B*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (
24 + 24*I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x
)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2
*A*d - 5*B*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin
[(3*(e + f*x))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.321.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2} - \frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(c}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(c}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2} - \frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(c}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-d^2(9Ac+15Bc+39Ad-95Bd)\sin^2(e+fx)a^2+c\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)a^2+\left(a^2d\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2cd(9Ac+15}{\frac{\sqrt{\sin(e+fx)a+a}}{4a^2}}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-d^2(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)^2a^2+c\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)a^2+\left(a^2d\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2cd(9Ac+15}{\frac{\sqrt{\sin(e+fx)a+a}}{4a^2}}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3502$$

$$\frac{2\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ad+}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ad+15}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ad+15}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3230$$

$$\frac{\frac{3a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}+\frac{2ad^2(9Ac+39Ac}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{3a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}+\frac{2ad^2(9Ac+39Ac}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3128$$

$$\frac{\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{6a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{3a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 219$$

$$\frac{\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{2}a^{5/2}(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{3a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^(5/2)) + (-1/2*(a*(3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^2)/(f*(a + a*Sin[e + f*x])^(3/2)) + ((2*a*d^2*(9*A*c + 15*B*c
+ 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + ((-3*Sqr
t[2]*a^(5/2)*(c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25
*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])
/f + (4*a^3*d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*d^2
))*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(281) = 562$.

Time = 3.99 (sec) , antiderivative size = 1157, normalized size of antiderivative = 3.76

method	result	size
parts	Expression too large to display	1157
default	Expression too large to display	1438

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/32*A*c^3/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a^2*sin(f*x+e)+6*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+
6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*
sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/32*c^2*(3*A*d+B*c)*(-5*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*(a-a*sin(
f*x+e))^(1/2)*a^(5/2)-10*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2
)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/32*d^2*(A*
d+3*B*c)/a^(9/2)*(-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*cos(f*x+e)^2+64*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*cos(f*x+e)^2+15
0*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+
e)-128*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+150*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2-204*(a-a*sin(f*x+e))^(1/2)*a^(3
/2)+42*(a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*
x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/96*d^3*B*(64*(a-a*sin(f*x+e))^
(3/2)*a^(1/2)*cos(f*x+e)^2+384*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*cos(f*x+e)^2
-489*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*co...
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(282) = 564$.

Time $= 0.30$ (sec) , antiderivative size $= 980$, normalized size of antiderivative $= 3.18$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A -
75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2
*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A
+ 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)
*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A
 - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 1
2*(5*A + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3
*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163
*B)*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(1
9*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos
(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*
sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - co
s(f*x + e) - 2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B
)*c^2*d - 36*(A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*
d^3)*cos(f*x + e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A
- 53*B)*c*d^2 + (53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(
A - 9*B)*c^2*d - 27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (
32*B*d^3*cos(f*x + e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c
*d^2 - 12*(A - B)*d^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 -...
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.321.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)
^(5/2), x)
```

3.321.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(282) = 564$.

Time $= 0.55$ (sec) , antiderivative size $= 751$, normalized size of antiderivative $= 2.44$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
1/192*(3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sqrt(a)*c^3 + 15*A*sqrt(a)*c^2*d +
 57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 - 225*B*sqrt(a)*c*d^2 - 75*A*sqrt
(a)*d^3 + 163*B*sqrt(a)*d^3)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)*(3*A*sqrt(a)*c^3 + 5*B*sq
rt(a)*c^3 + 15*A*sqrt(a)*c^2*d + 57*B*sqrt(a)*c^2*d + 57*A*sqrt(a)*c*d^2 -
 225*B*sqrt(a)*c*d^2 - 75*A*sqrt(a)*d^3 + 163*B*sqrt(a)*d^3)*log(-sin(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 6*
sqrt(2)*(3*A*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^
3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 15*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^3 - 39*B*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 39*
A*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 63*B*sqrt(a)*c*d^2*sin(
-1/4*pi + 1/2*f*x + 1/2*e)^3 + 21*A*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^3 - 29*B*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c
^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c^3*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) - 9*A*sqrt(a)*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 33*B*sqrt(a)
*c^2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 33*A*sqrt(a)*c*d^2*sin(-1/4*pi + 1
/2*f*x + 1/2*e) - 57*B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 19*A
*sqrt(a)*d^3*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 27*B*sqrt(a)*d^3*sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 128*sqrt(2)*(2*B*a^(13/2)*d^3*sin(-1/4*...
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^3}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/
2), x)
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.322.1 Optimal result . . . 2677
3.322.2 Mathematica [C] (verified) . . . 2678
3.322.3 Rubi [A] (verified) . . . 2678
3.322.4 Maple [B] (verified) . . . 2682
3.322.5 Fricas [B] (verification not implemented) . . . 2683
3.322.6 Sympy [F(-1)] . . . 2684
3.322.7 Maxima [F] . . . 2684
3.322.8 Giac [B] (verification not implemented) . . . 2685
3.322.9 Mupad [F(-1)] . . . 2686

3.322.1 Optimal result

Integrand size = 37, antiderivative size = 219

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}$$
$$-\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$
$$+\frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
-1/16*(c-d)*(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/
2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-1/32*(
B*(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*arctanh(1/2*cos(f*x+e)*a^
(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*(A-9*B)*d^2*co
s(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

3.322.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-11Ac^2\cos\left(\frac{1}{2}(e+f\right.\right.$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c
^2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2]
 + 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(
e + f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2
] + 26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d
^2*Cos[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e
+ f*x)/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d
*Sin[(e + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] +
 (2 + 2*I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 1
9*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin
[(3*(e + f*x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f
*x))/2] + 13*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] -
16*B*d^2*Sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.322.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3447$$

$$\frac{\int \frac{-a(A-9B)d^2\sin^2(e+fx)+(ad(3Ac+5Bc+4Ad-4Bd)-a(A-9B)cd)\sin(e+fx)+ac(3Ac+5Bc+4Ad-4Bd)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{-a(A-9B)d^2\sin(e+fx)^2+(ad(3Ac+5Bc+4Ad-4Bd)-a(A-9B)cd)\sin(e+fx)+ac(3Ac+5Bc+4Ad-4Bd)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3498$$

$$\frac{-\frac{\int -\frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2} - \frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}-$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}-$$

$$\downarrow 3230$$

$$\frac{\frac{a^2\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}-$$

$$\downarrow 3042$$

$$\frac{\frac{a^2\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}-$$

$$\downarrow 3128$$

$$\frac{\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 219$$

$$\frac{\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(}{2f(a\sin(e+fx)+a)^{3/2}}}{\frac{8a^2}{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^(5/2)) + (-1/2*(a*(c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/
(f*(a + a*Sin[e + f*x])^(3/2)) + (-((Sqrt[2]*a^(3/2)*(B*(5*c^2 + 38*c*d -
75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]])])/f) + (8*a^2*(A - 9*B)*d^2*Cos[e + f*x])/(
f*Sqrt[a + a*Sin[e + f*x]]))/(4*a^2))/(8*a^2)
```

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.322.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(196) = 392$.

Time = 3.22 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.89

method	result	size
parts	Expression too large to display	852
default	Expression too large to display	982

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/32*A*c^2/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a^2*sin(f*x+e)+6*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+
6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*
sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/32*c*(2*A*d+B*c)*(-5*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*(a-a*sin(f*
x+e))^(1/2)*a^(5/2)-10*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/
a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/32*d*(A*d+2*
B*c)/a^(9/2)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*cos(f*x+e)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*sin(f*x+e)+38*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2-44*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+26*(a-a*sin(f*x+e))^(3
/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(
f*x+e))^(1/2)/f+1/32*d^2*B/a^(9/2)*(-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+64*a^(3/2)*(a-a*sin(f*x+e))^(1/
2)*cos(f*x+e)^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*sin(f*x+e)-128*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)+150*...
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(196) = 392$.

Time $= 0.29$ (sec) , antiderivative size $= 744$, normalized size of antiderivative $= 3.40$

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))^2}{(a + a\sin(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\big(((3\,A+5\,B)c^2+2\,(5\,A+19\,B)cd+(19\,A-75\,B)d^2)\cos(fx+e)^3-4\,(3\,A+5\,B)c^2-8\,(5\,A+19\,B$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)
*cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)
*d^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*co
s(f*x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2
 - ((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)
^2 + 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
 - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
 e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 -
4*(A - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A
- 13*B)*c*d - (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A -
9*B)*c*d - 9*(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A
 - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13
*B)*c*d - (13*A - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
+ e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*s
in(f*x + e))
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.322.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)
^(5/2), x)
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(196) = 392.

Time = 0.43 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{128\sqrt{2}Bd^2\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{a^{\frac{5}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}(3A\sqrt{a}c^2+5B\sqrt{a}c^2+10A\sqrt{a}cd+38B}{a^3\mathrm{sgn}}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
1/64*(128*sqrt(2)*B*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(a^(5/2)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 +
10*A*sqrt(a)*c*d + 38*B*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)
*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 + 10*A*sqrt(a)*c*d +
 38*B*sqrt(a)*c*d + 19*A*sqrt(a)*d^2 - 75*B*sqrt(a)*d^2)*log(-sin(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt
(2)*(3*A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c^2*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 10*A*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x +
1/2*e)^3 - 26*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*A*sqrt(a
)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 21*B*sqrt(a)*d^2*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sq
rt(a)*c^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 6*A*sqrt(a)*c*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e) + 22*B*sqrt(a)*c*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*A*s
qrt(a)*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 19*B*sqrt(a)*d^2*sin(-1/4*pi +
 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^2}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/
2), x)
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.323.1 Optimal result 2687
3.323.2 Mathematica [C] (verified) 2687
3.323.3 Rubi [A] (verified) 2688
3.323.4 Maple [B] (verified) 2691
3.323.5 Fricas [B] (verification not implemented) 2691
3.323.6 Sympy [F(-1)] 2692
3.323.7 Maxima [F] 2692
3.323.8 Giac [B] (verification not implemented) 2693
3.323.9 Mupad [F(-1)] 2693

3.323.1 Optimal result

Integrand size = 35, antiderivative size = 151

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3Ac+5Bc+5Ad+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/4*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c+5*A*d+5*B
*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A*c+5*A*d+5*B*c+1
9*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5
/2)/f*2^(1/2)
```

3.323.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin(\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*
B*c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.323.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3042$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3498$$

$$-\frac{\int -\frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3229$$

$$\frac{\frac{1}{4}(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx - \frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3128$$

$$\frac{-\frac{(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2f} - \frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 219$$

$$\frac{-\frac{(3Ac+5Ad+5Bc+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}f} - \frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

input

```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2)
,x]
```

output

```
-1/4*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)) + (-1/2
*((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*Sqrt[a]*f) - (a*(3*A*c + 5*B*c + 5*A
*d - 13*B*d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)))/(8*a^2)
```

3.323. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}dx$

3.323.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

3.323.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(132) = 264.

Time = 2.59 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.97

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(3Ac+5dA+5Bc+19dB)(\cos^2(fx+e))+2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2$
parts	$-\frac{Ac\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(\cos^2(fx+e))+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6\sqrt{a-a\sin(fx+e)}\right.}{32a^{\frac{9}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{a+a\sin}}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2+2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+10*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+20*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c+12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-6*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c-10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+12*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c-44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c+26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(132) = 264.

Time = 0.27 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.55

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}(((3A+5B)c+(5A+19B)d)\cos(fx+e)^3+3((3A$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B
)*d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c +
(5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((
3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a
*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e)
 - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e
) - 2)) + 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c
 - 4*(A - B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c -
 4*(A - B)*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f)*sin(f*x + e))
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.323.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(
5/2), x)
```

3.323.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(132) = 264$.

Time = 0.35 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.41

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{\sqrt{2}(3A\sqrt{ac}+5B\sqrt{ac}+5A\sqrt{ad}+19B\sqrt{ad})\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
1/64*(sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19*B*sqrt(a
)*d)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a)*c + 5*B*sqrt(a)*c + 5*A*sqrt(a)*d + 19
*B*sqrt(a)*d)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x +
1/2*e)^3 + 5*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*sqrt
(2)*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 13*sqrt(2)*B*sqrt(a)*d*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*A*sqrt(a)*c*sin(-1/4*pi + 1/2
*f*x + 1/2*e) - 3*sqrt(2)*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*s
qrt(2)*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 11*sqrt(2)*B*sqrt(a)*d
*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2
*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2)
, x)
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.324.1 Optimal result . . . 2694
3.324.2 Mathematica [C] (verified) . . . 2694
3.324.3 Rubi [A] (verified) . . . 2695
3.324.4 Maple [B] (verified) . . . 2697
3.324.5 Fricas [B] (verification not implemented) . . . 2697
3.324.6 Sympy [F] . . . 2698
3.324.7 Maxima [F] . . . 2698
3.324.8 Giac [B] (verification not implemented) . . . 2699
3.324.9 Mupad [F(-1)] . . . 2699

3.324.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3A+5B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A+5*B)*cos(f*x+e)/a
/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A+5*B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^
(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

3.324.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+4(-A+B\right.}{\text{(illegible)}}$$

input

```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.324.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\downarrow 3229$$

$$\frac{(3A+5B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+5B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3129$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a} - \frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3128$$

$$\frac{(3A+5B)\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}} d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 219$$

$$\frac{(3A+5B)\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)) + ((3*A + 5*B)*
(-1/2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(
Sqrt[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))))/(8*a)
```

3.324.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.324.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(107) = 214$.

Time = 1.91 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.21

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)(\cos^2(fx+e))+2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)+20A\right.$
parts	$-\frac{A\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(\cos^2(fx+e))+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6\sqrt{a-a\sin(fx+e)}\right.$ s $32a^{\frac{9}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{a+a}$ si

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(3
*A+5*B)*cos(f*x+e)^2+2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^3*(3*A+5*B)+20*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-6*A*
(a-a*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a
-a*sin(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.324.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.11

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\big((3\,A+5\,B)\cos(fx+e)^3 + 3\,(3\,A+5\,B)\cos(fx+e)^2 - 2\,(3\,A+5\,B)\,c}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
 2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*
cos(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*co
s(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) + 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*
B)*cos(f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e)
 + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin
(f*x + e))
```

3.324.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{A + B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.324.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

3.324.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(107) = 214$.

Time $= 0.32$ (sec) , antiderivative size $= 236$, normalized size of antiderivative $= 1.87$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{\sqrt{2}(3A\sqrt{a}+5B\sqrt{a})\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(3A\sqrt{a}+5B\sqrt{a})\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{a^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(2)*(3*A*sqrt(a) + 5*B*sqrt(a))*log(sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(3*A*sqrt(a) +
 5*B*sqrt(a))*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))) - 2*(3*sqrt(2)*A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^3 + 5*sqrt(2)*B*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*sqrt(2)*
A*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*sqrt(2)*B*sqrt(a)*sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/((sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*a^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{A + B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

3.325.1 Optimal result 2700
3.325.2 Mathematica [C] (verified) 2701
3.325.3 Rubi [A] (verified) 2702
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3.325.7 Maxima [F] 2708
3.325.8 Giac [B] (verification not implemented) 2708
3.325.9 Mupad [F(-1)] 2709

3.325.1 Optimal result

Integrand size = 37, antiderivative size = 261

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx =$$
$$-\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f}$$
$$-\frac{2d^{3/2}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^3\sqrt{c+d}f}$$
$$-\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}}-\frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

output

```
-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c-11*A*d+5*
B*c+3*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(B*(5*c^2-34
*c*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2
)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)-2*d^(3/2)*(-A*d+B*c)*a
rctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5
/2)/(c-d)^3/f/(c+d)^(1/2)
```

3.325.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.74 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d
)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2
) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c
) + A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#
1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 +
 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[
(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c -
A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2
 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c
 + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*S
qrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e +
 f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*
Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*...
```

3.325.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{4a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{8a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{8a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3457$$

$$\frac{-\frac{\int -\frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a^2(c-d)} - \frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)} - \frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3464$$

$$\frac{\frac{a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{32ad^2(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{32ad^2(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 3128$$

$$\frac{\frac{32ad^2(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd}{2f(c-d)(a\sin(e+fx)}$$

$$\frac{8a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 219$$

$$\frac{\frac{32ad^2(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)}{2f(c-d)(a\sin(e+fx)+}$$

$$\frac{8a^2(c-d)}{\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}}$$

$$\downarrow 3252$$

3.325. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}dx$

$$\frac{-\frac{64a^2d^2(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{a(}{}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\downarrow 221$$

$$\frac{-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}-\frac{64a^{3/2}d^{3/2}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}}{4a^2(c-d)}-\frac{a(3A}{2}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]))
,x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)) + ((-((
Sqrt[2]*a^(3/2)*(B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))
*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c -
 d)*f)) - (64*a^(3/2)*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
 f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f))/(
4*a^2*(c - d)) - (a*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(2*(c -
 d)*f*(a + a*Sin[e + f*x])^(3/2)))/(8*a^2*(c - d))
```

3.325.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.325.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(228) = 456$.

Time = 1.32 (sec) , antiderivative size = 1418, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	1418

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/32*((64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)
*d^3-64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*
d^2-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-43*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-5*B*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+34*B
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*c*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2+2*sin(f*x+e)*(-64*A*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+64*B*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+3*A*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*A
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*c*d+43*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*d^2+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-34*B*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-3*B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*d^2)-128*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/...
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(228) = 456$.

Time = 3.69 (sec) , antiderivative size = 2577, normalized size of antiderivative = 9.87

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))^{5/2}(c + d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d
^2 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((
3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e
) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*c
os(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d
 - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a
*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2
)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(
d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 -
c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 -
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (
c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^
2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 +...
```

3.325.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.325.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
 + c)), x)
```

3.325.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(228) = 456$.

Time $= 0.43$ (sec) , antiderivative size $= 760$, normalized size of antiderivative $= 2.91$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/32*(64*sqrt(2)*(B*c*d^2 - A*d^3)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)/sqrt(-c*d - d^2))/((sqrt(2)*a^(5/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) - 3*sqrt(2)*a^(5/2)*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 3*sqrt(2)*a^(5/2)*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^
(5/2)*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - (3*A*sq
rt(a)*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d + 43*A*s
qrt(a)*d^2 - 3*B*sqrt(a)*d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqr
t(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c^2*d*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) +
(3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 14*A*sqrt(a)*c*d - 34*B*sqrt(a)*c*d +
 43*A*sqrt(a)*d^2 - 3*B*sqrt(a)*d^2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(sqrt(2)*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*sqrt(2)*a^3*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*sqrt(2)*a^3*c*d^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))) + 2*(3*A*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 5*B*sqrt(a)*c*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 11*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x +
1/2*e)^3 + 3*B*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 5*A*sqrt(a)*c*
sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B*sqrt(a)*c*sin(-1/4*pi + 1/2*f*x + 1/2
*e) + 13*A*sqrt(a)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*B*sqrt(a)*d*sin...
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)))
, x)
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

3.326.1 Optimal result . . . 2710
3.326.2 Mathematica [C] (warning: unable to verify) . . . 2711
3.326.3 Rubi [A] (verified) . . . 2711
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3.326.1 Optimal result

Integrand size = 37, antiderivative size = 395

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx &= \\ -\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^4 f} &\\ +\frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^4(c+d)^{3/2}f} &\\ -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} &\\ -\frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2 f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} &\\ -\frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} &\end{aligned}$$

output

```
d^(3/2)*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d
^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1
/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))-1/16*(
3*A*c-15*A*d+5*B*c+7*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c
+d*sin(f*x+e))-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*arc
tanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^
4/f*2^(1/2)-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*cos(f
*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.326.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 15.73 (sec) , antiderivative size = 1680, normalized size of antiderivative = 4.25

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])^2),x]
```

output
```
((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4
- 64*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(
-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d)
 - B*(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootS
um[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f
*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + T
an[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Ta
n[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*
d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^
4*(c + d)^(3/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5
*d)) + B*(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + R
ootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e
 + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1
 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1
+ 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(...
```

3.326.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a^2(c-d)}-\frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{\int \frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)} - \frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))} -$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)} - \frac{2a^2d\left(A\left(3c^2-16cd-35d^2\right)+\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}}$$

$$4a^2(c-d)$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)} - \frac{2a^2d\left(A\left(3c^2-16cd-35d^2\right)+\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}}$$

$$4a^2(c-d)$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)} - \frac{2a^2d\left(A\left(3c^2-16cd-35d^2\right)+\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}}$$

$$4a^2(c-d)$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3464$$

$$\frac{a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d} - \frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d} - \frac{2a^2d\left(A\left(3c^2-16\right.\right.}{f\left(c^2-d^2\right.}$$

$$4a^2(c-d)$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

3.326. $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}dx$

$$\frac{\frac{a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^2d\left(A\left(3c^2-16\right.\right.}{f\left(c^2-d\right.}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3128$$

$$\frac{-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+}}}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 219$$

$$\frac{-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 3252$$

$$\frac{\frac{32a^3d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\arctanh\left(\right.}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$$\downarrow 221$$

$$\frac{\frac{32a^{5/2}d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{2}}\right.}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*S
in[e + f*x])) + (-1/2*(a*(3*A*c + 5*B*c - 15*A*d + 7*B*d)*Cos[e + f*x])/((
c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) + ((-((Sqrt[2]*a
^(5/2)*(c + d)*(B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2)
)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c
- d)*f)) + (32*a^(5/2)*d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
f*x]])])/((c - d)*Sqrt[c + d]*f))/(a*(c^2 - d^2)) - (2*a^2*d*(A*(3*c^2 - 1
6*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*d^2))*Cos[e + f*x])/((c^2 - d^2)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(4*a^2*(c - d)))/(8*a^2*
(c - d))
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.326.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4091 vs. 2(358) = 716.

Time = 2.02 (sec) , antiderivative size = 4092, normalized size of antiderivative = 10.36

method	result	size
default	Expression too large to display	4092

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/32*(-84*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e
)*c^2*d^2+6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4-43
*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)^3*a^2*d^4+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4-43*B*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)*a^2*d^4-19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d+93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+115*A*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
/2))*a^2*c*d^3-53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-
1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d-101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2-43*B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c*d^3-10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*c^3*d-22*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*c^2*d^2-76*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f
*x+e)*c*d^3-32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*si...
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. 2(358) = 716.

Time = 8.10 (sec) , antiderivative size = 5151, normalized size of antiderivative = 13.04

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
Too large to include
```

3.326.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.326.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Timed out
```

3.326.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(358) = 716$.

Time $= 0.71$ (sec) , antiderivative size $= 1099$, normalized size of antiderivative $= 2.78$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/32*(16*sqrt(2)*(5*sqrt(2)*B*sqrt(a)*c^2*d^2 - 7*sqrt(2)*A*sqrt(a)*c*d^3
 + 5*sqrt(2)*B*sqrt(a)*c*d^3 - 5*sqrt(2)*A*sqrt(a)*d^4 + 2*sqrt(2)*B*sqrt(
a)*d^4)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/
((a^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^4*d*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) + 2*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 2*a^3*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c*d^4*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + a^3*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
*sqrt(-c*d - d^2)) - (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d
 - 58*B*sqrt(a)*c*d + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(sin(-1/4*p
i + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 4*sqrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)
*a^3*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))) + (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 22*A*sqrt(a)*c*d - 58
*B*sqrt(a)*c*d + 115*A*sqrt(a)*d^2 - 43*B*sqrt(a)*d^2)*log(-sin(-1/4*pi +
1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 - 4*sqrt(2)*a^3*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*sqrt(2)*a^3
*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*a^3*c*d^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*a^3*d^4*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))) + 64*(B*sqrt(a)*c*d^2*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*sqrt...
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
2), x)
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

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3.327.1 Optimal result

Integrand size = 37, antiderivative size = 519

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx =$$
$$-\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5 f}$$
$$+\frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{5/2}(c-d)^5(c+d)^{5/2}f}$$
$$-\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}$$
$$-\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2 f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}$$
$$-\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}$$
$$-\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2 f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
1/4*d^(3/2)*(3*A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d
^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+
e))^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*A*c-19*A*d+5*B*c+11*B*d)*cos(f*x+e)/a
/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/32*(B*(5*c^2-82*c*d
-115*d^2)+3*A*(c^2-10*c*d+73*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/
(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/f*2^(1/2)-1/16*d*(A*(3*c^2-20*c*d-
31*d^2)+B*(5*c^2+28*c*d+15*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f
*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/16*d*(3*A*(c^3-7*c^2*d-37*c*d^2-21*d^3)+
B*(5*c^3+73*c^2*d+79*c*d^2+35*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*
sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.327.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.01 (sec) , antiderivative size = 2465, normalized size of antiderivative = 4.75

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])^3),x]
```

output
```
((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)
*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(
e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5
 - 80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 +
 80*(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2))
- (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*
c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1
 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sq
rt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/
4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#
1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1
^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5
/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d +
13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[
(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-
(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f
*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1
 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*S
qrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)...
```

3.327.3 Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^3}dx$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3457$$

$$-\frac{\int -\frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{2a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$\frac{\int -\frac{2\left(\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3\right)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20cd\right.\right.}{f\left(c^2-d^2\right)}$$

$$\frac{}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{\int \frac{\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20cd-31d^2\right)\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(}}$$

$$\frac{}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20cd-31d^2\right)\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(}}$$

$$\frac{}{4a^2(c-d)}$$

$$8a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e+}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3464$$

$$\frac{\frac{a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx)a}}{c+d\sin(e+fx)}}{c-d}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\frac{a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx)a}}{c+d\sin(e+fx)}}{c-d}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3128$$

$$\frac{-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int \frac{1}{2a-\frac{a^2\cos}{\sin(e+}}}{f(c-d)}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 219$$

$$-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\arctan}{f(c-d)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{4}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 3252$$

$$\frac{8a^4d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-8\right.\right.}{f(c}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\downarrow 221$$

$$\frac{8a^{7/2}d^{3/2}\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-\right.\right.}{f(c-d)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{}{a\left(c^2-d^2\right)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^
3),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*S
in[e + f*x])^2) + (-1/2*(a*(3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])
/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + ((-2*a^2*
d*(A*(3*c^2 - 20*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x]
)/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + ((-((S
qrt[2]*a^(7/2)*(c + d)^2*(B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d
 + 73*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/((c - d)*f)) + (8*a^(7/2)*d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2)
 - B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f)
)/(a*(c^2 - d^2)) - (2*a^3*d*(3*A*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*
(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3))*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(a*(c^2 - d^2)))/(4*a^2*(c - d)
))/(8*a^2*(c - d))
```

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.327.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7321 vs. 2(476) = 952.

Time = 3.45 (sec) , antiderivative size = 7322, normalized size of antiderivative = 14.11

method	result	size
default	Expression too large to display	7322

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4135 vs. 2(476) = 952.

Time = 24.73 (sec) , antiderivative size = 8555, normalized size of antiderivative = 16.48

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.327.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

3.327.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2024 vs. $2(476) = 952$.

Time $= 1.00$ (sec) , antiderivative size $= 2024$, normalized size of antiderivative $= 3.90$

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/32*(4*sqrt(2)*(35*sqrt(2)*B*sqrt(a)*c^3*d^2 - 63*sqrt(2)*A*sqrt(a)*c^2*
d^3 + 70*sqrt(2)*B*sqrt(a)*c^2*d^3 - 90*sqrt(2)*A*sqrt(a)*c*d^4 + 67*sqrt(
2)*B*sqrt(a)*c*d^4 - 39*sqrt(2)*A*sqrt(a)*d^5 + 20*sqrt(2)*B*sqrt(a)*d^5)*
arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((a^3*c^
7*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*a^3*c^6*d*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + a^3*c^5*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*a^3*c^
4*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*a^3*c^3*d^4*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)) - a^3*c^2*d^5*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*
a^3*c*d^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^3*d^7*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) - (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2
- 30*A*sqrt(a)*c*d - 82*B*sqrt(a)*c*d + 219*A*sqrt(a)*d^2 - 115*B*sqrt(a)*
d^2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) - 5*sqrt(2)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)) + 10*sqrt(2)*a^3*c^3*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*
sqrt(2)*a^3*c^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*sqrt(2)*a^3*c*
d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*a^3*d^5*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))) + (3*A*sqrt(a)*c^2 + 5*B*sqrt(a)*c^2 - 30*A*sqrt(a)*c
*d - 82*B*sqrt(a)*c*d + 219*A*sqrt(a)*d^2 - 115*B*sqrt(a)*d^2)*log(-sin(-1
/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(2)*a^3*c^5*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) - 5*sqrt(2)*a^3*c^4*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10...
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
3), x)
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

3.328.1 Optimal result . . . 2732
3.328.2 Mathematica [F] . . . 2733
3.328.3 Rubi [A] (verified) . . . 2733
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3.328.1 Optimal result

Integrand size = 35, antiderivative size = 221

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{8\sqrt{2}a^2B\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c}{}\right.}{f\sqrt{1+\sin(e+fx)}}$$

$$-\frac{4\sqrt{2}a^2(A-B)\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx}{f\sqrt{1+\sin(e+fx)}}$$

output

```
-8*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1
+sin(f*x+e))^(1/2)-4*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(
c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*si
n(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

3.328.2 Mathematica [F]

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$= \int (a + a\sin(e+fx))^2(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

3.328.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3466, 3042, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^2(A + B\sin(e+fx))(c + d\sin(e+fx))^n dx$$
$$\downarrow 3466$$
$$(A-B)\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a+a)^3(c+d\sin(e+fx))^n dx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a+a)^3(c+d\sin(e+fx))^n dx}{a}$$
$$\downarrow 3263$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)+1)^{3/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+$$
$$\frac{a^2B\cos(e+fx)\int\frac{(\sin(e+fx)+1)^{5/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{(\sin(e+fx)+1)^{3/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+$$
$$\frac{a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{(\sin(e+fx)+1)^{5/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 155$$

$$-\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$
$$\frac{8\sqrt{2}a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)
*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*(
(c + d*Sin[e + f*x])/(c + d))^n)
```

3.328.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.328.4 Maple [F]

$$\int (a + a\sin(fx + e))^2 (A + B\sin(fx + e)) (c + d\sin(fx + e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.328.5 Fricas [F]

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2 (d\sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

3.328.7 Maxima [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)
^n, x)
```

3.328.8 Giac [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)
^n, x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2\,(c + d\,\sin(e + f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

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3.329.2 Mathematica [F] . . . 2739
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3.329.7 Maxima [F] . . . 2743
3.329.8 Giac [F] . . . 2744
3.329.9 Mupad [F(-1)] . . . 2744

3.329.1 Optimal result

Integrand size = 33, antiderivative size = 217

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{4\sqrt{2}aB\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{f\sqrt{1+\sin(e+fx)}}$$

$$-\frac{2\sqrt{2}a(A-B)\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

output

```
-4*a*B*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))
*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+s
in(f*x+e))^(1/2)-2*a*(A-B)*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d)
,1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*
x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

3.329.2 Mathematica [F]

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$= \int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
, x]
```

3.329.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3447, 3042, 3496, 3042, 3234, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)(A + B\sin(e+fx))(c + d\sin(e+fx))^n dx$$
$$\downarrow 3447$$
$$\int ((aA+aB)\sin(e+fx) + aA + aB\sin^2(e+fx))\,(c + d\sin(e+fx))^n dx$$
$$\downarrow 3042$$
$$\int ((aA+aB)\sin(e+fx) + aA + aB\sin(e+fx)^2)\,(c + d\sin(e+fx))^n dx$$
$$\downarrow 3496$$
$$a(A-B)\int (\sin(e+fx)+1)(c+d\sin(e+fx))^n dx + aB\int (\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\downarrow 3042$$

$$a(A-B)\int(\sin(e+fx)+1)(c+d\sin(e+fx))^n dx + aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\downarrow 3234$$

$$\frac{a(A-B)\cos(e+fx)\int\frac{\sqrt{\sin(e+fx)+1}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} + aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\downarrow 156$$

$$\frac{a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\sqrt{\sin(e+fx)+1}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} + aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\downarrow 155$$

$$aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx - \frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{f\sqrt{\sin(e+fx)+1}}\right.$$

$$\downarrow 3263$$

$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)+1)^{3/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} - \frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{f\sqrt{\sin(e+fx)+1}}\right.$$

$$\downarrow 156$$

$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{(\sin(e+fx)+1)^{3/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} - \frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{f\sqrt{\sin(e+fx)+1}}\right.$$

$$\downarrow 155$$

$$-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$

$$\frac{4\sqrt{2}aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*App
ellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
 d*Sin[e + f*x])/(c + d))^n)
```

3.329.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3234
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]]))   Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 -
(d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2,
0]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3496
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A - C)
  Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Simp[C   Int[(a +
 b*Sin[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B,
 C, m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

3.329.4 Maple [F]

$$\int (a + a\sin(fx + e))(A + B\sin(fx + e))(c + d\sin(fx + e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.329.5 Fricas [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(f*x + e) + c)^n, x)
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

3.329.7 Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

3.329.8 Giac [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n
, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,(c + d\,\sin(e + f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

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3.330.2 Mathematica [F] . . . 2746
3.330.3 Rubi [A] (verified) . . . 2746
3.330.4 Maple [F] . . . 2749
3.330.5 Fricas [F] . . . 2749
3.330.6 Sympy [F(-1)] . . . 2750
3.330.7 Maxima [F] . . . 2750
3.330.8 Giac [F] . . . 2750
3.330.9 Mupad [F(-1)] . . . 2751

3.330.1 Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(}{c+}\right.}{af\sqrt{1+\sin(e+fx)}}$$
$$-\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d}{}\right.}{\sqrt{2}af\sqrt{1+\sin(e+fx)}}$$

output
```
-1/2*(A-B)*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+
e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)
/(1+sin(f*x+e))^(1/2)-B*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2
-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+
e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

3.330.2 Mathematica [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a + a\sin(e + fx)}\,dx = \int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a + a\sin(e + fx)}\,dx$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
]),x]
```

output
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
]), x]
```

3.330.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3466, 3042, 3144, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a\sin(e + fx) + a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a\sin(e + fx) + a}dx$$

$$\downarrow 3466$$

$$(A - B)\int \frac{(c + d\sin(e + fx))^n}{\sin(e + fx)a + a}dx + \frac{B\int (c + d\sin(e + fx))^n dx}{a}$$

$$\downarrow 3042$$

$$(A - B)\int \frac{(c + d\sin(e + fx))^n}{\sin(e + fx)a + a}dx + \frac{B\int (c + d\sin(e + fx))^n dx}{a}$$

$$\downarrow 3144$$

$$(A - B)\int \frac{(c + d\sin(e + fx))^n}{\sin(e + fx)a + a}dx + \frac{B\cos(e + fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}d\sin(e + fx)}{af\sqrt{1 - \sin(e + fx)}\sqrt{\sin(e + fx) + 1}}$$

$$\downarrow 156$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx + \frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 155$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx - \frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 3263$$

$$\frac{(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} - \frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 156$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}} - \frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 155$$

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{\sqrt{2}af\sqrt{\sin(e+fx)+1}} - \frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

output
```
-((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin
[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/
2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e +
f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*S
in[e + f*x])/(c + d))^n)
```

3.330.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.330.4 Maple [F]

$$\int \frac{(A + B\sin(fx+e))(c + d\sin(fx+e))^n}{a + a\sin(fx+e)}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

3.330.5 Fricas [F]

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))^n}{a + a\sin(e+fx)}\,dx$$
$$= \int \frac{(B\sin(fx+e) + A)(d\sin(fx+e) + c)^n}{a\sin(fx+e) + a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

3.330.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

3.330.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

3.330.8 Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a + a\sin(e + fx)}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(c + d\,\sin(e + f\,x))^n}{a + a\,\sin(e + f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

3.331.1 Optimal result . . . 2752
3.331.2 Mathematica [F] . . . 2753
3.331.3 Rubi [A] (verified) . . . 2753
3.331.4 Maple [F] . . . 2756
3.331.5 Fricas [F] . . . 2756
3.331.6 Sympy [F(-1)] . . . 2756
3.331.7 Maxima [F] . . . 2757
3.331.8 Giac [F] . . . 2757
3.331.9 Mupad [F(-1)] . . . 2757

3.331.1 Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{B\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+f}{c+d}\right.}{\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}}$$
$$-\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d}{}\right.}{2\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}}$$

output

```
-1/2*B*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(
1+sin(f*x+e))^(1/2)-1/4*(A-B)*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+
d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+
e))/(c+d))^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

3.331.2 Mathematica [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2}\,dx$$
$$= \int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2}\,dx$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^2, x]
```

3.331.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3466, 3042, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a\sin(e + fx) + a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a\sin(e + fx) + a)^2}dx$$
$$\downarrow 3466$$
$$(A - B)\int \frac{(c + d\sin(e + fx))^n}{(\sin(e + fx)a + a)^2}dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx}{a}$$
$$\downarrow 3042$$
$$(A - B)\int \frac{(c + d\sin(e + fx))^n}{(\sin(e + fx)a + a)^2}dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx}{a}$$
$$\downarrow 3263$$

$$\frac{(A-B)\cos(e+fx)\int\frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{5/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+$$
$$\frac{B\cos(e+fx)\int\frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 156$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{5/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+$$
$$\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\downarrow 155$$

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$
$$\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
-((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2,
5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c
 + d*Sin[e + f*x])/(c + d))^n)
```

3.331.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.331.4 Maple [F]

$$\int \frac{(A + B\sin(fx + e))(c + d\sin(fx + e))^n}{(a + a\sin(fx + e))^2} dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

3.331.5 Fricas [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2} dx$$
$$= \int \frac{(B\sin(fx + e) + A)(d\sin(fx + e) + c)^n}{(a\sin(fx + e) + a)^2} dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2
 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2} dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.331.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

3.331.8 Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x
)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,
x)
```

3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

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3.332.1 Optimal result

Integrand size = 37, antiderivative size = 427

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$
$$-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)}$$
$$+\frac{2a^2(A-B)(c-d(5+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^2B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

output
```
-2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e)
)^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(
4*n^2+16*n+15)/(a+a*sin(f*x+e))^(1/2)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e)
*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/
(3+2*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*B*(3*c^2
-2*c*d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],
d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(4*n^2+16*n+15)/(((c+d*si
n(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e
))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

3.332.2 Mathematica [A] (verified)

Time = 11.79 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.57

$$\int (a + a\sin(e+fx))^{3/2}(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx =$$

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(5+4n))\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^n,x]
```

output
```
-1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*
n))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] +
6*B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/
(c + d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hyper
geometric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e
 + f*x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*
f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

3.332.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {3042, 3466, 3042, 3242, 27, 2011, 3042, 3255, 80, 79, 3460, 3042, 3255, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n dx$$

$$\downarrow 3466$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a+a)^{5/2}(c+d\sin(e+fx))^n dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a+a)^{5/2}(c+d\sin(e+fx))^n dx}{a}$$

$$\downarrow 3242$$

$$(A-B)\left(\frac{2\int -\frac{(c+d\sin(e+fx))^n\left((c-5d-4dn)a^2+(c-5d-4dn)\sin(e+fx)a^2\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{d(2n+3)} - \frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right) + \frac{B\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx}{df(2n+5)}\right)}{a}$$

$$\downarrow 27$$

$$(A-B)\left(-\frac{\int \frac{(c+d\sin(e+fx))^n\left((c-5d-4dn)a^2+(c-5d-4dn)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{d(2n+3)} - \frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right) + \frac{B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right)}{a}$$

$$\downarrow 2011$$

$$(A - B)\left(-\frac{a(c-4dn-5d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)} - \frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right) + B\left(\frac{\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right)$$

$$\downarrow 3042$$

$$(A - B)\left(-\frac{a(c-4dn-5d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)} - \frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right) + B\left(\frac{\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right)$$

$$\downarrow 3255$$

$$B\left(\frac{\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right)$$

$$(A - B)\left(-\frac{a^3(c-4dn-5d)\cos(e+fx)\int\frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)$$

$$\downarrow 80$$

$$B\left(\frac{\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)} - \frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right)$$

$$(A - B)\left(-\frac{a^3(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} - \frac{2a^2}{}\right)$$

$$\downarrow 79$$

$$\frac{B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n}{df(2n+5)}\right.}{a}$$

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\downarrow$ 3460

$$\frac{B\left(\frac{\frac{a^2\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{}}{}\right.}{a}$$

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\downarrow$ 3042

$$\frac{B\left(\frac{\frac{a^2\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{}}{}\right.}{a}$$

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\downarrow$ 3255

$$\frac{B\left(\frac{\frac{a^4\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e+f}{}\right.}{a}$$

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\downarrow$ 80

3.332. $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

$$B\left(\frac{\frac{a^4\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-d(4n+11))\cos(}{df(2n+3)\sqrt{}}}{d(2n+5)}\right)$$

$$\frac{}{a}$$

$$(A-$$

$$B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e-}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 79$$

$$(A-$$

$$B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e-}{c+d}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$B\left(\frac{\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeo}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}\right)$$

$$\frac{}{a}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
,x]
```

output
```
(A - B)*((-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)
*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(c - 5*d - 4*d*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c +
d))^n)) + (B*((-2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
 f*x])^(1 + n))/(d*f*(5 + 2*n)) + ((2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x
]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a^3*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin
[e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x]
)/(c + d))^n))/(d*(5 + 2*n))))/a
```

3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.332.4 Maple [F]

$$\int (a + a\sin(fx+e))^{\frac{3}{2}} (A + B\sin(fx+e)) (c + d\sin(fx+e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.332.5 Fricas [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

3.332.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

3.332.7 Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^n, x)
```

3.332.8 Giac [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^n, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \int (A + B\sin(e + fx))(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n
, x)
```

3.333 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

3.333.1 Optimal result 2769
3.333.2 Mathematica [F] 2770
3.333.3 Rubi [A] (verified) 2770
3.333.4 Maple [F] 2772
3.333.5 Fricas [F] 2773
3.333.6 Sympy [F] 2773
3.333.7 Maxima [F] 2773
3.333.8 Giac [F] 2774
3.333.9 Mupad [F(-1)] 2774

3.333.1 Optimal result

Integrand size = 37, antiderivative size = 167

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a(Ad(3+2n)-B(c-2d(1+n)))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin$$
$$df(3+2n)\sqrt{a+a\sin(e+fx)}$$

output

```
-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2
)-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d
*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(
c+d))^n)/(a+a*sin(f*x+e))^(1/2)
```

3.333.2 Mathematica [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^n,x]
```

output
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^n, x]
```

3.333.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3042, 3460, 3042, 3255, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^n dx$$
$$\downarrow 3460$$
$$-\frac{(-Ad(2n+3)+Bc-2Bd(n+1))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$
$$\downarrow 3042$$
$$-\frac{(-Ad(2n+3)+Bc-2Bd(n+1))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3255$$

$$-\frac{a^2\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))\int\frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 80$$

$$-\frac{a^2\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 79$$

$$\frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
 a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x
]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*S
in[e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```

3.333.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.333.4 Maple [F]

$$\int \sqrt{a + a\sin(fx+e)}\,(A + B\sin(fx+e))\,(c + d\sin(fx+e))^n\,dx$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.333.5 Fricas [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

3.333.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}(A+B\sin\left(e+fx\right))\left(c+d\sin\left(e+fx\right)\right)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**n, x)
```

3.333.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n, x)
```

3.333.8 Giac [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{a\sin(fx + e) + a}(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n, x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,\sqrt{a + a\,\sin(e + f\,x)}\,(c + d\,\sin(e + f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n
, x)
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.334.1 Optimal result 2775
3.334.2 Mathematica [A] (warning: unable to verify) 2776
3.334.3 Rubi [A] (verified) 2776
3.334.4 Maple [F] 2780
3.334.5 Fricas [F] 2780
3.334.6 Sympy [F] 2781
3.334.7 Maxima [F] 2781
3.334.8 Giac [F(-1)] 2781
3.334.9 Mupad [F(-1)] 2782

3.334.1 Optimal result

Integrand size = 37, antiderivative size = 220

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{(A-B)\,\text{AppellF1}\left(1+n,\frac{1}{2},1,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

output

```
-2*B*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin
(f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-(A-B)*App
ellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x
+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)/f/(1+n)/(1
-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

3.334.2 Mathematica [A] (warning: unable to verify)

Time = 8.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{\cos(e+fx)\sqrt{a(1+\sin(e+fx))}(c+d\sin(e+fx))^n\left(-\left((A+B)\,\text{AppellF1}\left(1,\frac{1}{2},-n,2,\frac{1}{2}(1+\sin(e+\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B
)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c
 + d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A
 - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)
/(d + d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 +
 2*n)*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

3.334.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3466, 3042, 3255, 80, 79, 3267, 27, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\downarrow 3466$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx + \frac{B\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{a}$$

$$\downarrow 3255$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{aB\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 80$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 79$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 154$$

$$\frac{a(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int\frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 153$$

$$-\frac{a(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\,\text{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+f}{c-d}\right.}{f(n+1)(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

output
```
-((a*(A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d),
(c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c
+ d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(a - a*Sin[e + f*x]
)*Sqrt[a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n
, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a +
 a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

3.334.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.334.4 Maple [F]

$$\int \frac{(A + B\sin(fx+e))(c + d\sin(fx+e))^n}{\sqrt{a + a\sin(fx+e)}}dx$$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

output

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

3.334.5 Fricas [F]

$$\int \frac{(A + B\sin(e+fx))(c + d\sin(e+fx))^n}{\sqrt{a + a\sin(e+fx)}}\,dx$$
$$= \int \frac{(B\sin(fx+e) + A)(d\sin(fx+e) + c)^n}{\sqrt{a\sin(fx+e) + a}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

3.334.6 Sympy [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a + a\sin(e + fx)}}\,dx = \int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a(\sin(e + fx) + 1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

3.334.7 Maxima [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a + a\sin(e + fx)}}\,dx = \int \frac{(B\sin(fx + e) + A)(d\sin(fx + e) + c)^n}{\sqrt{a\sin(fx + e) + a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e)
+ a), x)
```

3.334.8 Giac [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a + a\sin(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/
2), x)
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.335.1 Optimal result 2783
3.335.2 Mathematica [B] (warning: unable to verify) 2784
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3.335.8 Giac [F(-1)] 2788
3.335.9 Mupad [F(-1)] 2789

3.335.1 Optimal result

Integrand size = 37, antiderivative size = 269

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{B\,\text{AppellF1}\left(1+n,\frac{1}{2},1,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{(A-B)d\,\text{AppellF1}\left(1+n,\frac{1}{2},2,2+n,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+$$

$$(c-d)^2 f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}$$

output

```
-B*AppellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*c
os(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/
(1+n)/(1-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+(A-B)*d*AppellF1(1+n,2,1/2,2+n
,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)^2/f/(1+n)/(a-a*sin(f*x+e))/(a
+a*sin(f*x+e))^(1/2)
```

3.335.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 603 vs. $2(269) = 538$.

Time $= 16.63$ (sec) , antiderivative size $= 603$, normalized size of antiderivative $= 2.24$

$$\int \frac{(A + B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(a$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
(Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[
1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt
[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n
 - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1
[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e +
 f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e
 + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*Appel
lF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*
Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d)
)^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*Appel
lF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[
e + f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin
[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n
^2)*(1 + (c - d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e +
f*x])])
```

3.335.3 Rubi [A] (verified)

Time $= 0.69$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 0.99$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used $= \{3042, 3466, 3042, 3267, 27, 154, 153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\downarrow 3466$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a^2(\sin(e+fx)+1)^2\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)^2\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{B\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 154$$

$$\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)^2\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 153$$

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\,\text{AppellF1}\left(n+1,\frac{1}{2},2,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right.}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$
$$\frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\,\text{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*S
in[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c
 + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a
 + a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Si
n[e + f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1
 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 +
 n)*(a - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.335.4 Maple [F]

$$\int \frac{(A+B\sin(fx+e))(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

3.335.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

3.335.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^n}{(a\,(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1
))**(3/2), x)
```

3.335.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)
^(3/2), x)
```

3.335.8 Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/
2), x)
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

3.336.1 Optimal result 2790
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3.336.1 Optimal result

Integrand size = 35, antiderivative size = 351

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\,(Acd(3+m)+B(c^2+d^2+cdm)))\cos(e+fx)(a+a\sin(e+$$
$$-\frac{2^{\frac{1}{2}+m}(A(3+m)\,(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2))+B(d^2m(5+3m+m^2)+c^2m$$
$$-\frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$
$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$

output

```
(d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2)))*cos(f*
x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2
+m)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+
2*c*d*(m^3+4*m^2+4*m+3)))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*
sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(3+m)/(m^2+3*m+2)
-d*(A*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+
m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

3.336.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.35 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.43

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{i(a(1+\sin(e+fx)))^m(1-i\cos(e+fx)+\sin(e+fx))^{-2m}\left(\frac{8Ac^2\,\text{Hypergeometric2F1}(-2m,-m,1-m,i\cos(e+fx)-\sin}{m}\right.$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
((I/8)*(a*(1 + Sin[e + f*x]))^m*((8*A*c^2*Hypergeometric2F1[-2*m, -m, 1 -
m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (8*B*c*d*Hypergeometric2F1[-2*m, -m
, 1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*A*d^2*Hypergeometric2F1[-2
*m, -m, 1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*B*c^2*Hypergeometric
2F1[1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x]
+ Sin[e + f*x]))/(-1 + m) + (8*A*c*d*Hypergeometric2F1[1 - m, -2*m, 2 - m,
 I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 +
m) + (3*B*d^2*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e
 + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 + m) + (4*B*c^2*Hypergeom
etric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*x]
+ Sin[e + f*x]))/(1 + m) + (8*A*c*d*Hypergeometric2F1[-1 - m, -2*m, -m, I*
Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (3
*B*d^2*Hypergeometric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*
(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) - (2*d*(2*B*c + A*d)*Hypergeometr
ic2F1[-2 - m, -2*m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x
)] - I*Sin[2*(e + f*x)]))/(2 + m) - (4*B*c*d*Hypergeometric2F1[2 - m, -2*m
, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f
*x)]))/(-2 + m) - (2*A*d^2*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*Cos[e +
 f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m) -
(I*B*d^2*Hypergeometric2F1[-3 - m, -2*m, -2 - m, I*Cos[e + f*x] - Sin[e...
```

3.336.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3462, 3042, 3447, 3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2 dx$$

$$\downarrow 3462$$

$$\frac{\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))(a(Ac(m+3)+B(2d+cm))+a(Ad(m+3)+B(2c+dm))\sin(e+fx$$
$$\frac{a(m+3)}{\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))(a(Ac(m+3)+B(2d+cm))+a(Ad(m+3)+B(2c+dm))\sin(e+fx$$
$$\frac{a(m+3)}{\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}}$$

$$\downarrow 3447$$

$$\frac{\int (\sin(e+fx)a+a)^m\left(ad(Ad(m+3)+B(2c+dm))\sin^2(e+fx)+(ad(Ac(m+3)+B(2d+cm))+ac(Ad(m+\right.}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^m\left(ad(Ad(m+3)+B(2c+dm))\sin(e+fx)^2+(ad(Ac(m+3)+B(2d+cm))+ac(Ad(m+\right.}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3502$$

$$\frac{\int (\sin(e+fx)a+a)^m \left(a^2(c(m+2)(Ac(m+3)+B(2d+cm))+d(m+1)(Ad(m+3)+B(2c+dm)))-a^2\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)\right.\right.\right.}{a(m+2)}$$

$$a(m+3)$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^m \left(a^2(c(m+2)(Ac(m+3)+B(2d+cm))+d(m+1)(Ad(m+3)+B(2c+dm)))-a^2\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)\right.\right.\right.}{a(m+2)}$$

$$a(m+3)$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3230$$

$$\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a^2\cos}{a(m+2)}$$

$$a(m$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3042$$

$$\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a^2\cos}{a(m+2)}$$

$$a(m$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3131$$

$$\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)}{m+1}$$

$$a(m+2)$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\downarrow 3042$$

$$\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)}{m+1}$$

$$a(m+2)$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

↓ 3130

$$\frac{a^2\cos(e+fx)\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)+B\left(c^2+cdm+d^2\right)\right)\right)(a\sin(e+fx)+a)^m}{f(m+1)} - \frac{a^2 2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m
))) + (-((d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(1 + m))/(f*(2 + m))) + ((a^2*(d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 +
m)*(A*c*d*(3 + m) + B*(c^2 + d^2 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m)/(f*(1 + m)) - (2^(1/2 + m)*a^2*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1
 + m + m^2) + c^2*(2 + 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 +
 5*m + m^2) + 2*c*d*(3 + 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2
F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*
(a + a*Sin[e + f*x])^m)/(f*(1 + m)))/(a*(2 + m)))/(a*(3 + m))
```

3.336.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.336.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c + d\sin(fx + e))^2 \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

3.336.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2\, dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^2 (a\sin(fx + e) + a)^m\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*
d^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e
) + a)^m, x)
```

3.336.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2\, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx)) (c + d\sin(e + fx))^2\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

3.336.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2\, dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^2 (a\sin(fx + e) + a)^m\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)
^m, x)
```

3.336.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^2 (a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)
^m, x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c + d\,\sin(e + f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

3.337.1 Optimal result . . . 2798
3.337.2 Mathematica [C] (warning: unable to verify) . . . 2799
3.337.3 Rubi [A] (verified) . . . 2799
3.337.4 Maple [F] . . . 2802
3.337.5 Fricas [F] . . . 2802
3.337.6 Sympy [F] . . . 2803
3.337.7 Maxima [F] . . . 2803
3.337.8 Giac [F] . . . 2804
3.337.9 Mupad [F(-1)] . . . 2804

3.337.1 Optimal result

Integrand size = 33, antiderivative size = 199

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}$$
$$- \frac{2^{\frac{1}{2}+m}(A(2+m)(c+cm+dm)+B(cm(2+m)+d(1+m+m^2)))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{f(1+m)(2+m)}$$
$$- \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}$$

output
```
(B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m
)*(A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/
2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e
))^m/f/(m^2+3*m+2)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

3.337.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.75 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{(a(1+\sin(e+fx)))^m(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{2(2Ac+Bd)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+fx}{m}\right.$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
-1/4*((a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*((-2*
(2*A*c + B*d)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - Sin[e +
f*x]])/m - ((2*I)*(B*c + A*d)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x] -
 Sin[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((2*I)*(B*c + A*
d)*Hypergeometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[
e + f*x] + I*Sin[e + f*x]))/(-1 + m) + (B*d*Hypergeometric2F1[1, -1 + m, -
1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x
)]))/(2 + m) + (B*d*Hypergeometric2F1[1, 3 + m, 3 - m, I*Cos[e + f*x] - Si
n[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m)))/f
```

3.337.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\downarrow 3447$$

$$\int (a\sin(e+fx)+a)^m\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\downarrow 3502$$

$$\frac{\int (\sin(e+fx)a+a)^m(a(Bd(m+1)+Ac(m+2))-a(Bd-(Bc+Ad)(m+2))\sin(e+fx))dx}{a(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3042$$

$$\frac{\int (\sin(e+fx)a+a)^m(a(Bd(m+1)+Ac(m+2))-a(Bd-(Bc+Ad)(m+2))\sin(e+fx))dx}{a(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3230$$

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}}{a(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3042$$

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}}{a(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3131$$

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int (\sin(e+fx)+1)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(}{f(}}{a(m+2)} - \frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3042$$

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(}{f(}}{a(m+2)}$$

$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\downarrow 3130$$

$$\frac{\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)+1}{f(m+1)}}{a(m+2)}$$

$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-((B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))) + ((a*(B*
d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
 - (2^(1/2 + m)*a*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*(1 + m +
 m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x]
)/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)))/(a
*(2 + m))
```

3.337.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.337.4 Maple [F]

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e)) \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

3.337.5 Fricas [F]

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) (c + d \sin(e + fx)) \, dx$$
$$= \int (B \sin(fx + e) + A)(d \sin(fx + e) + c)(a \sin(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*s
in(f*x + e) + a)^m, x)
```

3.337.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

3.337.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m
, x)
```

3.337.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,(c + d\,\sin(e + f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

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3.338.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}$$
$$-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+$$
$$f(1+m)$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*
hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(
a+a*sin(f*x+e))^m/f/(1+m)
```

3.338.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+\sin$$
$$f$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
(2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin
[e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin
[e + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

3.338.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\downarrow 3230$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3042$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3131$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3042$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\downarrow 3130$$

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2}\right.}{f(m+1)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
 A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e +
 f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)
)
```

3.338.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.338.4 Maple [F]

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

3.338.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m (A+B\sin(e+fx))\, dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.338.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\, dx = \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

3.338.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.338.8 Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

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3.339.7 Maxima [F] . . . 2815
3.339.8 Giac [F] . . . 2815
3.339.9 Mupad [F(-1)] . . . 2816

3.339.1 Optimal result

Integrand size = 35, antiderivative size = 191

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}(Bc-Ad)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin}{(c-d)df(1+2m)\sqrt{1-\sin(e+fx)}}$$
$$-\frac{2^{\frac{1}{2}+m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a}{df}$$

output
```
-2^(1/2+m)*B*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(
1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/f-(-A*d+B*c)*AppellF1(1/2+m,1,
1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(
f*x+e))^m*2^(1/2)/(c-d)/d/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

3.339.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$=\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]), x]
```

3.339.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\downarrow 3465$$

$$\frac{B\int(\sin(e+fx)a+a)^m dx}{d} - \frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$$\downarrow 3042$$

$$\frac{B\int(\sin(e+fx)a+a)^m dx}{d} - \frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$$\downarrow 3131$$

$$\frac{B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d} - \frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$$\downarrow 3042$$

$$\frac{B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d} - \frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$$\downarrow 3130$$

$$-\frac{(Bc-Ad)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} -$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f\right.}{df}$$

$$\downarrow 3267$$

$$-\frac{a^2(Bc-Ad)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} -$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f\right.}{df}$$

$$\downarrow 154$$

$$-\frac{a^2(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{\sqrt{2}df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}} -$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f\right.}{df}$$

$$\downarrow 27$$

$$-\frac{a^2(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}} -$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f\right.}{df}$$

$$\downarrow 153$$

$$-\frac{\sqrt{2}a(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m \text{ AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+\right.}{df(2m+1)(c-d)(a-a\sin(e+fx))}$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f\right.}{df}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
-((2^(1/2 + m)*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Si
n[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
) - (Sqrt[2]*a*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e +
 f*x]]*(a + a*Sin[e + f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])
)
```

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

3.339.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{c+d\sin(fx+e)}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

3.339.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

3.339.7 Maxima [F]

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx \\ &= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
, x)
```

3.339.8 Giac [F]

$$\begin{aligned}&\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx \\ &= \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

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3.340.1 Optimal result

Integrand size = 35, antiderivative size = 293

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \frac{\sqrt{2}(Ad(c(1-m)-dm)-B(d^2-c^2m-cdm))\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+}{(c-d)^2d(c+d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$
$$+\frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))}{d\,(c^2-d^2)\,f}$$
$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))}$$

output

```
2^(1/2+m)*(-A*d+B*c)*m*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin
(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)/f-(-A*d+B*
c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))+(A*d*(c*(1-m
)-d*m)-B*(-c^2*m-c*d*m+d^2))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/
(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/d/
(c+d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

3.340.2 Mathematica [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\, dx = \int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\, dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2, x]
```

3.340.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3463, 25, 3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e + fx) + a)^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\, dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e + fx) + a)^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\, dx$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{(\sin(e+fx)a+a)^m (a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}\, dx}{a\left(c^2 - d^2\right)} - \frac{(Bc - Ad)\cos(e + fx)(a\sin(e + fx) + a)^m}{f\left(c^2 - d^2\right)(c + d\sin(e + fx))}$$

$$\downarrow 25$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m (a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}\, dx}{a\left(c^2 - d^2\right)} - \frac{(Bc - Ad)\cos(e + fx)(a\sin(e + fx) + a)^m}{f\left(c^2 - d^2\right)(c + d\sin(e + fx))}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}dx}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3465$$

$$\frac{\frac{a(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} - \frac{am(Bc-Ad)\int(\sin(e+fx)a+a)^m dx}{d}}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{a(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} - \frac{am(Bc-Ad)\int(\sin(e+fx)a+a)^m dx}{d}}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3131$$

$$\frac{\frac{a(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} - \frac{am(Bc-Ad)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d}}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$\frac{\frac{a(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} - \frac{am(Bc-Ad)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d}}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3130$$

$$\frac{\frac{a(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d} + \frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hy}}{df}}{a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3267$$

$$\frac{a^3\cos(e+fx)(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+f}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 154$$

$$\frac{a^3\sqrt{1-\sin(e+fx)}\cos(e+fx)(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{\sqrt{2}df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e-}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 27$$

$$\frac{a^3\sqrt{1-\sin(e+fx)}\cos(e+fx)(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e-}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 153$$

$$\frac{\sqrt{2}a^2\sqrt{1-\sin(e+fx)}\cos(e+fx)(Ad(c(1-m)-dm)-B(c^2(-m)-cdm+d^2))(a\sin(e+fx)+a)^m\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1),-\frac{d(s}{df(2m+1)(c-d)(a-a\sin(e+fx))}\right.$$

$$a\left(c^2-\right.$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-(((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*
Sin[e + f*x]))) + ((2^(1/2 + m)*a*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(d*f) + (Sqrt[2]*a^2*(A*d*(c*(1 - m) - d*m) - B*
(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x
])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x
]]*(a + a*Sin[e + f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])))/(
a*(c^2 - d^2))
```

3.340. $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

3.340.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

3.340.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^2}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

3.340.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.340.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

3.340.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^2, x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,
x)
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

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3.341.1 Optimal result

Integrand size = 35, antiderivative size = 467

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-3m+m^2)}{\sqrt{2}(c-d)^3d(c+}$$

$$-\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-r}{d\left(c^2-d^2\right)^2 f}$$

$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$+\frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))}$$

output

```
-2^(-1/2+m)*m*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hyp
ergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a
*sin(f*x+e))^m/d/(c^2-d^2)^2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^
m/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)
-c*d*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+1/2*
(B*(2*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A*d*(2*c*d*(2-m
)*m-c^2*(m^2-3*m+2)-d^2*(m^2-m+1)))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f
*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c-d)^3/d/(
c+d)^2/f/(1+2*m)*2^(1/2)/(1-sin(f*x+e))^(1/2)
```

3.341.2 Mathematica [F]

$$\int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx = \int \frac{(a + a\sin(e+fx))^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3, x]
```

3.341.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3463, 25, 3042, 3463, 3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m (A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx$$

$$\downarrow 3463$$

$$-\frac{\int -\frac{(\sin(e+fx)a+a)^m (a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m (a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3463$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)^m\left(a^2((Bc-Ad)(1-m)(d-cm)-(c-dm)(2Ac+Bmc-}{c+}}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{\int \frac{(\sin(e+fx)a+a)^m\left(a^2((Bc-Ad)(1-m)(d-cm)-(c-dm)(2Ac+Bmc-}{c+}}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3465$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)\int(\sin(e+fx)a+a)}{d}}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)\int(\sin(e+fx)a+a)}{d}}{2a\left(c^2-d^2\right)} - \frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3131$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(\sin(e+fx)+1)^{-n}}{d}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3042$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(\sin(e+fx)+1)^{-n}}{d}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3130$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(s}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 3267$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(s}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 154$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(s}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 27$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(s}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\downarrow 153$$

$$\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))} - \frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(s}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

output
```
-1/2*((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^2) + ((a*(A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1 - m)
- c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e
 + f*x])) - ((2^(1/2 + m)*a^2*m*(A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1
 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin
[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
- (Sqrt[2]*a^3*(B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3
- 3*m + m^2)) - A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m +
m^2)))*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e
+ f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])))/(a*(c^2 - d^2)))/
(2*a*(c^2 - d^2))
```

3.341.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

3.341.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^3}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

3.341.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m (A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e
)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

3.341.7 Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{(d\sin(fx + e) + c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^3, x)
```

3.341.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^3, x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,
x)
```

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

3.342.1 Optimal result . . . 2834
3.342.2 Mathematica [F] . . . 2835
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3.342.4 Maple [F] . . . 2838
3.342.5 Fricas [F] . . . 2839
3.342.6 Sympy [F(-1)] . . . 2839
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3.342.9 Mupad [F(-1)] . . . 2840

3.342.1 Optimal result

Integrand size = 37, antiderivative size = 284

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{2}(A-B)(c-d)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$+\frac{\sqrt{2}B(c-d)\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}}{af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

output

```
(A-B)*(c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*
sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f
/(1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*(c-d)*Appel
lF1(3/2+m,-3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f
*x+e)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1
-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

3.342.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2), x]
```

3.342.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}dx$$

$$\downarrow 3466$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^{3/2}dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{3/2}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^{3/2}dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{3/2}dx}{a}$$

$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 157$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+$$
$$f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}$$
$$\frac{aB(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}a(A-B)(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$\frac{\sqrt{2}B(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{3}{2},m+\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
,x]
```

output
```
(Sqrt[2]*a*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[
e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[
e + f*x]]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*(a
 - a*Sin[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d
)*AppellF1[3/2 + m, 1/2, -3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Si
n[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e +
f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*
Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.342.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c + d\sin(fx+e))^{\frac{3}{2}}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

3.342.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^{\frac{3}{2}}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt
(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.342.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^{\frac{3}{2}}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

3.342.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)^{\frac{3}{2}}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = \int (A + B\sin(e + fx))\,(a + a\sin(e + fx))^m\,(c + d\sin(e + fx))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2), x)
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+$

3.343.1 Optimal result 2841
3.343.2 Mathematica [F] 2842
3.343.3 Rubi [A] (verified) 2842
3.343.4 Maple [F] 2845
3.343.5 Fricas [F] 2846
3.343.6 Sympy [F] 2846
3.343.7 Maxima [F] 2846
3.343.8 Giac [F] 2847
3.343.9 Mupad [F(-1)] 2847

3.343.1 Optimal result

Integrand size = 37, antiderivative size = 274

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+$$
$$f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}$$
$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx$$
$$af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}$$

output

```
(A-B)*AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*
x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*
m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*AppellF1(3/2+m,-1
/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*s
in(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))
^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

3.343.2 Mathematica [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))\sqrt{c + d\sin(e+fx)}\,dx$$
$$= \int (a + a\sin(e+fx))^m (A + B\sin(e+fx))\sqrt{c + d\sin(e+fx)}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f
*x]],x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f
*x]], x]
```

3.343.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m (A + B\sin(e+fx))\sqrt{c + d\sin(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m (A + B\sin(e+fx))\sqrt{c + d\sin(e+fx)}dx$$
$$\downarrow 3466$$
$$(A-B)\int (\sin(e+fx)a+a)^m\sqrt{c + d\sin(e+fx)}dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}\sqrt{c + d\sin(e+fx)}dx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int (\sin(e+fx)a+a)^m\sqrt{c + d\sin(e+fx)}dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}\sqrt{c + d\sin(e+fx)}dx}{a}$$
$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 157$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-\frac{1}{2},m+\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{1}{2},m+\frac{5}{2},\frac{1}{2}(\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x
]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]
]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*(a - a*Sin
[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 +
 m, 1/2, -1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
- d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*Sq
rt[c + d*Sin[e + f*x]])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*Sqrt[(c + d*Sin[
e + f*x])/(c - d)])
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.343.4 Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) \sqrt{c + d\sin(fx+e)}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

3.343.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

3.343.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
 f*x)), x)
```

3.343.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

3.343.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m\,\sqrt{c + d\,\sin(e + f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2)
, x)
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

3.344.1 Optimal result . . . 2848
3.344.2 Mathematica [F] . . . 2849
3.344.3 Rubi [A] (verified) . . . 2849
3.344.4 Maple [F] . . . 2852
3.344.5 Fricas [F] . . . 2852
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3.344.7 Maxima [F] . . . 2853
3.344.8 Giac [F] . . . 2854
3.344.9 Mupad [F(-1)] . . . 2854

3.344.1 Optimal result

Integrand size = 37, antiderivative size = 274

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{1}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{1}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))}{af(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

output

```
(A-B)*AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/
f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,1/2
,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin
(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/f/(3+2*m)/(1-sin(f
*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

3.344.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
 f*x]],x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
 f*x]], x]
```

3.344.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$\downarrow 3466$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 157$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(si\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{1}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e+\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]]
,x]
```

3.344. $\int\frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]
*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*(
a - a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 +
m, 1/2, 1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c -
d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[(c + d*Sin[e + f*x])/(c - d)])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*Sqrt[c +
 d*Sin[e + f*x]])
```

3.344.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.344.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c+d\sin(fx+e)}}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

3.344.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

3.344.6 Sympy [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\,dx = \int \frac{(a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
 f*x)), x)
```

3.344.7 Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{\sqrt{d\sin(fx + e) + c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e)
 + c), x)
```

3.344.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e)
+ c), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/
2), x)
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

3.345.1 Optimal result 2855
3.345.2 Mathematica [F] 2855
3.345.3 Rubi [A] (verified) 2856
3.345.4 Maple [F] 2859
3.345.5 Fricas [F] 2859
3.345.6 Sympy [F] 2860
3.345.7 Maxima [F] 2860
3.345.8 Giac [F] 2860
3.345.9 Mupad [F(-1)] 2861

3.345.1 Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},\frac{3}{2},\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+\right.}{(c-d)f(1+2m)\sqrt{1-}}$$
$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},\frac{3}{2},\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1+}}{a(c-d)f(3+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

output
```
(A-B)*AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/
(c-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2
+m,3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a
+a*sin(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/(c-d)/f/(3+2
*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

3.345.2 Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^(3/2),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^(3/2), x]
```

3.345.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3466$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{(c+d\sin(e+fx))^{3/2}}dx + \frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{(c+d\sin(e+fx))^{3/2}}dx}{a}$$

$$\downarrow 3042$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{(c+d\sin(e+fx))^{3/2}}dx + \frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{(c+d\sin(e+fx))^{3/2}}dx}{a}$$

$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}} +$$
$$\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 157$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}} +$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}d\sin(e+fx)}{f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}d\sin(e+fx)}{f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\frac{3}{2},m+\frac{3}{2},\frac{1}{2}(si\right.}{f(2m+1)(c-d)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{3}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e+\right.}{f(2m+3)(c-d)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/
2),x]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]
*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1
+ 2*m)*(a - a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF
1[3/2 + m, 1/2, 3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]
))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 +
 m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(3 + 2*m)*(a - a*Sin[e
+ f*x])*Sqrt[c + d*Sin[e + f*x]])
```

3.345.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.345.4 Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^{\frac{3}{2}}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

3.345.5 Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m (A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}} dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}} dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

3.345.6 Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x
))**(3/2), x)
```

3.345.7 Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^(3/2), x)
```

3.345.8 Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^(3/2), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/
2), x)
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

3.346.1 Optimal result . . . 2862
3.346.2 Mathematica [F] . . . 2863
3.346.3 Rubi [A] (verified) . . . 2863
3.346.4 Maple [F] . . . 2866
3.346.5 Fricas [F] . . . 2866
3.346.6 Sympy [F(-1)] . . . 2867
3.346.7 Maxima [F] . . . 2867
3.346.8 Giac [F] . . . 2867
3.346.9 Mupad [F(-1)] . . . 2868

3.346.1 Optimal result

Integrand size = 35, antiderivative size = 270

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \frac{\sqrt{2}(A-B)\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$
$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},-n,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^n}{af(3+2m)\sqrt{1-\sin(e+fx)}}$$

output

```
(A-B)*AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((
c+d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,-n,1/2,5/2
+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))
^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)
/(1-sin(f*x+e))^(1/2)
```

3.346.2 Mathematica [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n \, dx$$
$$= \int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n \, dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

3.346.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx) + a)^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n \, dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx) + a)^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n dx$$
$$\downarrow 3466$$
$$(A-B)\int (\sin(e+fx)a + a)^m (c + d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a + a)^{m+1}(c + d\sin(e+fx))^n dx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int (\sin(e+fx)a + a)^m (c + d\sin(e+fx))^n dx + \frac{B\int (\sin(e+fx)a + a)^{m+1}(c + d\sin(e+fx))^n dx}{a}$$
$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 157$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 27$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 156$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)}{\sqrt{1-\sin(e+fx)}}}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$
$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(}{f(2m+1)(a-a\sin(e+fx))}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(m+\right.}{f(2m+3)(a-a\sin(e+fx))}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/
2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*
(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*(a - a*Sin[e +
 f*x])*((c + d*Sin[e + f*x])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/
2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*C
os[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*(c + d*Sin
[e + f*x])^n)/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*((c + d*Sin[e + f*x])/(c -
 d))^n)
```

3.346.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.346.4 Maple [F]

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.346.5 Fricas [F]

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) (c + d \sin(e + fx))^n \, dx$$
$$= \int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

3.346.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

3.346.7 Maxima [F]

$$\begin{aligned}&\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx \\ &= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.346.8 Giac [F]

$$\begin{aligned}&\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx \\ &= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx$$
$$= \int (A + B \sin(e + f x)) (a + a \sin(e + f x))^m (c + d \sin(e + f x))^n \, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

3.347.1 Optimal result . . . 2869
3.347.2 Mathematica [F] . . . 2870
3.347.3 Rubi [A] (verified) . . . 2870
3.347.4 Maple [F] . . . 2874
3.347.5 Fricas [F] . . . 2874
3.347.6 Sympy [F(-1)] . . . 2875
3.347.7 Maxima [F] . . . 2875
3.347.8 Giac [F] . . . 2875
3.347.9 Mupad [F(-1)] . . . 2876

3.347.1 Optimal result

Integrand size = 39, antiderivative size = 277

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx))^{-1+m}}{(c+d)f}$$

$$+\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{3}{2}+m,\frac{1}{2},1+m,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+$$

$$a(c-d)f(3+2m)\sqrt{1-\sin(e+fx)}$$

output

```
-2^(1/2+m)*a*(A-B)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-si
n(f*x+e))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/
(c+d*sin(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)+B*AppellF1(3/2+m,1+
m,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*si
n(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)/a/(c-d)/f/(3+2*m)/((c+d
*sin(f*x+e))^m)/(1-sin(f*x+e))^(1/2)
```

3.347.2 Mathematica [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m}\,dx$$
$$= \int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(-1 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(-1 - m), x]
```

3.347.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3466, 3042, 3267, 142, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-m-1}\,dx$$
$$\downarrow 3042$$
$$\int (a\sin(e+fx)+a)^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-m-1} dx$$
$$\downarrow 3466$$
$$(A-B)\int (\sin(e+fx)a+a)^m (c + d\sin(e+fx))^{-m-1} dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}(c + d\sin(e+fx))^{-m-1} dx}{a}$$
$$\downarrow 3042$$
$$(A-B)\int (\sin(e+fx)a+a)^m (c + d\sin(e+fx))^{-m-1} dx + \frac{B\int (\sin(e+fx)a+a)^{m+1}(c + d\sin(e+fx))^{-m-1} dx}{a}$$
$$\downarrow 3267$$

$$\frac{a^2(A-B)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+$$
$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow 142$$

$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometric}}{f(c+d)}$$

$$\downarrow 157$$

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometric}}{f(c+d)}$$

$$\downarrow 27$$

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometric}}{f(c+d)}$$

$$\downarrow 156$$

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{m}\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{-m-1}}{\sqrt{1-\sin(e+fx)}}d$$
$$f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometric}}{f(c+d)}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{m}\text{AppellF1}\left(m+\right.}{f(2m+3)(c-d)(a-a\sin(e+fx))}$$

$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometric}}{f(c+d)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
-((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 ((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x
])^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))
/((c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1
 + m, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Co
s[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin
[e + f*x])/(c - d))^m)/((c - d)*f*(3 + 2*m)*(a - a*Sin[e + f*x])*(c + d*Si
n[e + f*x])^m)
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

3.347.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c + d\sin(fx + e))^{-1-m}\, dx$$

input

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

output

```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

3.347.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{-1-m}\, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^{-m-1}\, dx$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output

```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{-1-m}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
```

output
```
Timed out
```

3.347.7 Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

3.347.8 Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))^{-1-m}\, dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^m}{(c + d\,\sin(e + f\,x))^{m+1}}\, dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m
+ 1), x)
```

3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

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3.348.1 Optimal result

Integrand size = 36, antiderivative size = 132

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$= \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},-n,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}}{f(1+2m)}$$

output
```
2*AppellF1(1/2+m,-n,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))
*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)*(1-sin(f*x+e
))^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)
```

3.348.2 Mathematica [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$= \int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^n, x]
```

3.348.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3487, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n dx$$

$$\downarrow 3487$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx)$$
$$f$$

$$\downarrow 157$$

$$\frac{\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{2}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 27$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 156$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int \sqrt{1-\sin(e+fx)}(\sin($$
$$f\sqrt{1-\sin(e+fx)}$$

$$\downarrow 155$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(m+\right.}{af(2m+1)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

output
```
(2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d
*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + 2*m)*Sqrt[1 - Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c - d))^n)
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.348.4 Maple [F]

$$\int (a - a\sin(fx+e))\,(a + a\sin(fx+e))^m\,(c + d\sin(fx+e))^n\,dx$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

3.348.5 Fricas [F]

$$\int (a - a\sin(e+fx))(a + a\sin(e+fx))^m(c + d\sin(e+fx))^n\,dx$$
$$= \int -(a\sin(fx+e) - a)(a\sin(fx+e) + a)^m(d\sin(fx+e) + c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^n \, dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

3.348.7 Maxima [F]

$$\begin{aligned}&\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^n \, dx \\ &= \int -(a\sin(fx + e) - a)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx\end{aligned}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^n, x)
```

3.348.8 Giac [F]

$$\begin{aligned}&\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^n \, dx \\ &= \int -(a\sin(fx + e) - a)(a\sin(fx + e) + a)^m (d\sin(fx + e) + c)^n \, dx\end{aligned}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^n, x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$= \int (a + a\,\sin(e+f\,x))^m\,(a - a\,\sin(e+f\,x))\,\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

output
```
int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.349 $\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$

3.349.1 Optimal result . . . 2883
3.349.2 Mathematica [F] . . . 2883
3.349.3 Rubi [A] (verified) . . . 2884
3.349.4 Maple [F] . . . 2886
3.349.5 Fricas [F] . . . 2886
3.349.6 Sympy [F(-1)] . . . 2887
3.349.7 Maxima [F] . . . 2887
3.349.8 Giac [F] . . . 2887
3.349.9 Mupad [F(-1)] . . . 2888

3.349.1 Optimal result

Integrand size = 40, antiderivative size = 139

$$\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,-\frac{1}{2},1+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)\sqrt{1-\sin(e+f}}{(c-d)f(1+2m)}$$

output
```
2*AppellF1(1/2+m,1+m,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e)
)*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)*(1-
sin(f*x+e))^(1/2)/(c-d)/f/(1+2*m)/((c+d*sin(f*x+e))^m)
```

3.349.2 Mathematica [F]

$$\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \int (a - a\sin(e+fx))(a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$$

input
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^(-1 - m),x]
```

output
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^(-1 - m), x]
```

3.349.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3487, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m (c+d\sin(e+fx))^{-m-1}\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))(a\sin(e+fx)+a)^m (c+d\sin(e+fx))^{-m-1}dx$$

$$\downarrow 3487$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx)}{f}$$

$$\downarrow 157$$

$$\frac{\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{2}}d\sin(e+f}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 27$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{-}}{f\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 156$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\int \sqrt{1-\sin(e+fx)}(\sin(}{f(c-d)\sqrt{1-\sin(e+fx)}}$$

$$\downarrow 155$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{m}\text{AppellF1}\left(m+\right.}{af(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
(2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -
((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c - d)*f*(1 +
 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.349.4 Maple [F]

$$\int (a - a\sin(fx+e))\,(a + a\sin(fx+e))^m\,(c + d\sin(fx+e))^{-1-m}\,dx$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

output
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

3.349.5 Fricas [F]

$$\int (a - a\sin(e+fx))(a + a\sin(e+fx))^m(c + d\sin(e+fx))^{-1-m}\,dx$$
$$= \int -(a\sin(fx+e) - a)(a\sin(fx+e) + a)^m(d\sin(fx+e) + c)^{-m-1}\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

output
```
Timed out
```

3.349.7 Maxima [F]

$$\begin{aligned}&\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx \\ &= \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx\end{aligned}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^(-m - 1), x)
```

3.349.8 Giac [F]

$$\begin{aligned}&\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx \\ &= \int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx\end{aligned}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^(-m - 1), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int (a - a\sin(e+fx))(a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \int \frac{(a+a\sin(e+fx))^m (a - a\sin(e+fx))}{(c+d\sin(e+fx))^{m+1}}\,dx$$

input
```
int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m
+ 1), x)
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

3.350.1 Optimal result 2889
3.350.2 Mathematica [A] (verified) 2889
3.350.3 Rubi [A] (verified) 2890
3.350.4 Maple [F] 2891
3.350.5 Fricas [A] (verification not implemented) 2891
3.350.6 Sympy [F(-1)] 2891
3.350.7 Maxima [F] 2892
3.350.8 Giac [F(-1)] 2892
3.350.9 Mupad [B] (verification not implemented) 2892

3.350.1 Optimal result

Integrand size = 55, antiderivative size = 39

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

output
```
-cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

3.350.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d
)*m + (c + (c - d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

3.350.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-2}((m(c-d)+c)\sin(e+fx)-m(c-d)+d)\,dx$$

$$\downarrow 3042$$

$$\int (a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-2}((m(c-d)+c)\sin(e+fx)-m(c-d)+d)dx$$

$$\downarrow 3453$$

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m +
(c + (c - d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

3.350.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

3.350.4 Maple [F]

$$\int (a+a\sin(fx+e))^m (c+d\sin(fx+e))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(fx+e))\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(
f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(
f*x+e)),x)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))(a\sin(fx+e)+a)^m (d\sin(fx+e)+c)^{-m-2}}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)
*m)*sin(f*x+e)),x)
```

output
```
Timed out
```

3.350.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$$
$$=\int -((c-d)m-((c-d)m+c)\sin(fx+e)-d)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(((c - d)*m - ((c - d)*m + c)*sin(f*x + e) - d)*(a*sin(f*x + e)
+ a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

3.350.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d$$
$$-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.350.9 Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx=$$
$$-\frac{(a(\sin(e+fx)+1))^m\left(d\sin(2e+2fx)-2c\left(2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)\right)}{f(c+d\sin(e+fx))^m\left(d^2\left(2\sin(e+fx)^2-1\right)+2c^2+d^2+4cd\sin(e+fx)\right)}$$

input
```
int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d)))
)/(c + d*sin(e + f*x))^(m + 2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)
^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 +
d^2 + 4*c*d*sin(e + f*x)))
```

3.350.
$\int (a + a\sin(e + fx))^m (c + d\sin(e + fx))^{-2-m} (d - (c - d)m + (c + (c - d)m)\sin(e + fx))\, dx$

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

3.351.1 Optimal result . . . 2894
3.351.2 Mathematica [A] (verified) . . . 2894
3.351.3 Rubi [A] (verified) . . . 2895
3.351.4 Maple [F] . . . 2896
3.351.5 Fricas [A] (verification not implemented) . . . 2896
3.351.6 Sympy [F(-1)] . . . 2896
3.351.7 Maxima [F] . . . 2897
3.351.8 Giac [F(-1)] . . . 2897
3.351.9 Mupad [B] (verification not implemented) . . . 2897

3.351.1 Optimal result

Integrand size = 51, antiderivative size = 40

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

output
```
-cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

3.351.2 Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d
)*m + (c + (c + d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

3.351.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-m-2}((m(c+d)+c)\sin(e+fx) + m(c+d) + d)\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-m-2}((m(c+d)+c)\sin(e+fx) + m(c+d) + d)dx$$

$$\downarrow 3453$$

$$-\frac{\cos(e+fx)(a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-m-1}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m +
(c + (c + d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

3.351.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

3.351.4 Maple [F]

$$\int (a - a\sin(fx+e))^m (c + d\sin(fx+e))^{-2-m} (d + (c+d)m + (c + (c+d)m)\sin(fx+e))\,dx$$

input
```
int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(
f*x+e)),x)
```

output
```
int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(
f*x+e)),x)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-2-m} (d + (c+d)m + (c + (c+d)m)\sin(e+fx))\,dx = -\frac{(d\cos(fx+e)\sin(fx+e) + c\cos(fx+e))(-a\sin(fx+e)+a)^m (d\sin(fx+e)+c)^{-m-2}}{f}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d
*sin(f*x + e) + c)^(-m - 2)/f
```

3.351.6 Sympy [F(-1)]

Timed out.

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-2-m} (d + (c+d)m + (c + (c+d)m)\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)
*m)*sin(f*x+e)),x)
```

output
```
Timed out
```

3.351.7 Maxima [F]

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-2-m}(d + (c+d)m + (c + (c+d)m)\sin(e+fx))\,dx$$
$$= \int ((c+d)m + ((c+d)m + c)\sin(fx+e) + d)(-a\sin(fx+e) + a)^m (d\sin(fx+e) + c)^{-m-2}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((c + d)*m + ((c + d)*m + c)*sin(f*x + e) + d)*(-a*sin(f*x + e)
+ a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

3.351.8 Giac [F(-1)]

Timed out.

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-2-m}(d + (c+d)m + (c + (c+d)m)\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.351.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int (a - a\sin(e+fx))^m (c + d\sin(e+fx))^{-2-m}(d + (c+d)m + (c + (c+d)m)\sin(e+fx))\,dx =$$
$$-\frac{(-a(\sin(e+fx)-1))^m \left(d\sin(2e+2fx) - 2c\left(2\sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1\right)\right)}{f(c + d\sin(e+fx))^m \left(d^2\left(2\sin(e+fx)^2 - 1\right) + 2c^2 + d^2 + 4cd\sin(e+fx)\right)}$$

input
```
int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d))
)/(c + d*sin(e + f*x))^(m + 2),x)
```

output
```
-((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2
)^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 +
 d^2 + 4*c*d*sin(e + f*x)))
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

3.352.1 Optimal result . . . 2899
3.352.2 Mathematica [A] (verified) . . . 2899
3.352.3 Rubi [A] (verified) . . . 2900
3.352.4 Maple [A] (verified) . . . 2904
3.352.5 Fricas [B] (verification not implemented) . . . 2904
3.352.6 Sympy [F(-1)] . . . 2905
3.352.7 Maxima [F(-2)] . . . 2906
3.352.8 Giac [B] (verification not implemented) . . . 2906
3.352.9 Mupad [B] (verification not implemented) . . . 2907

3.352.1 Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(2Bc^3-Ac^2d-3Bcd^2+2Ad^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

output

```
-b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*
d+2*A*d^3+2*B*c^3-3*B*c*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1
/2))/d^3/(c^2-d^2)^(3/2)/f-b^2*B*cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*
cos(f*x+e)/d^2/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

3.352.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{b(-2bBc+Abd+2aBd)(e+fx)+\frac{2(bc-ad)\left(ad^2(-Ac+Bd)+b(2Bc^3-Ac^2d-3Bcd^2+2Ad^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}-b}{d^3f}$$

input
```
Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c)
+ B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (
d*(b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e
 + f*x])))/(d^3*f)
```

3.352.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3467, 25, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\downarrow 3467$$

$$\frac{\int -\frac{-b^2Bd(c^2-d^2)\sin^2(e+fx)+b(bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)+d(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))}{c+d\sin(e+fx)}dx}{d^2\left(c^2-d^2\right)} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 25$$

$$-\frac{\int \frac{-b^2Bd(c^2-d^2)\sin^2(e+fx)+b(bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)+d(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))}{c+d\sin(e+fx)}dx}{d^2\left(c^2-d^2\right)} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{-b^2Bd\left(c^2-d^2\right)\sin(e+fx)^2+b(bBc-Abd-2aBd)\left(c^2-d^2\right)\sin(e+fx)+d\left(B(bc-ad)^2-Ad\left(ca^2-2bda+b^2c\right)\right)}{c+d\sin(e+fx)}dx}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3502$$

$$-\frac{\frac{\int \frac{\left(B(bc-ad)^2-Ad\left(ca^2-2bda+b^2c\right)\right)d^2+b(2bBc-Abd-2aBd)\left(c^2-d^2\right)\sin(e+fx)d}{c+d\sin(e+fx)}dx}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\frac{\int \frac{\left(B(bc-ad)^2-Ad\left(ca^2-2bda+b^2c\right)\right)d^2+b(2bBc-Abd-2aBd)\left(c^2-d^2\right)\sin(e+fx)d}{c+d\sin(e+fx)}dx}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3214$$

$$-\frac{\frac{(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3042$$

$$-\frac{\frac{(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 3139$$

$$-\frac{\frac{\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 1083$$

3.352. $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$

$$-\frac{\frac{bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)-\frac{4(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+\right.\right.\right.}{f}}{d}}{d^2\left(c^2-d^2\right)}$$

$$\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$$\downarrow 217$$

$$-\frac{\frac{\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}}+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+f}{f}}{d^2\left(c^2-d^2\right)}$$

$$\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

input
```
Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-(((b*(2*b*B*c - A*b*d - 2*a*B*d)*(c^2 - d^2)*x + (2*(b*c - a*d)*(a*d^2*(A
*c - B*d) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(2*d + 2*c
*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(Sqrt[c^2 - d^2]*f))/d + (b^2*B*(
c^2 - d^2)*Cos[e + f*x])/f)/(d^2*(c^2 - d^2))) - ((b*c - a*d)^2*(B*c - A*d
)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

3.352.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.352.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{2b\left(-\frac{Bbd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Abd+2dBa-2Bbc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{d^2\left(A\,a^2d^3-2Aabc\,d^2+A\,b^2c^2d-B\,a^2d^2c+2Bab\,c^2d-c^3\right.}{\left(c^2-d^2\right)c}\right.}{\left(\tan^2\right.}$
default	$\frac{2b\left(-\frac{Bbd}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}+(Abd+2dBa-2Bbc)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{d^3}+\frac{2\left(\frac{d^2\left(A\,a^2d^3-2Aabc\,d^2+A\,b^2c^2d-B\,a^2d^2c+2Bab\,c^2d-c^3\right.}{\left(c^2-d^2\right)c}\right.}{\left(\tan^2\right.}$
risch	Expression too large to display

input
```
int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/f*(2*b/d^3*(-B*b*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*b*d+2*B*a*d-2*B*b*c)*arct
an(tan(1/2*f*x+1/2*e)))+2/d^3*((d^2*(A*a^2*d^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B
*a^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+d*(A*a^
2*d^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-
d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(A*a^2*c*d^3-2*A*a
*b*d^4-A*b^2*c^3*d+2*A*b^2*c*d^3-B*a^2*d^4-2*B*a*b*c^3*d+4*B*a*b*c*d^3+2*B
*b^2*c^4-3*B*b^2*c^2*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))))
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(194) = 388.

Time = 0.36 (sec) , antiderivative size = 1308, normalized size of antiderivative = 6.57

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b
^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (
2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b
 + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c
^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (
B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x
+ e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f
*x + e) - c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c
^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c
^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d
 - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*
b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2
*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 +
d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4
*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b +
A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3
*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*
a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2
)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*...
```

3.352.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

3.352.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.352.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(194) = 388.

Time = 0.50 (sec) , antiderivative size = 749, normalized size of antiderivative = 3.76

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{2\left(2Bb^2c^4-2Babc^3d-Ab^2c^3d-3Bb^2c^2d^2+Aa^2cd^3+4Babcd^3+2Ab^2cd^3-Ba^2d^4-2Aabd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right.}{(c^2d^3-d^5)\sqrt{c^2-d^2}}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
(2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*
d^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1
/2*f*x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan
(1/2*f*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1
/2*f*x + 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan
(1/2*f*x + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d
^2*tan(1/2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c
*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b
*c^2*d^2*tan(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2
*c*d^3*tan(1/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2
*c*d^3*tan(1/2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4
 - 2*B*a*b*c^3*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c
^2*d^2 - A*a^2*c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*t
an(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2
*e) + c)) - (2*B*b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

3.352.9 Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 16312, normalized size of antiderivative = 81.97

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x
)
```

output

```
((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2
*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2
*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*
a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A
*a^2*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b
*c^2*d))/(c*d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^
3 + A*b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^
2*d))/(c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f
*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (atan((((
b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d
^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c
^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6
*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a
*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^
2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4
*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10
 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^1
4 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b
*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^
9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B...
```

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

3.353.1 Optimal result 2909
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3.353.1 Optimal result

Integrand size = 39, antiderivative size = 840

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\sqrt{c+d}(2Ab^2c-2abBc-2aAbd+3a^2Bd-b^2Bd}{}$$
$$+\frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(b}{(}}}{b^3\sqrt{a+b}f}$$
$$+\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$
$$-\frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$
$$+\frac{\sqrt{a+b}(2Ab(b(c-2d)+ad)-B(3a^2d-6abd+b^2(2c+d)))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(}{(a-b)(}\right)}{(a-b)b^3\sqrt{c+d}f}$$

output
```
(c-d)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*EllipticE((a+b)^(
1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d
)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)
*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(
c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+(2*A*b*d-3
*B*a*d+3*B*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*se
c(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a
+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(
1/2)/b^3/f/(a+b)^(1/2)+(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c+
d)))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x
+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a
+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)
+2*(A*b-B*a)*(-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a
+b*sin(f*x+e))^(1/2)-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2*a*b*c+b^2*d))*cos(f*x
+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)
```

3.353.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2042 vs. $2(840) = 1680$.

Time = 8.06 (sec) , antiderivative size = 2042, normalized size of antiderivative = 2.43

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^{3/2}}{(a + b\sin(e + fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
 f*x])^(3/2),x]
```

output

```
(-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] +
a^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a +
 b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b
^2*c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/
((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-
(b*c) + a*d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*
b^2*d^2 + 2*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d
)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]
*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt
[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)...
```

3.353.3 Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3468, 27, 3042, 3540, 25, 3042, 3532, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3468$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$2\int \frac{c(Bc+2Ad)b^2-a(2Bcd+A(c^2+d^2))b+a^2Bd^2-d(2Ab(bc-ad)-B(-3da^2+2bca+b^2d))\sin^2(e+fx)-(A(c^2-d^2)b^2+B(2cda^2-b(c^2-d^2)a-2b^2$$
$$\frac{}{2\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 27$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$\int \frac{c(Bc+2Ad)b^2-a(2Bcd+A(c^2+d^2))b+a^2Bd^2-d(2Ab(bc-ad)-B(-3da^2+2bca+b^2d))\sin^2(e+fx)-(A(c^2-d^2)b^2+B(2cda^2-b(c^2-d^2)a-2b^2$$
$$\frac{}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 3042$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$\int \frac{c(Bc+2Ad)b^2-a(2Bcd+A(c^2+d^2))b+a^2Bd^2-d(2Ab(bc-ad)-B(-3da^2+2bca+b^2d))\sin(e+fx)^2-(A(c^2-d^2)b^2+B(2cda^2-b(c^2-d^2)a-2b^2$$
$$\frac{}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 3540$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$\frac{\int -\frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin^2(e+fx)+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin(e+fx)+\left(a^2-b^2\right)d\left(2Abc^2-Bd(bc-ad)\right)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{2d} + \frac{\cos(e+fx)(2Ab(bc-ad)-}{f}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 25$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$\frac{\cos(e+fx)(2Ab(bc-ad)-B(-3a^2d+2abc+b^2d))\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{\int \frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin^2(e+fx)+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin(e}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{2d}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 3042$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$
$$\frac{\cos(e+fx)(2Ab(bc-ad)-B(-3a^2d+2abc+b^2d))\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{\int \frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin(e+fx)^2+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin(e}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{2d}$$
$$b\left(a^2-b^2\right)$$

$$\downarrow 3532$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}+\int -\frac{\left(a^2-b^2\right)d(bc-ad)\left(\right.}{2d}}{b\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}-\int \frac{\left(a^2-b^2\right)d(bc-ad)\left(\right.}{2d}}{b\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}-\int \frac{\left(a^2-b^2\right)d(bc-ad)\left(\right.}{2d}}{b\left(a^2-b^2\right)}$$

$$\downarrow 3290$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-s}{(c+d)(a+b\,si}}}{}$$

$$\downarrow 3477$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-s}{(c+d)(a+b\,si}}}{}$$

$$\downarrow 3042$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-s}{(c+d)(a+b\,si}}$$

$$\downarrow 3297$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-s}{(c+d)(a+b\,si}}$$

$$\downarrow 3475$$

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} -$$

$$\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} - \frac{2\left(a^2-b^2\right)d\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e-}}{\sqrt{c+d}\sqrt{a+b\sin(e-}}\right.\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])
^(3/2),x]
```

output
```
(2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2
- b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (((2*A*b*(b*c - a*d) - B*(2*a*b*c - 3
*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[
e + f*x]]) - ((2*(a^2 - b^2)*d*Sqrt[c + d]*(3*b*B*c + 2*A*b*d - 3*a*B*d)*E
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)
*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
 b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) - ((-2*b*Sqrt
[a + b]*(c - d)*d*Sqrt[c + d]*(2*A*b*(b*c - a*d) - B*(2*a*b*c - 3*a^2*d +
b^2*d))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x
])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))
]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) - (2*(a + b)^(3/2)*d*(2*A*b*(b*(c
- 2*d) + a*d) - B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(S
qrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]]
)], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1
 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/...
```

3.353.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

3.353. $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.353.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 114.64 (sec) , antiderivative size = 1442707, normalized size of antiderivative = 1717.51

method	result	size
parts	Expression too large to display	1442707
default	Expression too large to display	1467567

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.353.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
Timed out
```

3.353.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^{\frac{3}{2}}}{(a+b\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f
*x))**(3/2), x)
```

3.353.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e)
+ a)^(3/2), x)
```

3.353.8 Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e)
+ a)^(3/2), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^{3/2}}{(a+b\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))
^(3/2), x)
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

3.354.1 Optimal result 2920
3.354.2 Mathematica [B] (warning: unable to verify) 2921
3.354.3 Rubi [A] (verified) 2921
3.354.4 Maple [B] (warning: unable to verify) 2925
3.354.5 Fricas [F] 2925
3.354.6 Sympy [F] 2926
3.354.7 Maxima [F] 2926
3.354.8 Giac [F] 2926
3.354.9 Mupad [F(-1)] 2927

3.354.1 Optimal result

Integrand size = 39, antiderivative size = 630

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(c}{(c}\right.$$

$$+\frac{2\sqrt{a+b}(Ab-aB)(c-d)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx}{(a+b)(c+d\sin(e+fx}}}{(a-b)b\sqrt{c+d}(bc-ad)f}$$

$$+\frac{2\sqrt{a+b}B\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(b}{(a}}$$

$$b^2\sqrt{c+d}f$$

output

```
2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)
/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))
^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/(-
a*d+b*c)/f/(a+b)^(1/2)+2*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*
x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(
1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(
a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*
x+e)))^(1/2)/(a-b)/b/(-a*d+b*c)/f/(c+d)^(1/2)+2*B*EllipticPi((c+d)^(1/2)*(
a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin
(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)
```

3.354.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1901 vs. $2(630) = 1260$.

Time = 15.54 (sec) , antiderivative size = 1901, normalized size of antiderivative = 3.02

$$\int \frac{(A + B\sin(e+fx))\sqrt{c + d\sin(e+fx)}}{(a + b\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f
*x])^(3/2),x]
```

output
```
(-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)
*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
+ d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +...
```

3.354.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 3471, 3042, 3274, 3042, 3290, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3471$$

$$\frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$$\downarrow 3042$$

$$\frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$$\downarrow 3274$$

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$$\downarrow 3042$$

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$$\downarrow 3290$$

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{b^2f\sqrt{c+d}}$$

$$\downarrow 3297$$

$$(Ab-aB)\left(\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

$$\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{b^2f\sqrt{c+d}}$$

$$\downarrow 3475$$

$$(Ab-aB)\left(\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right.\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

$$\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{b^2f\sqrt{c+d}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(
3/2),x]
```

output
```
(2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*S
qrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f
*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d
]*f) + ((A*b - a*B)*((2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b
)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f
*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[
a + b]*(b*c - a*d)*f) + (2*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
+ b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sq
rt[c + d]*(b*c - a*d)*f)))/b
```

3.354.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3471
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> S
imp[B/b   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + S
imp[(A*b - a*B)/b   Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2)
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

3.354.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 51.58 (sec) , antiderivative size = 637252, normalized size of antiderivative = 1011.51

method	result	size
parts	Expression too large to display	637252
default	Expression too large to display	646989

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.354.5 Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

3.354.6 Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x)
)**(3/2), x)
```

3.354.7 Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) +
a)^(3/2), x)
```

3.354.8 Giac [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) +
a)^(3/2), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c+d\,\sin(e+f\,x)}}{(a+b\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))
^(3/2), x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

3.355.1 Optimal result . . . 2928
3.355.2 Mathematica [B] (warning: unable to verify) . . . 2929
3.355.3 Rubi [A] (verified) . . . 2929
3.355.4 Maple [B] (warning: unable to verify) . . . 2932
3.355.5 Fricas [F] . . . 2932
3.355.6 Sympy [F] . . . 2933
3.355.7 Maxima [F] . . . 2933
3.355.8 Giac [F] . . . 2933
3.355.9 Mupad [F(-1)] . . . 2934

3.355.1 Optimal result

Integrand size = 39, antiderivative size = 417

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\right|$$
$$+\frac{2\sqrt{a+b}(A-B)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}{(a-b)\sqrt{c+d}(bc-ad)f}$$

output

```
2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)
/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))
^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*
d+b*c)^2/f/(a+b)^(1/2)+2*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2
)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(
f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*
sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/
2)/(a-b)/(-a*d+b*c)/f/(c+d)^(1/2)
```

3.355.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1949 vs. 2(417) = 834.

Time = 6.76 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.67

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[
e + f*x]]),x]
```

output
```
(-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((
a^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d
)*(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(
b*c) + a*d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[...
```

3.355.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx$$

$$\downarrow 3477$$

$$\frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b} - \frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\downarrow 3042$$

$$\frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b} - \frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\downarrow 3297$$

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c-}}{\sqrt{a-}}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

$$\frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\downarrow 3475$$

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c-}}{\sqrt{a-}}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

$$\frac{2(c-d)\sqrt{c+d}(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{f(a-b)\sqrt{a+b}(bc-ad)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*
x]]),x]
```

output
```
(2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c +
d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d)
)/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/
((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b
*c - a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]
))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))
/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c +
d]*(b*c - a*d)*f)
```

3.355.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

3.355.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 87093 vs. $2(387) = 774$.

Time = 14.80 (sec) , antiderivative size = 87094, normalized size of antiderivative = 208.86

method	result	size
parts	Expression too large to display	87094
default	Expression too large to display	88800

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.355.5 Fricas [F]

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d
*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

3.355.6 Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(
e + f*x))), x)
```

3.355.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x
+ e) + c)), x)
```

3.355.8 Giac [F]

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x
+ e) + c)), x)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+b\,\sin(e+f\,x))^{3/2}\,\sqrt{c+d\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(1/2)), x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

3.356.1 Optimal result 2935
3.356.2 Mathematica [B] (warning: unable to verify) 2936
3.356.3 Rubi [A] (verified) 2936
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3.356.9 Mupad [F(-1)] 2941

3.356.1 Optimal result

Integrand size = 39, antiderivative size = 544

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+$$

$$-\frac{2(A(a^2d^2+b^2(c^2-2d^2))-B(a^2cd-b^2cd+ab(c^2-d^2)))\,E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3f}$$

$$+\frac{2(Abc+bBc-aAd-2Abd+aBd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad}{(a+b)(}}$$

output

```
-2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*(c^2-d^2)))*Ellipti
cE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1
-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-
b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*
(-A*a*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^
(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+
e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)
/(-a*d+b*c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)
/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

3.356.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2266 vs. $2(544) = 1088$.

Time = 7.24 (sec) , antiderivative size = 2266, normalized size of antiderivative = 4.17

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^(3/2)),x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
 - a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x
])) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2
- d^2)*(c + d*Sin[e + f*x])))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*
B*c^3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 +
b^3*B*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[
((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2
*d - 2*a^2*b*B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B
*c*d^2 + 2*a*b^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqr
t[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e...
```

3.356.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 3479, 27, 3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}dx$$

$$\downarrow 3479$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 27$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3042$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3477$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int \frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d} - \frac{(a-b)(-aAd+aBd+Abc-2Abd+bBc)\int \frac{}{\sqrt{a-}}}{c-d}}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3042$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int \frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d} - \frac{(a-b)(-aAd+aBd+Abc-2Abd+bBc)\int \frac{}{\sqrt{a-}}}{c-d}}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3297$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} -$$
$$\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int\frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d} - \frac{2(a-b)\sqrt{a+b}\sec(e+fx)(-aAd+aBd+Abc-$$
$$\left(a^2-b^2\right)(bc-$$

$$\downarrow 3475$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} -$$
$$-\frac{2(a-b)\sqrt{a+b}\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx}{(a-b)(c+d\sin(e+}}$$
$$f(c-d)\sqrt{c+d}(bc-ad)^2$$

input
```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
(3/2)),x]
```

output
```
(2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((-2*(a - b)*Sqrt[a + b]*(a^2*B*c*d -
 b^2*B*c*d - a^2*A*d^2 - A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Si
n[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*
c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) - (2*(a - b)*Sqrt[a + b]*(A*
b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/
((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d
]*(b*c - a*d)*f))/((a^2 - b^2)*(b*c - a*d))
```

3.356.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

3.356.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 174461 vs. 2(510) = 1020.

Time = 18.71 (sec) , antiderivative size = 174462, normalized size of antiderivative = 320.70

method	result	size
parts	Expression too large to display	174462
default	Expression too large to display	177559

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.356.5 Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*
d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^
2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*
x + e)), x)
```

3.356.6 Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{\frac{3}{2}}(c+d\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e +
f*x))**(3/2)), x)
```

3.356.7 Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(3/2)), x)
```

3.356.8 Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(3/2)), x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+b\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(3/2)), x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

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3.357.1 Optimal result

Integrand size = 39, antiderivative size = 858

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+f$$

$$+\frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(a^2-b^2)(bc-ad)^2(c^2-d^2)f(c+d\sin(e+fx))^{3/2}}$$

$$+\frac{2(B(2a^2bcd(3c^2-d^2)-2b^3cd(3c^2-d^2)-a^3d^2(c^2+3d^2)+ab^2(3c^4-5c^2d^2+6d^4))+A(4a^3cd^3-4ab^2cd$$

$$-\frac{2(B(a^2d^2(c+3d)-b^2c(3c^2+3cd-2d^2)-6abd(c^2-d^2))-A(a^2d^2(3c+d)-6abd(c^2-d^2)+b^2(3c^3-$$

output
```
2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+
b*sin(f*x+e))^(1/2)+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*
d+3*a*b*(c^2-d^2)))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)
^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^
3*c*d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*
a^3*c*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^
4)))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x
+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin
(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/
(a+b)^(1/2)-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2
-d^2))-A*(a^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d
^3)))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(
(-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+si
n(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f
/(a+b)^(1/2)
```

3.357.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2837 vs. $2(858) = 1716$.

Time = 8.07 (sec) , antiderivative size = 2837, normalized size of antiderivative = 3.31

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^(5/2)),x]
```

output

```
(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x
] - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*
x])) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2
*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*
A*b*c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e +
f*x] - 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e
 + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*
(-(b*c) + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4
*c^4*d - 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^
4*B*c^3*d^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 +
10*a^3*b*B*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^
4 - 4*a^4*B*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*
A*b^2*d^5 - 8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d...
```

3.357.3 Rubi [A] (verified)

Time = 3.34 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}dx$$

$$\downarrow 3479$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin^2(e+fx)+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 27$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin^2(e+fx)+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3042$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin(e+fx)^2+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$$\downarrow 3534$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2\int -\frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(Ac-Bd)\left(c^2-2d^2\right)a-b^3\left(3Bc^3-9Adc^2-2Bd^2c+8Ad^3\right)+\left(B\left(cd^2a^3-3bd\left(2c^2-d^2\right)a^2-b^2c\left(3c^2-2d^2\right)a+3b^3\right.\right.}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}}{3(c^2-d^2)(bc-ad)}}{(a$$

$$\downarrow 27$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f(c^2-d^2)(bc-ad)(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(Ac-}{(a^2}$$

$$\downarrow 3042$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f(c^2-d^2)(bc-ad)(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(Ac-}{(a^2}$$

$$\downarrow 3477$$

3.357. $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} -$$

$$\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}} - \frac{-}{}$$

$$\downarrow 3042$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} -$$

$$\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}} - \frac{-}{}$$

$$\downarrow 3297$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} -$$

$$\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}} - \frac{-}{}$$

$$\downarrow 3475$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} -$$

$$\frac{2d\left(-Ad^2a^2+Bcda^2+3bB\left(c^2-d^2\right)a-b^2Bcd-Ab^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2(a-b)\sqrt{a+b}\left(B\left(-d^2\left(c^2+3d^2\right)a^3+2bcd\left(3c^2-d^2\right)a^2+b^2\right.\right.}{}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
(5/2)),x]
```

3.357. $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

output
```
(2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - ((2*d*(a^2*B*c*d - b^2*B*c*d - a^2*
A*d^2 - A*b^2*(3*c^2 - 4*d^2) + 3*a*b*B*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - ((2*(a - b)*Sqrt[a + b]*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^
2 - d^2) - a^3*d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*
(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15
*c^2*d^2 + 8*d^4)))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
 - (2*(a - b)*Sqrt[a + b]*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*
d^2) - 6*a*b*d*(c^2 - d^2)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) +
 b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*S
qrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f
*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c
 + d]*(b*c - a*d)*f))/(3*(b*c - a*d)*(c^2 - d^2)))/((a^2 - b^2)*(b*c - ...
```

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.357.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 33.72 (sec) , antiderivative size = 726985, normalized size of antiderivative = 847.30

method	result	size
parts	Expression too large to display	726985
default	Expression too large to display	742054

input

```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output

```
result too large to display
```

3.357.5 Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4
+ (a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d
^3 + 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a
*b*c^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*
d + 6*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

3.357.7 Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(5/2)), x)
```

3.357.8 Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(5/2)), x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+b\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(5/2)), x)
```

3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

3.358.1 Optimal result . . . 2953
3.358.2 Mathematica [N/A] . . . 2953
3.358.3 Rubi [N/A] . . . 2954
3.358.4 Maple [N/A] (verified) . . . 2955
3.358.5 Fricas [N/A] . . . 2955
3.358.6 Sympy [F(-2)] . . . 2955
3.358.7 Maxima [N/A] . . . 2956
3.358.8 Giac [N/A] . . . 2956
3.358.9 Mupad [N/A] . . . 2957

3.358.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \text{Int}((a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n, x)$$

output
```
Unintegrable((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.358.2 Mathematica [N/A]

Not integrable

Time = 14.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

3.358.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int (A + B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n dx$$

$$\downarrow 3486$$

$$\int (A + B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n dx$$

input
```
Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
$Aborted
```

3.358.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.358.4 Maple [N/A] (verified)

Not integrable

Time = 1.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+b\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^n \, dx$$

input
```
int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

3.358.5 Fricas [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m (d\sin(fx+e)+c)^n \, dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

3.358.6 Sympy [F(-2)]

Exception generated.

$$\int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.358.7 Maxima [N/A]

Not integrable

Time = 29.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.358.8 Giac [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.358.9 Mupad [N/A]

Not integrable

Time = 18.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,(a+b\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 2958

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```